



# Short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system

Chun-Jen Chung<sup>a</sup>, Hui-Ming Wee<sup>b,\*</sup>

<sup>a</sup> Department of Information Application, Aletheia University, Pei-Shih Li, Matou, Tainan 72147, Taiwan, ROC

<sup>b</sup> Department of Industrial Engineering, Chung Yuan Christian University, Chungli 32023, Taiwan, ROC

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## ABSTRACT

Due to global warming, environmental consciousness and shortening product life-cycles, more attentions have been paid to ecological protection and resource utilization. Green products and production process designs significantly influence the environment and resource re-usage. The relevant EU regulations, such as WEEE and EuP, have reduced negative effects by controlling the disposals and the resource re-usage. In this study, green product designs and remanufacturing efforts are investigated when we develop an integrated production inventory model with short life-cycles. A numerical example is provided to illustrate the theory. We have shown that new technology evolution, remanufacturing ratios and system's holding costs are critical factors affecting decision making in a green supply chain inventory control system.

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## 1. Introduction

In the last decade, due to environmental and ecological responsibility, enterprises are trying to reuse, remanufacture and recycle the used products to reduce the negative impact on environment, especially the manufacturers of the electrical consumer products. Therefore, the reverse manufacturing problem, which is strongly related to all stages of a product development, nowadays is a critical problem to all level of the electrical and computer industry. This paper considers and simplifies the reverse manufacturing problem from an electrical industry. Green product design and systems collaboration have become major issues faced by organizations. For greening issues, several countries at all levels are developing waste handling prohibitions, regulations, or incentive programs to encourage alternative disposition of electronic waste, and ensuring that producers or consumers of such products are more responsible for their safe disposal (Boks et al., 1998). Governments have begun implementing regulations that impose various requirements on manufacturers with respect to their end-of-life (EOL) products. Owing to some regulations and international proposals, such as European Union's proposal for a directive on Waste Electrical and Electronic Equipment (WEEE) and directives of energy using Product (EuP), an increasing number of manufacturers engage in modifying product designs and incorporate EOL product reuse

concepts into product and component design to reduce recovery and remanufacturing costs (Toffel, 2002). Such regulations seek to reduce both the volume and toxicity of waste by increasing the incentives for manufacturers to fully incorporate EOL concerns into product design (Fishbein, 2000; Toffel, 2002). Hence, green product design has become hot board of supply chain management and received increasing attention recently since proper product design can significantly influence the cost of energy usage, disassembly, component inspection and repair, remanufacturing, recycling and waste disposal.

Consequently, the goal of this paper is to develop a production inventory policy considering green product design with the new technology evolution and remanufacturing. The optimal inventory system is developed to comprehend the importance of related factors in the policy and to find the influence of costs in a green supply chain. A short life-cycle product with a stationary demand is considered.

Prior to delving into this study, a brief description of the major influencing factors must be noted for clarification.

First, life-cycle design seeks to maximize the life-cycle value of a product at the early stages of design, while minimizing cost and environmental impact. Ishii et al. (1994) introduced the concept of the life-cycle value and illustrated a prototype computer tool of Design for Product Retirement (DFPR). Their paper focused on product retirement and advanced planning for material recycling. For the issue of designing for remanufacturing or recycling, Klausne and Wolfgang (1999) outlined a concept to integrate product repair and product take-back. They showed that the replacement of a large share of conventional repairs with remanufacturing and reconditioning would result in a higher

\* Corresponding author. Tel.: +886 03 2654409.

E-mail addresses: [cjchungboy@gmail.com](mailto:cjchungboy@gmail.com) (C.-J. Chung), [weehm@cycu.edu.tw](mailto:weehm@cycu.edu.tw) (H.-M. Wee).

service level in product repair. Moreover, the regulation of Eco-Design Requirement for Energy-using Products (EuP) was announced by EU in 2005. The purpose of these regulations is to reduce the usage of energy and resource by incorporating ecological considerations along with product design from a product life-cycle perspective. The environmental regulations highlighted have significantly influenced the industries at all levels. Some researchers find that product design has critical link with environmental issues and business contracts. For example, Bovea and Vidal (2004) considered an evaluation method to identify the product value with the consideration of environmental factor. They proposed a model that allows user to add value for customer to a product, by means of the integration of the environmental, cost and customer valuation during its design process. Their model combines three methodologies: Life-Cycle Assessment (LCA), Life-Cycle Cost (LCC), and Contingent Valuation (CV) to quantify the customer's value in terms of customer's Willingness-To-Pay (WTP) for a product that incorporates certain environmental improvements. Lin et al. (2010) presented a proposed model which addresses the drivers of innovation in channel integration in supply chain management. Their results indicate that a significant relationship has been established between market orientation and supply chain performance. Their findings also confirmed that value co-creation and the value constellations as the drivers of innovation in channel integration are positively associated with supply chain performance. Hua et al. (2011) investigated the optimal product design strategy of a manufacturer in a two-stage supply chain that consists of an upstream manufacturer and a downstream retailer. It is found that the incentive contract can perfectly coordinate the distribution channel in the product design problem.

For issues in the greening process, Nahmias and Rivera (1979) have studied an EPQ variant of Schraday's model (1967) with a finite recovery rate. Koh et al. (2002) assumed an infinite production rate and finite recovery rate. They did not limit the recovery rate. White et al. (2003) presented a generalized overview of product recovery. The purpose of their paper described the recovery of computers as a step-by-step process, and framed an environmental research agenda for recovery management of computer industry. Bonney et al. (2003) examined some of the changes that are occurring in manufacturing companies and in the market. Changes include the product design process, reduction in product design time, new technology, new materials and production methods, the availability of better quality data, organization change including changes in techniques and tools used for planning and control. De Brito (2004) provided an extensive literature reviews while Bayındır et al. (2006) investigated the level of the desired recovery effort when the recovery process is not perfect. Tagaras and Zikopoulos (2008) assumed that demand is satisfied only by remanufactured items and studied a single period model with return in which inspection/sorting schemes are incorporated. Nenes et al. (2010) investigated alternative policies for a system where both demand of new products and returns of used products are stochastic. The expected cost of each policy for a real application problem is computed and the best policy is proposed.

Finally, there are numerous researches on just-in-time implementation with closer collaboration of the supplier-buyer integration as one of the keys to successful JIT implementation. Considerable researches have been done on the integrated inventory model and the JIT implementation. Banerjee (1986) derived a joint economic lot size model for a single vendor, single buyer system with the finite vendor's production rate. Hill (1999) and Kim and Ha (2003) presented a cooperative policy for multiple deliveries. The phenomenon of deterioration is prevalent and should not be neglected in the integrated model development.

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, and loss of entity or loss of marginal value of a commodity that results in decreasing usefulness from the original one (Wee, 1993). Bhunia and Maiti (1998) studied the deteriorating inventory model with shortages and time-dependent demand, and considered functional relations of the replenishment rate and the on-hand inventory. Yang and Wee (2002) developed an integrated deteriorating inventory model considering multiple buyers. Balkhi and Benkherouf (2004) presented an inventory model for deteriorating items with stock dependent and time-varying demand rates for a finite time planning horizon. Lin et al. (2006) studied a production-inventory model with continuous deterioration. The problem of the paper is to schedule multiple products to be manufactured on a single machine repetitively over an infinite planning horizon. He et al. (2010) examined the issue of the selling season between geographically dispersed markets with an insightful production-inventory model of a deteriorating items manufacturer selling goods to multiple-markets with different selling seasons. Therefore, the factor of deterioration cannot be neglected in an inventory model development.

This study considers green product design and remanufacturing with re-usage concept by the development of an integrated production inventory model with short life-cycle. This study is organized as follows: In Section 2, assumptions and notation are provided for model development. In Section 3, the study develops an integrated buyer-supplier deteriorating model considering JIT deliveries, design costs for product function and gas emission, reverse-manufacturing costs and other costs. A simple algorithm to derive an optimal solution is also provided. A numerical example is presented in Section 4. Conclusion and remarks are shown in Section 5.

## 2. Notation and assumptions

### 2.1. Notation for the forward manufacturing

#### 2.1.1. Notation for the supplier

$P$	production rate
$D$	demand rate
$B$	production lot size per cycle time
$T_1$	production time interval (year)
$T_2$	time interval after production time (year)
$\Psi_{S1}(t_1)$	inventory level during the production period
$\Psi_{S2}(t_2)$	inventory level after the production period
$I_{Sm}$	maximum inventory level during the production period
$C_S$	the supplier's setup cost per cycle time
$C_{Is}$	the supplier's unit item cost
$F_S$	the cost of less flexibility per cycle time
$C_{if}$	fixed inspection cost per year
$C_{io}$	variable inspection cost per setup per year
$U_{inv}$	the unit variable inspection cost per year
$H_S$	holding cost per unit per unit time
$TC_S$	the total cost function of the supplier

#### 2.1.2. Notation for the buyer

$I_b(t)$	the buyer's inventory level
$H_b$	holding cost per unit per unit time for the buyer
$T_b$	delivery cycle time per batch for the buyer (year)
$O$	ordering cost per production cycle for the buyer
$N$	the number of deliveries per cycle time
$q$	delivery size per delivery

$C_{bd}$	the buyer's unit deteriorating cost
$TC_B$	the total cost function of the buyer
2.2. Notation for the reverse manufacturing considering green design of resource re-usage	
$I_{r1}(t_{r1})$	inventory level during the collecting process for the reverse manufacturing
$I_{r2}(t_{r2})$	inventory level during the collected items consuming process for the reverse manufacturing
$\Phi_R(t_R)$	inventory level during the remanufacturing process for the reverse manufacturing
$h_{RC}$	holding cost per unit per unit time during the collecting and consuming process for the reverse manufacturing
$h_R$	holding cost per unit per unit time during the remanufacturing process for the reverse manufacturing
$P_{RC}$	recovery rate
$R_C$	collecting rate
$F_{CL}$	fixed cost including cleaning and disassembly cost during the collecting process
$C_{CL}$	variable cost including cleaning and disassembly cost during the collecting process
$C_0$	fixed component-life-cycle design cost rate for the function upgrading design of components.
$a_0$	fixed component-life-cycle design cost rate of the resource re-usage and green design for reducing waste and energy usage
$b_0$	variable component-life-cycle design cost rate of the resource re-usage and green design for reducing waste and energy usage.
$d_0$	variable component-life-cycle design cost rate for the function upgrading design of components.
$R_j$	reliability of the sub-function $j$
$F_m$	fixed cost during the remanufacturing process
$C_m$	variable cost during the remanufacturing process
$\delta_m$	the arrival rate of the failed returned items
$C_{VF}$	fixed convertibility cost
$V_{CV}$	variable convertibility cost
$\delta_C$	the convertibility parameter for technology evolution of the returned items
$F_{rp}$	fixed repair cost
$C_{rp}$	variable repair cost
$\lambda_1$	the arrival rate of the returned-items needing to be repaired
$S_{av}$	salvage for the unusable items after cleaning, disassembly and sorting
$L$	number of the life-cycles before the component is recycled or disposed off
$\xi$	the green handling ratio of the reusable and take-back items
$r_s$	the ratio for remanufacturing process after cleaning, disassembly and sorting
$r_m$	the ratio for the items which are to be remanufactured during the remanufacturing distribution
$C_d$	component-life-cycle design cost scale for the green design

2.3. Assumptions

From practical experience, it is assumed that  $H_b \geq H_s$ ,  $C_{bd} \geq C_{is}$  and  $P > D$ . The assumption is reasonable and will apply in most contexts. A single item with a constant rate of deterioration  $\theta$  is considered. Deterioration of the units is considered only after they have been received into the inventory.

The additional assumptions are used in the development of the production inventory model:

- (a) Production rate and demand rate are independent of the production lot size and are constant.
- (b) Shortages are not allowed for the supplier and the buyer.
- (c) The model does not consider wait-in-process and defective items.
- (d) The deteriorating items are not replaced.
- (e) The lead time is known and constant.
- (f) The players have complete information of each other.
- (g) The number of the supplier's shipment is an integer number.
- (h) A single item with a constant rate of deterioration  $\theta$  is considered.
- (i) Deterioration of the units is considered only after they have been received into the inventory.

3. Model development and analysis

3.1. Modeling

The supplier integrates with the buyer considering the product life-cycle, just-in-time (JIT) delivery contract and remanufacturing in a semi-closed supply chain (SC). Some opportunity and flexibility to change SC players' policies are lost because of the JIT contract. Owing to the growing environmental conscious, greening and re-usage have become critical issues in decade. This paper considers that the reverse manufacturing processes begins with cleaning and sorting distribution. The remanufacturing products collected from the users are proportional to the production lot size  $B$  with a ratio of  $\xi$ . After collection, sorting and disassembly procedure, ratio  $r_s$  of the collected items is salvaged and ratio  $(1 - r_s)$  of the collected items is to be remanufactured and repaired (the upper section of Fig. 1). Our study incorporates inspection cost, the transportation cost and the cost of less flexibility, and considers the green product design with function upgrade and re-usage concept. From the supplier's point of view, the supplier's production advances the first batch of the buyer's need and is continuous till the production lot satisfies the production-cycle-time demand. Our paper use supplier's Time-Weighted-Inventory (TWI) to derive the saw-tooth holding cost. From Fig. 1, the supplier's and buyer's differential equations are:

$$\begin{aligned} \frac{d\Psi_{S1}(t_1)}{dt_1} &= P - \theta \Psi_{S1}(t_1) \quad 0 \leq t_1 \leq T_1 \\ \frac{d\Psi_{S2}(t_2)}{dt_2} &= -\theta \Psi_{S2}(t_2) \quad 0 \leq t_2 \leq T_2 \\ \frac{dI_B(t)}{dt} &= -D - \theta I_B(t) \quad 0 \leq t \leq T_b \end{aligned} \tag{1a}$$

The differential equations can be solved using the various boundary conditions,  $\Psi_{S2}(T_2) = B = Nq$ ,  $\Psi_{S1}(0) = 0$  and  $I_B(T_b) = 0$  as follows:

$$\begin{aligned} \Psi_{S1}(t_1) &= \frac{P}{\theta} \{1 - \exp(-\theta t_1)\} \\ \Psi_{S2}(t_2) &= B \exp[\theta(T_2 - t_2)] \\ \text{and} \\ I_B(t) &= \frac{D}{\theta} \{ \exp[\theta(T_b - t)] - 1 \} \end{aligned} \tag{1b}$$

From (1a), the delivery batch size is

$$q = D[\exp(\theta T_b) - 1] / \theta \tag{1c}$$

The relationship of  $T_1$  and  $T_2$  is derived from the boundary conditions  $\Psi_{S1}(T_1) = \Psi_{S2}(0)$  and the following equality:

$$P\{1 - \exp(-\theta T_1)\} / \theta = B \exp[\theta T_2]$$

when  $\theta \ll 1$  and  $T_j < 1$ ,  $j = 1, 2$ , since the terms higher than  $(\theta T_1)^2$  and  $(\theta T_2)^2$  are very small,  $\exp(-\theta T_1)$  and  $\exp(-\theta T_2)$  can be replaced by  $1 - \theta T_1 + \frac{1}{2}!(\theta T_1)^2$  and  $1 - \theta T_2 + \frac{1}{2}!(\theta T_2)^2$ , respectively.

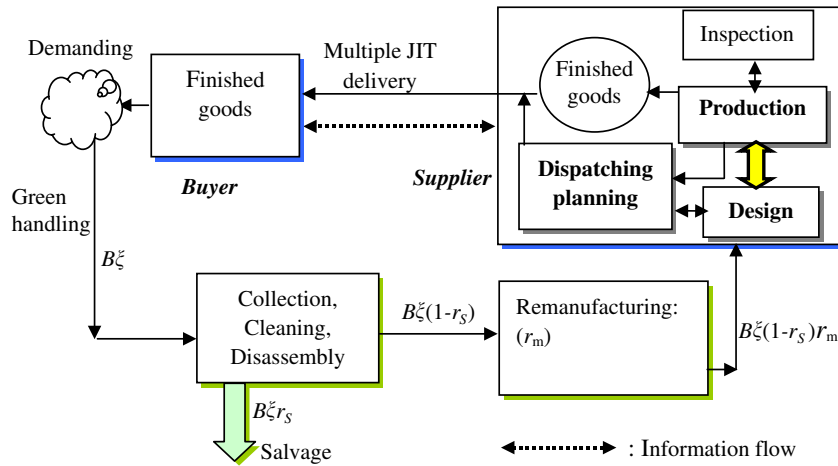


Fig. 1. The behavior of the production-inventory green-handling chain.

Neglecting the terms equal or higher than  $(\theta^2 T_j)$ ,  $j=1$  and  $2$ , one has

$$\frac{P}{\theta} \left\{ 1 - 1 + \theta T_1 - \frac{1}{2} (-\theta T_1)^2 \right\} \approx B[1 + \theta T_2]$$

and the result is

$$T_1 \approx \frac{B[1 + \theta T_2]}{[P - \theta B/2]} \quad (2)$$

Substituting (2) and  $B(=Nq) \approx NDT_b[1 + (\theta T_b/2)]$  into  $T_1 = T_2 = (N-1)T_b + (q/P)$  and solving for  $T_2$  and  $T_1$ , one has

$$T_2 = \frac{T_b[(N-2)D(1 + \theta T_b) - 2P(N-1)]}{ND\theta T_b(2 + \theta T_b) + 2P} \quad (3)$$

Since the single order and multiple deliveries are considered, the buyer's total cost per unit time is

$$\frac{O}{T} + \frac{NH_b}{T} \int_{t=0}^{T_b} I_B(t) dt + \frac{NC_{db}}{T} \left\{ \frac{D}{\theta} [\exp(\theta T_b) - 1] - DT_b \right\} \quad (4)$$

3.1.1. The supplier's forward manufacturing

From Fig. 2, the supplier's holding cost considering TWI can be derived using Eq. (1b):

$$\left\{ \int_{t_1=0}^{T_1} \Psi_{S1}(t_1) dt_1 + \int_{t_2=0}^{T_2} \Psi_{S2}(t_2) dt_2 - T_b(q + 2q + \dots + (N-1)q) \right\} \frac{H_S}{T} \quad (5)$$

The total cost of forward manufacturing consists of the supplier's setup cost per cycle, the supplier's total item cost per cycle, the supplier's cost of less flexibility of implementing JIT delivery, and the inspection cost is

$$\{C_S + C_{iS}PT_1 + NF_S + (C_{if}/N + NC_{io} + C_{ins}PT_1)\}/T \quad (6)$$

3.1.2. The supplier's remanufacturing

From the upper areas in Fig. 2, the bold-dash line illustrates the collecting process and remanufacturing inventory level. The differential equations of the collecting process, the collected-item-consuming process and can be represented as follows:

$$\begin{aligned} \frac{dI_{r1}(t_{r1})}{dt_{r1}} &= R_C - \theta I_{r1}(t_{r1}), \quad 0 \leq t_{r1} \leq T_1 \\ \frac{dI_{r2}(t_{r2})}{dt_{r2}} &= -P_{RC} - \theta I_{r2}(t_{r2}), \quad T_1 \leq t_{r2} \leq (T_1 + T_2) + (T_b - q/P) \\ \frac{d\Phi_R(t_R)}{dt_R} &= P_{RC} - \theta \Phi_R(t_R), \quad T_1 \leq t_R \leq (T_1 + T_2) + (T_b - q/P) \end{aligned} \quad (7a)$$

where  $T_b = T/N$  and  $T_2 + T_b - q/P = T - T_1$ .

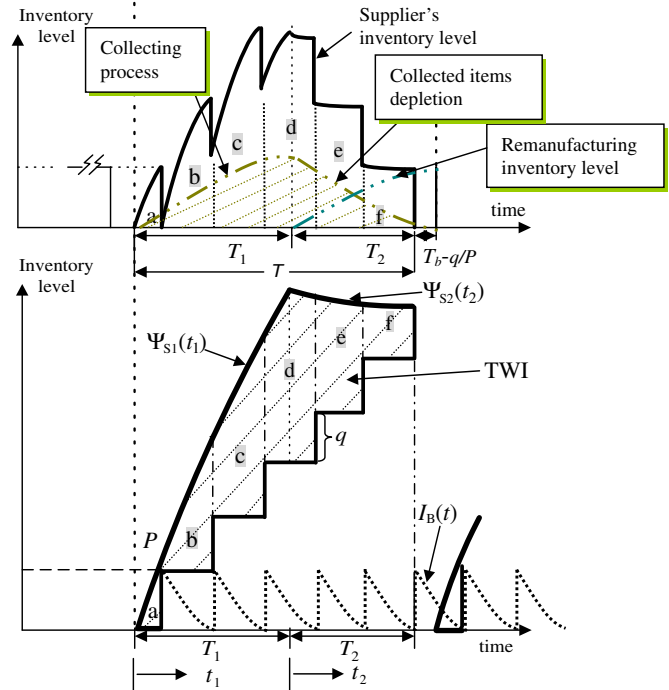


Fig. 2. Supplier's time-weighted inventory (TWI) level and remanufacturing inventory for the production-inventory deteriorating model.

Using the various boundary conditions,  $I_{r1}(T_1) = B\zeta$ ,  $I_{r2}(0) = B\zeta$ , and  $\Phi_R(T - T_1) = q$ , the differential equations of the collecting process, the collected-item-consuming process can be derived as follows:

$$\begin{aligned} I_{r1}(t_{r1}) &= \left( B\zeta - \frac{R_C}{\theta} \right) \left[ \exp(\theta(T_1 - t_{r1})) \right] + \frac{R_C}{\theta} \\ I_{r2}(t_{r2}) &= \frac{P_{RC}}{\theta} \left[ \exp(-\theta t_{r2}) - 1 \right] + B\zeta \exp(-\theta t_{r2}) \\ \text{and} \\ \Phi_R(t_R) &= \left( q - \frac{P_{RC}}{\theta} \right) \left[ \exp\{\theta(T - T_1 - t_R)\} \right] + \frac{P_{RC}}{\theta} \end{aligned} \quad (7b)$$

The supplier's total holding cost of remanufacturing consists of the collecting process, the collected-items-consumption and the



remanufacturing process can be derived referring to Eq. (7):

$$Th_R = \frac{h_{RC}}{T} \int_{t_{r1}=0}^{T_1} I_{r1}(t_{r1}) dt_{r1} + \frac{h_{RC}}{T} \int_{t_{r2}=0}^{T_2+T_b-q/P} I_{r2}(t_{r2}) dt_{r2} + \frac{h_R}{T} \int_{t_R=0}^{T_2+T_b-q/P} \Phi_R(t_R) dt_R \tag{8}$$

The supplier's total reverse manufacturing cost consists of cleaning and disassembly cost, remanufacturing cost and convertibility cost. The reverse manufacturing cost is dependent on the product-design cycle time. We assume the component has an exponential re-condition distribution with a mean of  $1/\delta_m$ . Winters et al. (2004) propose that a continuous development cycle allows for the introduction of new technology on a regular basis in order to improve the capabilities of the next generation products. Therefore the cost of convertibility has a strong relationship with reusable technology. It is straightforward to assume that the cost of convertibility is proportional to the time between technological evolutions. The cost of convertibility is valid that the new evolutions increase exponentially in various situations in product development. We assume the mean time between technological evolutions is an exponential pattern. The supplier's reverse manufacturing cost per unit time can be derived as

$$\{F_{CL} + C_{CL}B\zeta\}/T + \{F_m/L + LB\zeta(1-r_s)r_m C_m[1 - \exp(-\delta_m T)]\}/T + \{C_{CF}/L + LB\zeta(1-r_s)V_{CV}[1 - \exp(-\delta_C T)]\}/T \tag{9}$$

Note that no remanufacturing cost and convertibility cost occur when  $L=1$ , but the cleaning, sorting and disassembly cost occur due to regulations and legislations.

Weibull distribution is assumed for component functional life and hazard rate, as a commonly available in the literature. Various researches and product data bases compiled by computer and electronics industries have shown this functional-life distribution.  $LT$  is the component-design life or the characteristic life and  $\beta$  is the shape parameter. Assuming the hazard rate items is Weibull distributed with the form of  $\beta$ , the mean failure rate is  $F_h = \int_0^{LT} \lambda_1^\beta t^{\beta-1} dt = (\lambda_1 LT)^\beta$ . The supplier's repair cost per unit time when the partial functions are of malfunction is

$$F_{rp}/(LT) + \{C_{rp}F_h B\zeta(1-r_s)\}/T = \{F_{rp}/L + C_{rp}(\lambda_1 LT)B\zeta(1-r_s)\}/T \tag{10}$$

Assuming the items to be salvaged decreases due to the product design with the consideration of function upgrade design, the supplier's salvage per unit time after cleaning, disassembly, sorting and identifying in the collecting distribution is

$$(1 - k_s b_0)S_{av}\{B\zeta r_s\}/T \tag{11}$$

### 3.1.3. Product design considering product function upgrade

The supplier's product design cost,  $Y(L)$  is a function of  $L$ . Although there are many parameters influencing the design and production cost of a component, from the product-design life point of view, it is appropriate to take it as a function increasing with product-design life, i.e.  $dY(L)/dL > 0$ . The supplier's total product design cost considering short life-cycle is

$$Y(L) = C_d \{a_0/L + Lb_0 \prod_{j=1}^K R_j\}/T + C_d \{C_0/L + Ld_0 \prod_{j=1}^K R_j\}/T \tag{12}$$

where the first cost term is design cost of green design and the second cost term is design cost of product function upgrade.  $R_j$  is the reliability of subcomponent  $j$ .

### 3.2. Optimization

The total relevant cost of the green supply chain per unit time is  $TC(T_1, T_2, T_b, N, L) = \{\text{total cost of forward manufacturing}\} + \{\text{total cost of reverse manufacturing}\}$ .

The purpose of this study is to derive the optimal number of inspection, deliveries, component-design life, and the delivery cycle time by determining the optimal values of  $N, L$  and  $T_b$  that minimize  $TC(N, L, T_b)$ . Using Taylor's series expansion, we have  $T_1$  and  $T_2$  in the forms of  $T_b$ :

$$T_1 = NDT_b/P + ND\theta([1 + [2ND + 2(P-D)(N-1)]/P]T_b^2)/P$$

$$T_2 = (N-1)(P-D)T_b/P + (D\theta(N-1)[1 - [2(P-D)N]/P]T_b^2)/P \tag{13}$$

Since  $\theta T_b$  is very small, substituting (13) into the total cost function in (12) and neglecting the cost terms higher than  $\theta T_b^2$ , one has the series represented by using the power of  $T_b$  (see Appendix A) denoted as

$$TC(N, L, T_b) \tag{14}$$

For any given positive integer  $N$  and  $L$ , we can rewrite the cost function (14) as follows:

$$TC_{N,L}(T_b) = g_1/T_b + g_2 T_b + g_3 T_b^2 + C_k \tag{15}$$

From (14) and (15), for given  $N$  and  $L$ , the second derivative of  $TC(N, L, T_b)$  with respect to  $T_b$  can be derived as follows:

$$\partial^2 TC(N, L, T_b) / \partial T_b^2 = d^2 TC_{N,L}(T_b) / dT_b^2 = 2g_1/T_b^3 + 2g_3 \tag{16}$$

**Property 1.** For given  $N$  and  $L, N > 1, L > 1$ , when  $g_1, g_2$  and  $g_3$  are strictly positive, the cost function of the green supply chain inventory model is convex in  $T_b$ .

**Proof.** Since the second derivative of  $TC(N, L, T_b)$  is  $d^2 TC_{N,L}(T_b) / dT_b^2 = 2g_1/T_b^3 + 2g_3 > 0$ , for  $N > 1$  and  $L > 1, TC_{N,L}(T_b)$  is convex.

(1) When  $g_1, g_2$  and  $g_3$  are strictly positive,  $TC_{N,L}(T_b)$  has a unique, finite minimum, given by the solution of

$$dTC_{N,L}(T_b) / dT_b = (-g_1)/T_b^2 + g_2 + 2g_3 T_b = 0 \tag{17}$$

The iterative scheme is

$$T_{b,n+1} = \sqrt{g_1 / (g_2 + 2g_3 T_{b,n})} \tag{18}$$

(2) When  $g_2$  is negative, from Eq. (17), one has  $-g_1 + g_2 T_b^2 + 2g_3 T_b^3 = 0$ .

Rearranging Eq. (17), the result is derived as

$$2g_3 T_b^3 = g_1 - g_2 T_b^2$$

Therefore, the optimal  $T_b$  can be obtained by iterative procedure using the following equation:

$$T_{b,n+1} = \sqrt[3]{(g_1 - g_2 T_{b,n}^2) / (2g_3)} \tag{19}$$

□

**Property 2.** For a given  $N$  and  $T_b, N > 1, T_b > 0$ , the cost function of the green supply chain inventory model is convex in  $L$ .

Since the second derivative of  $TC(N, L, T_b)$  with respect to  $L$  is  $d^2 TC_{N,T_b}(L) / dL^2 = \{2[C_{VF} + F_m + F_{rp} + C_d(a_0 + C_0)] / (NL^3)\} / T_b > 0$

for  $N > 1, T_b > 0, TC_{N,T_b}(L)$  is convex.

Consequently, one obtain the optimal condition for deriving optimum  $L$  value by using cost difference method of  $TC(N, L, T_b)$  with respect to  $L$ . The optimal condition is

$$(L^*)(L^* - 1) \leq (L^*)^2 \leq (L^*)(L^* + 1) \tag{20}$$

and

$$L^* = 2\sqrt{(F_{rp} + F_m + C_{VF} + C_d(a_0 + C_0)) / (2\varpi)}$$

where

$$\varpi = (1-r_s)D\zeta N^2 T_b^2 \{(-\theta T_b + 2)(C_{rp}\lambda_1 + V_{CV}/(NT_b)) - (\theta T_b + 2)(V_{CV}\delta_C - r_m C_m \delta_m) + NT_b(r_m C_m \delta_m^2 - V_{CV}\delta_C^2)\} + 2C_d(b_0 + d_0) \prod_j R_j$$

Obviously, the optimal value of  $L$  is influenced mainly by the fixed cost of remanufacturing, repair, cleaning, sorting, disassembly and product design parameters. Due to the complexity in solving the symbolic solutions of  $T_b$ , the integer  $L$  and  $N$ , we develop a simple solution procedure to derive the relevant optimal values.

To estimate the delivery-time interval and provide relevant remanufacturing plan for original design manufacturer (ODM), the proposed model develops a simple approach when the effects of the component-life value design and remanufacturing are considered. Due to the complexity in solving the symbolic solutions of  $T_b$ ,  $L$  and the integer  $N$  simultaneously, we develop a solution procedure to derive the relevant optimal values. When the total cost function,  $TC$ , of the multiple-level production-inventory in a green supply chain is obtained, the iterative equations and procedure are used to derive the optimal solution considering the take-back product with component-value design and remanufacturing.

Solution procedure

Step 1-1: For a range of  $L$ , initialize  $L$  and start from  $L_{fea} = 1$ .

Step 2-1: For a range of  $N$ , from Eq. (15):

- (1) When  $g_2 > 0$ , initialize  $N$  and determine  $T_b$  from Eq. (18). When  $g_2 < 0$ , determine  $T_b$  from Eq. (19).
- (2) Substitute  $N$  and  $T_b$  derived from (1) of Step 2-1 into Eq. (20) to derive a new input parameter of  $L_{fea}$ . If new  $L_{fea}$  = old  $L_{fea}$ , go to (3); otherwise, using new  $L_{fea}$  and go to (1).
- (3) For the new  $L_{fea}$  derived from (2) and each  $N$ , denote the value of  $T_b$  as  $T_b(N, L_{fea})$  which minimizes  $TC$ .

Step 3:

- (1) The optimal values of  $N$ ,  $L$  and  $T_b$  satisfying the optimal conditions of Hessian matrix  $H$  can be derived.

where the Hessian matrix is  $H_m = \begin{bmatrix} \frac{\partial^2 TC}{\partial T_b^2} & \frac{\partial^2 TC}{\partial T_b \partial N} & \frac{\partial^2 TC}{\partial T_b \partial L} \\ \frac{\partial^2 TC}{\partial N \partial T_b} & \frac{\partial^2 TC}{\partial N^2} & \frac{\partial^2 TC}{\partial N \partial L} \\ \frac{\partial^2 TC}{\partial L \partial T_b} & \frac{\partial^2 TC}{\partial L \partial N} & \frac{\partial^2 TC}{\partial L^2} \end{bmatrix}$

The first principal minor determinant of  $H_m$  is  $|H_{11}| > 0$ , the second principal minor determinant of  $H_m$  is  $|H_{22}| > 0$  and the third principal minor determinant of  $H_m$  is  $|H_{33}| > 0$ .

- (2) For  $L^* =$  new  $L_{fea}$  values, the total cost of the green supply chain inventory model is optimal when  $TC(T_b(N^* - 1, L^*), N^* - 1, L^*) \geq TC(T_b(N^*, L^*), N^*, L^*) \leq TC(T_b(N^* + 1, L^*), N^* + 1, L^*)$ , and  $TC(T_b(N^*, L^*), N^*, L^*)$  is minimum value of the all  $TC(T_b(N, L), N, L)$  for various  $N$  and  $L$  values.

Step 4: Find the optimal values of  $T_1$ ,  $T_2$  and  $T$  from Eqs. (2) and (3), and the delivery batch size from Eq. (1c).

#### 4. Numerical example and sensitivity analysis

The preceding theory can be illustrated by the following example. The parameter inputs are:

$$\theta = 0.01 \quad C_{bd} = \$4 \quad a_0 = 5 \quad C_{rp} = \$2 \quad \zeta = 0.85 \\ P = 18\,000 \quad C_{IS} = \$3.5 \quad C_0 = 3 \quad S_{av} = \$5.7 \quad \lambda_1 = 0.001$$

units

$D = 5000$ units	$F_S = \$100$	$b_0 = 0.6$	$C_d = \$50$	$P_{RC} = 4000$ units
$O = \$100$	$C_{if} = \$2800$	$d_0 = 0.4$	$C_{CL} = \$ 0.5$	$R_C = 3500$ units
$C_S = \$600$	$C_{io} = \$250$	$\delta_C = 0.002$	$h_{RC} = \$0.8$	$F_m = \$3000$
$H_b = \$1.4$	$C_{ins} = \$ 0.7$	$\delta_m = 0.002$	$h_R = \$1.0$	$F_{CV} = \$3000$
$H_S = \$1.0$	$R_1 = 0.999$	$C_m = \$2$	$r_m = 0.9$	$F_{CL} = \$3000$
$k_s = 0.5$	$R_2 = 0.98$	$C_{CV} = \$2$	$r_s = 0.9$	$F_{TP} = \$3000$

Using the fast solution procedure stated in Section 3.2, the optimal solution considering deterioration is derived as  $(N^*, L^*, T_b^*, B^*, TC^*) = (6 \text{ times}, 3 \text{ times}, 7073 \text{ units}, 0.2358 \text{ years}, \$24,589.34)$ , respectively. If deterioration approximates to zero, the solution is derived as in Table 1. The percentage of integrated total cost difference is defined as  $PICD = (TC - TC^*)/TC^*$ .

When the parameter increases, the relationships between the known parameters, the decision variable and the percentage integrated total cost difference are derived. “\*” is the optimal total cost and “{•}” is the base column.

The main conclusions drawn from the sensitivity analysis are as follows:

- (a) When the deterioration rate and convertibility parameter increase, the delivery time interval decrease. In practice, when the deteriorating rate increases and the new technologies evolve frequently, the delivery time interval should be shortened, i.e., more frequent deliveries may be implemented (Tables 1 and 2). The total cost is sensitive to the holding cost  $\{H_S, H_b, h_{RC}, h_R\}$ . When the system holding cost increase, the number of delivery interval increases resulting in an increase in the total cost (Table 4).
- (b) When the convertibility parameter and green handling ratio increase, the total cost tends to decrease. That the new technologies evolve frequently and the take-back items increase will benefit the green supply chain inventory system. When the convertibility parameter increases, the number of deliveries increases resulting in a decrease in the total cost. When the green handling ratio increases, the number of deliveries decreases resulting in a decrease in the total cost. When a manager considers a dispatching plan, there is a trade-off between the convertibility parameter and green

**Table 1**  
Sensitivity analysis when the deteriorating rate changes.

$\theta$	0.0001	0.001	{0.01}	0.05	0.1
$L$	3	3	3	3	3
$N$	6	6	6	7	7
$T_b (=T/n)$	0.2372	0.2370	0.2358	0.2028	0.1969
$B$	7116	7112	7073	7134	6959
$TC$	245 21.87	24528.03	24 589.34*	24 840.09	25 133.53
$PICD$	-0.27%	-0.25%	0.00%	1.02%	2.21%

\* The optimal total cost; {}: the base column.

**Table 2**  
Sensitivity analysis when convertibility parameter of technology evolution changes.

$\delta_C$	0.0002	0.0004	{0.002}	0.02	0.2
$L$	3	3	3	3	3
$N$	6	6	6	6	7
$T_b (=T/n)$	0.2358	0.2358	0.2358	0.2359	0.2119
$B$	7116	7112	7073	7085	7425
$TC$	24 609.17	24 609.17	24 589.34*	24 608.07	24 503.66
$PICD$	0.08%	0.08%	0.00%	0.08%	-0.35%

\* The optimal total cost; {}: the base column.

**Table 3**  
Sensitivity analysis when the green handling ratio of take-back items changes.

$\xi$	0.765	0.808	{0.85}	0.893	0.935
$L$	3	3	3	3	3
$N$	7	7	6	6	6
$T_b (=T/n)$	0.2164	0.2122	0.2358	0.2318	0.2280
$B$	7582	7437	7073	6961	6848
$TC$	25 599.96	25 103.92	24 589.34*	24 096.49	23 591.84
$PICD$	4.11%	2.09%	0.00%	-2.00%	-4.06%

\* The optimal total cost; {}: the base column.

**Table 4**  
Sensitivity analysis when the holding costs change.

$H_b; H_s;$ $h_{RC}; h_R$	1.12; 0.80;	1.26; 0.90;	{1.4}; {1.0};	1.54; 1.10;	1.68; 1.2;
	0.64; 0.80	0.72; 0.90	{0.8}; {1.0}	0.88; 1.10	0.96; 1.20
$L$	3	3	3	3	3
$N$	6	6	6	6	6
$T_b (=T/n)$	0.2628	0.2480	0.2358	0.2247	0.2152
$B$	7895	7450	7073	6748	6464
$TC$	22 183.36	23 405.74	24 589.34*	25 739.80	26 861.52
$PICD$	-9.78%	-4.81%	0.00%	4.68%	9.24%

\* The optimal total cost; {}: the base column.

handling ratio in determining the optimal number of deliveries (Tables 2 and 3).

- (c) When the deterioration rate  $\theta$  and convertibility parameter  $\delta_C$  increase, the delivery time interval  $T_b$  decrease. In practice, when the deteriorating rate  $\theta$  increases and the new technologies evolve frequently, the delivery time interval  $T_b$  should be shortened, i.e., more frequent deliveries may be implemented (Tables 1 and 2).
- (d) When the convertibility parameter  $\delta_C$  and green handling ratio  $\xi$  increase, the total cost tends to decrease. That the new technologies evolve frequently and the take-back items increase will benefit the green supply chain inventory system (Tables 2 and 3).
- (e) The optimal number of deliveries  $N$  is sensitive to the parameters: demand rate  $D$ , the ratio for remanufacturing process  $r_s$  and the cost of less flexibility  $F_S$ . The total cost is less sensitive to demand rate  $D$ , the ratio for remanufacturing process  $r_s$ , the green handling ratio of the reusable and take-back items  $\xi$ , the unit inspection cost  $U_{inv}$  ..., unit variable cost  $C_{CL}$  including cleaning, sorting and disassembly cost and the variable inspection cost per setup  $C_{io}$  (Tables 3, 6, 7, 10, 11 and 12). When the demand, the holding costs and the ratio for remanufacturing after cleaning, disassembly and sorting increase, a frequent delivery policy is encouraged to be implemented (Tables 3, 10, 11).
- (f) When the ratio for remanufacturing process  $r_s$  after sorting, cleaning and disassembly decreases, a shorter component life-cycle strategy and a frequent delivery policy are encouraged to be executed, i.e. when a product is not popular in the market place or is different to converted/upgrade to a newer generation function, the product design policy should consider the trade-off between the factors of the re-manufacturability and the component life-cycle of product design (Table 11). In the point view of systematic design in first product-design stage, the product design process should provide good design quality for increasing the function and higher rate of re-manufacturability, i.e. an excellent design for remanufacturing (DFR) (Table 11).
- (g) From (b), a critical policy that the manager should increase the frequency of the new technology evolution and take-back items will benefit green production-inventory control system to reduce cost.

**Table 5**  
Sensitivity analysis when the design cost changes.

$C_d$	40	45	{50}	55	60
$L$	3	3	3	3	3
$N$	6	6	6	6	6
$T_b (=T/n)$	0.2348	0.2351	0.2358	0.2358	0.2369
$B$	7053	7063	7073	7084	7094
$TC$	24 549.62	24 569.50	24 589.34*	24 609.16	24 628.94
$PICD$	-0.16%	-0.08%	0.00%	0.08%	0.16%

\* The optimal total cost; {}: the base column.

**Table 6**  
Sensitivity analysis when the cost of less flexibility changes.

$F_S$	80	90	{100}	110	120
$L$	3	3	3	3	3
$N$	7	6	6	6	6
$T_b (=T/n)$	0.2066	0.2347	0.2358	0.2362	0.2369
$B$	7240	7051	7073	7094	7117
$TC$	24 499.12	24 546.81	24 589.34*	24 631.74	24 674.01
$PICD$	-0.37%	-0.17%	0.00%	0.17%	0.34%

\* The optimal total cost; {}: the base column.

**Table 7**  
Sensitivity analysis when unit variable inspection and variable cleaning, disassembly and sorting costs change.

$U_{inv}$ $C_{CL}$	0.56 0.40	0.63 0.45	{0.7} {0.5}	0.77 0.55	0.84 0.60
$L$	3	3	3	3	3
$N$	6	6	6	6	6
$T_b (=T/n)$	0.2359	0.2358	0.2358	0.2358	0.2358
$B$	7053	7063	7073	7084	7094
$TC$	24 332.00	24 470.58	24 589.34*	24 747.32	24 886.31
$PICD$	-1.05%	-0.48%	0.00%	0.64%	1.21%

\* The optimal total cost; {}: the base column.

**Table 8**  
Sensitivity analysis when the fixed component-life-cycle design cost rate changes.

$a_0$	4.0	4.5	{5}	5.5	6.0
$L$	3	3	3	3	3
$N$	6	6	6	6	6
$T_b (=T/n)$	0.2353	0.2354	0.2358	0.2356	0.2357
$B$	7067	7070	7073	7076	7079
$TC$	24 577.54	24 583.44	24 589.34*	24 595.24	24 601.13
$PICD$	-0.05%	-0.02%	0.00%	0.02%	0.05%

\* The optimal total cost; {}: the base column.

**Table 9**  
Sensitivity analysis when the collecting rate changes.

$R_C$	2800	3150	{3500}	3850	4200
$L$	3	3	3	3	3
$N$	6	6	6	6	6
$T_b (=T/n)$	0.2350	0.2352	0.2358	0.2358	0.2360
$B$	7057	7065	7073	7081	7089
$TC$	24 619.97	24 604.67	24 589.34*	24 574.00	24 558.64
$PICD$	0.12%	0.06%	0.00%	-0.06%	-0.12%

\* The optimal total cost; {}: the base column.

**Table 10**  
Sensitivity analysis when the cost of less flexibility changes.

D	4000	4500	{5000}	5500	6000
L	3	3	3	3	3
N	7	7	6	6	6
T <sub>b</sub> (=T/n)	0.2458	0.2243	0.2358	0.2216	0.2104
B	6890	7075	7073	7322	7583
TC	20 096.57	22 410.33	245 89.34*	26 646.52	28 622.73
PICD	-18.27%	-8.86%	0.00%	8.37%	16.40%

\* The optimal total cost; {} the base column.

**Table 11**  
Sensitivity analysis when the ratio for remanufacturing process after cleaning, disassembly and sorting changes.

r <sub>s</sub>	0.8	0.85	{0.9}	0.95	0.99
L	2	2	3	4	4
N	7	7	6	6	6
T <sub>b</sub> (=T/n)	0.2235	0.2235	0.2358	0.2264	0.2264
B	6890	7075	7073	7322	7583
TC	28 154.04	26 451.09	24 589.34*	22 356.41	20 312.63
PICD	14.50%	7.57%	0.00%	-9.08%	-17.39%

\* The optimal total cost; {} the base column.

**Table 12**  
Sensitivity analysis when the variable inspection cost per setup changes.

C <sub>io</sub>	200	225	{250}	275	300
L	3	3	3	3	3
N	7	7	6	6	6
T <sub>b</sub> (=T/n)	0.2044	0.2063	0.2358	0.2373	0.2391
B	7161	7227	7073	7128	7182
TC	24 353.14	24 474.90	24 589.34*	24 695.09	24 800.04
PICD	-0.96%	-0.47%	0.00%	0.43%	0.86%

\* The optimal total cost; {} the base column.

(h) The production lot size of the supplier is positively sensitive to the convertibility parameter of technology evolution  $\delta_c$ , demand rate  $D$  and the ratio for remanufacturing after cleaning, disassembly and sorting  $r_s$ ; and slightly sensitive to design cost  $C_d$ , fixed component-life-cycle design cost  $a_0$  and the collecting rate  $R_C$  (see Tables 5, 8 and 9). Moreover, the variable cleaning, disassembly and sorting costs  $C_{CL}$  and collecting rate  $R_C$  slightly influence the production lot size of the supplier (see Tables 7 and 9). From the managerial view point, when the variable cleaning cost of the supply chain's collecting process and collecting rate  $R_C$  increase, the system does not change very much; it may encourage the management to invest more on cautious activities in the collecting process to support the next remanufacturing actions.

**5. Conclusion**

The green-component life-cycle value design is an innovative approach for manufacturer to maximize the environmental benefits of the supply chain. An integrated production-inventory deteriorating model considering the greening operation processes over a finite planning horizon is developed from an end-of-life perspective and reduction design is considered in the supplier's forward and remanufacturing processes. This study incorporates inspection cost, transportation cost, cost of less flexibility,

green-component life-cycle value design, green design cost of reducing gas emission and reverses manufacturing.

An optimal replenishment policy for an integrated supplier-buyer deteriorating inventory model is derived. The result of the analysis shows that the re-manufacturability and the component life-cycle of product design are interrelated. We have shown that it is significant to consider new technology evolution, green handling ratio of take-back items, higher rate of re-manufacturability and the system's holding costs. Further research can be extended to consider the issue of multi-objective optimization and revenue sharing mechanisms for the integrated model under green supply chain.

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**Appendix A**

$$\begin{aligned}
 TC(N,L,T_b) = & T_b^2 \left( (1-r_s)LND\xi \left\{ \frac{-V_{CV}\delta_c}{2}(\theta+N\delta_c) + \frac{C_m\delta_m r_m}{2}(\theta-N\delta_m) \right. \right. \\
 & + \left. \left. \frac{C_{rp}\lambda_1\theta}{2} \right\} + ND\theta \left( 1 - \frac{D}{P} \right) \left[ \left( \frac{P_{RC}}{P} h_R + h_{RC} \left( \frac{P_{RC}}{P} - \frac{N\xi}{2} \right) \right) \right] \right. \\
 & + D\theta \left[ \frac{h_{RC}N}{2} \left( \xi - \frac{2DR_C}{P^2} \right) + \frac{C_{bd}}{2} \right] \\
 & + \frac{h_R D}{2} \left[ (N+1) \left( 1 - \frac{2D}{P} \right) - \frac{D}{P} \left( 1 - \frac{ND}{P} \right) \right] \\
 & + \frac{H_S D}{2} \left[ (N-1) \left( \frac{\theta}{2} (2N-1) - \frac{2D}{P} \right) \right] \\
 & + T_b \left\{ LD\xi(1-r_s) \left[ N(C_{rp}\lambda_1 + C_m r_m \delta_m) + \frac{V_{CV}}{2}(\theta-2N\delta_c) \right] \right. \\
 & + \frac{H_S D}{2} \left[ (N-1) + \theta \left( 2 - \frac{DN}{P} \right) \right] + \frac{P_{RC}ND}{2P} \left[ 2 - \left( \frac{P}{D} + \frac{D}{P} \right) \right] (h_{RC} + h_R) \\
 & + D \left[ h_R \left( 1 - \frac{D}{P} \right) + \frac{H_b}{2} + \frac{h_{RC}N}{2} \left( 2\xi - \frac{D}{P^2} R_C \right) \right] \\
 & \left. + D\theta \left( C_{IS} - \frac{\xi(1-k_s b_0)S_{av} r_s}{2} + \frac{\xi C_{CL}}{2} + U_{inv} \right) \right\} \\
 & + \frac{1}{T_b} \left\{ \frac{1}{N} \left[ (F_{CL} + O + C_S + LC_d(b_0 + d_0)) \prod_j R_j \right] + \frac{1}{L} (F_{rp} + C_{VF} + C_d(a_0 \right. \right. \\
 & \left. \left. + C_0) + F_m) \right] + F_S + C_{io} + \frac{C_{if}}{N^2} \right\} \\
 & + D \left[ (H_S + C_{IS} + U_{inv}) - \xi((1-k_s b_0)S_{av} r_s + C_{CL}) + LV_{CV}\xi(1-r_s) \right] \\
 = & g_3 T_b^2 + g_2 T_b + g_1 / T_b + C_K
 \end{aligned}$$

where

$$\begin{aligned}
 g_1 = & \left\{ \frac{1}{N} \left[ (F_{CL} + O + C_S + LC_d(b_0 + d_0)) \prod_j R_j \right] + \frac{1}{L} (F_{rp} + C_{VF} \right. \\
 & \left. + C_d(a_0 + C_0) + F_m) \right] + F_S + C_{io} + \frac{C_{if}}{N^2} \right\} \\
 g_2 = & \left\{ LD\xi(1-r_s) \left[ N(C_{rp}\lambda_1 + C_m r_m \delta_m) + \frac{V_{CV}}{2}(\theta-2N\delta_c) \right] \right. \\
 & + \frac{H_S D}{2} \left[ (N-1) + \theta \left( 2 - \frac{DN}{P} \right) \right] \\
 & \left. + \frac{P_{RC}ND}{2P} \left[ 2 - \left( \frac{P}{D} + \frac{D}{P} \right) \right] (h_{RC} + h_R) \right\}
 \end{aligned}$$



$$\begin{aligned}
& + D \left[ h_R \left( 1 - \frac{D}{P} \right) + \frac{H_b}{2} + \frac{h_{RC} N}{2} \left( 2\xi - \frac{D}{P^2} R_C \right) \right] \\
& + D \theta \left( C_{IS} - \frac{\xi(1-k_s b_0) S_{av} r_S}{2} + \frac{\xi C_{CL}}{2} + U_{inv} \right) \Big\} \\
g_3 = & \left( (1-r_S) LND \xi \left\{ \frac{-V_{CV} \delta_C}{2} (\theta + N \delta_C) + \frac{C_m \delta_m r_m}{2} (\theta - N \delta_m) \right. \right. \\
& \left. \left. + \frac{C_{TP} \lambda_1 \theta}{2} \right\} + ND \theta \left( 1 - \frac{D}{P} \right) \left[ \left( \frac{P_{RC}}{P} h_R + h_{RC} \left( \frac{P_{RC}}{P} - \frac{N \xi}{2} \right) \right) \right] \right. \\
& \left. + D \theta \left[ \frac{h_{RC} N}{2} \left( \xi - \frac{2DR_C}{P^2} \right) + \frac{C_{bd}}{2} \right] \right. \\
& \left. + \frac{h_R D}{2} \left[ (N+1) \left( 1 - \frac{2D}{P} \right) - \frac{D}{P} \left( 1 - \frac{ND}{P} \right) \right] \right. \\
& \left. + \frac{H_S D}{2} \left[ (N-1) \left( \frac{\theta}{2} (2N-1) - \frac{2D}{P} \right) \right] \right) \\
C_K = & D[(H_S + C_{IS} + U_{inv}) - \xi((1-k_s b_0) S_{av} r_S + C_{CL}) + LV_{CV} \xi(1-r_S)]
\end{aligned}$$

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