Ann Oper Res
DOI 10.1007/s10479-012-1146-x

Strategies for protecting supply chain networks against facility and transportation disruptions: an improved Benders decomposition approach

Nader Azad · Georgios K.D. Saharidis · Hamid Davoudpour · Hooman Malekly · Seyed Alireza Yektamaram

Abstract Disruptions rarely occur in supply chains, but their negative financial and technical impacts make the recovery process very slow. In this paper, we propose a capacitated supply chain network design (SCND) model under random disruptions both in facility and transportation, which seeks to determine the optimal location and types of distribution centers (DC) and also the best plan to assign customers to each opened DC. Unlike other studies in the extent literature, we use new concepts of reliability to model the strategic behavior of DCs and customers at the network: (1) Failure of DCs might be partial, i.e. a disrupted DC might still be able to serve with a portion of its initial capacity (2) The lost capacity of a disrupted DC shall be provided from a non-disrupted one and (3) The lost capacity fraction of a disrupted DC depends on its initial investment amount in the design phase.

In order to solve the proposed model optimally, a modified version of Benders' Decomposition (BD) is applied. This modification tackles the difficulties of the BD's master

N. Azad · H. Davoudpour
Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran

N. Azad
e-mail: n.azad@aut.ac.ir

H. Davoudpour
e-mail: hamidp@aut.ac.ir

G.K.D. Saharidis (✉)
Kathikas Institute of Research & Technology, Columbus, MO, USA
e-mail: saharidis@gmail.com

G.K.D. Saharidis
University of Thessaly, Leoforos Athinon, Pedion Areos, 38834 Volos, Greece

H. Malekly
School of Industrial Engineering, Islamic Azad University–South Tehran Branch, Tehran, Iran
e-mail: h_malekly@azad.ac.ir

S.A. Yektamaram
Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran
e-mail: a_yektamaram@ie.sharif.edu

Published online: 16 May 2012

problem (MP), which ultimately improves the solution time of BD significantly. The classical BD approach results in low density cuts in some cases, Covering Cut Bundle (CCB) generation addresses this issue by generating a bundle of cuts instead of a single cut, which could cover more decision variables of the MP. Our inspiration to improve the CCB generation led to a new method, namely Maximum Density Cut (MDC) generation. MDC is based on the observation that in some cases CCB generation is cumbersome to solve in order to cover all decision variables of the MP rather than to cover part of them. Thus the MDC method generates a cut to cover the remaining decision variables which are not covered by CCB. Numerical experiments demonstrate the practicability of the proposed model to be promising in the SCND area, also the modified BD approach decreases the number of BD iterations and improves the CPU times, significantly.

Keywords Supply chain network · Facility location · Random disruption risks · Benders decomposition

1 Introduction

Risk management is a structured approach to manage threatening uncertain events through a sequence of human activities. These activities consist of risk assessment, strategies development to manage it, and mitigation of its impact using managerial resources. The strategies include risk transferring (usually to another party), risk avoidance, negative effect reduction, and the acceptance of some or all consequences of a particular risk. The objective of risk management is to reduce the impacts of different risks in a preselected domain, to a level accepted by the decision maker. Risks may refer to numerous types of threats caused by environment, technology, humans, organizations and politics. Some risk management strategies tackle risks of disruptions in the network, which might be caused by natural sources or planned operations (e.g. earthquakes, terrorist attacks, etc.).

In this regard, supply chain risk management attempts to reduce supply chain's vulnerability by a coordinated comprehensive approach, involving all supply chain stakeholders, which identifies and analyzes the risk of failure points within the supply chain.

An example of disruptions caused by natural disasters, which is the focus of this study, is the recent tragedy in Japan.

On March 11, 2011, Tokyo was struck by a 9.0-magnitude earthquake, followed by a massive tsunami. As of early April, the death count was over 14000, with an additional 13500 persons still missing. The tsunami also triggered the most serious nuclear incident since the 1986 Chernobyl disaster as it caused a disruption in the power supply of a cooling system at a major reactor complex at Fukushima. Damage to public infrastructure and private capital stock, as estimated by Japanese authorities, is roughly $200–300 billion, or 3–5 percent of Japanese GDP. The supply chains of many international companies were also affected dramatically, including those in the steel, automotive, electronics and chemical industries, e.g. General Motors had to halt the production of vehicles at several plants, due to parts shortages from Japanese suppliers. Also Toyota had to suspend production of parts in the mother country that were intended to be shipped overseas. Finally, most Japanese automotive assembly plants remain closed. The resulting slowdowns and cessation of operations by so many companies has raised questions regarding supply chain disruption risk and how to manage it.

As can be seen, disruption risks in supply chain might impose great impacts in today's economical environment. Companies are often used to deal with operational risks of supply chains; whereas many suffer much heavily from supply chain disruptions risks.

Within the literature, there are some mitigation approaches to deal with supply chain disruptions: (i) Tactical plans, which are mostly propounded as a particular driver of the supply chain. For instance, Parlar and Berkin (1991), Parlar (1997) and Mohebbi (2004) recommended inventory control strategies. Kulkarni et al. (2004), Tomlin and Wang (2005) and Tomlin (2006) considered flexible strategies in the configurations of supply chain and Kraiselburd et al. (2004), Martinez-de-Albeniz and Simchi-Levi (2005) and Chopra et al. (2007) endorsed some procurement strategies for contraction in the supply chain. In fact, common methods of this category employ buffer-oriented methods to manage disruption risks, although buffering may concern time, capacity or inventory. These methods represent only a shield against risks and contribute to raise overall costs. (ii) Strategic plans, which consist of a wide range of managerial strategies (e.g. demand, product and information management). Many researchers such as Chopra and Sodhi (2004), Kleindorfer and Saad (2005), Elkins et al. (2005), Wagner and Bode (2006), Tang (2006), Craighead et al. (2007) and Oke and Gopalakrishnan (2009), have studied this issue. Plans in this category more effectively reduce supply chain's disruption risks. Also they deeply analyze risk sources which are crucial to the choice of disruption mitigation strategies. Sources of disruption risks can be segmented into two categories: *random disruption risks* which may occur at any physical point of supply chain network, e.g. natural hazards (earthquakes, etc.), whereas, *premeditated disruption risks* are deliberately planned to inflict the supply chain with maximum damages. Terrorist attacks and labor union strikes are the examples of these risks. Random disruption risks can be modeled reasonably to follow a probability distribution, whereas premeditated disruption risks need to be modeled in a different manner reflecting the "game" that is played between the interdictor of the network and the defender of the network. The structural design of the supply chain greatly affects how these kinds of disruption risks influence the network. Consequently, in order to mitigate both kinds of risks, undertaking appropriate strategic plans to improve structural design of the supply chain, becomes more important.

Thus, we turn the scope of this paper to explore the supply chain network design (SCND) under random disruption risks and propose strategies reduce their impacts. To hedge against network disruptions, providing a robust-designed network is one of the most striking ways.

Therefore, this paper aims to make a contribution to the area of random disruption risk management in SCNDs. Resilience (the ability to adapt to disruptive phenomena), is a key feature of a SCND that equips it with the ability to withstand the adverse effects of disruptive events. In this paper we emphasize on a mathematical model which aims to make the components of SCND collaborate to deal with random risk topic. Therefore, we first lay out a theoretical framework to mathematically characterize the notion of this strategy, and analyze the advantages it offers over an existing strategy. Besides underlining the importance of this strategy to achieve resilience in SCND, this research has resulted in the identification of new research questions along the way. The pursuit of these questions should help shedding further light on the challenges associated with random disruption risk management.

The remainder of the paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 presents the mathematical formulation of the problem. Section 4 describes a decomposition based solution approach for the model. Section 5 presents the computational experiments and results. Section 6 concludes the paper and presents the future work perspectives.

2 Literature review

In this section, we present an overview of the literature for SCND under random disruption risks (for premeditated disruption risks, we address, Church et al. 2004; Church and Scaparra 2006; Scaparra and Church 2008; Altner et al. 2010; Liberatore et al. 2011, 2012). The objective is to design a supply chain network which operates efficiently with the lowest possible cost, both at normal and disruption situations. The origination of SCND under random disruption risks looks back to network reliability theory (Colbourn 1987; Shier 1991 and Shooman 2002), which is concerned with maximizing the probability that a link remains connected after random failures. The primary purpose in designing reliable networks is often to maximize the demand coverage. For a review of SCND problems considering disruption risks see Snyder et al. (2006) and Klibi et al. (2010).

Drezner (1987) is one of the papers that studied the facility location under random disruption risks and suggested two models. In the first one, a reliable classical p-median problem was assumed, which considers a given probability for the failure of facilities. The objective was to minimize the expected demand-weighted travel distance. The second model, is called the (p, q)-center problem; in which p facilities must be located by a means of a minimax objective cost function with the assumption that at most q facilities may fail. In both problems, customers are assumed to be chosen from the nearest non-disrupted facility. He proposed a neighborhood search heuristic approach for both problems. An efficient method based on space filling curves to solve the reliable p-median problem is proposed in Lee (2001).

Snyder and Daskin (2005) studied SCND under random disruptions, based on classical facility location problems, in which a distribution center (DC) may fail (because of a disruption occurrence) with a given probability. They assumed that when a DC fails, it can no more provide any product, and the customers assigned to it, must be reassigned to a non-disrupted DC. Their objective was to minimize a weighted sum of nominal costs by ignoring disruptions and also the expected cost of disruption situation where the expected additional transportation cost was accounted for disrupted DCs. In their model customers are assigned to several DCs, one of which is the "primary" DC that serves it under regular situation (without disruption), the others serve it if the primary DC fails and so on (Non-linear probability term). They made the simplifying assumption that all DCs have the same disruption probability. This assumption allows the expected transportation cost to be expressed as a linear function of the decision variables. They used the Lagrangian relaxation algorithm to solve the problem.

One important simplifying assumption of Snyder and Daskin (2005) model is that all DCs have the same disruption probability. Without this assumption, calculating expected transportation cost in disruption situation becomes significantly complicated. Snyder and Daskin (2006) developed their previous work and considered the site dependent disruption probability. They used the scenario approach to formulate the problem. Also they introduced the concept of stochastic p-robustness where the relative regret is always less than p for any possible scenario. As the scenario approach enumerates all of the disruption scenarios, one of its issues is the exponential growth of the problem size by an increase in the number of DCs. Berman et al. (2007) proposed a p-median problem, in which the objective function was to minimize the demand-weighted transportation cost. They considered site dependent disruption probabilities in DCs. The resulting model, which they call the median problem with unreliable facilities uses non-linear terms to calculate the expected transportation cost in disruption situation. The authors presented a greedy heuristic to solve the problem. Berman et al. (2009) developed their previous work and assumed that customers do not know which

DCs are disrupted and must travel from a DC to another until they find a non-disrupted one. They applied a heuristic algorithm to solve the problem.

Cui et al. (2010) also proposed a formulation for the problem with site-dependent disruption probabilities. Unlike the model by Berman et al. (2007) which consists of compound multiplied decision variables, the only non-linear term of their model is a product of a single continuous and a single discrete decision variable and continuum approximation (CA) was used to formulate the model. In this approximation approach customers are scattered uniformly throughout some geographical area, and the parameters are expressed as a function of the location. Replacing explicit disruption probabilities with probabilities depending on the location allows the expected transportation cost or distance to be calculated without using any assignment decision. Lagrangian relaxation was used to solve the model. Qi et al. (2010) studied the SCND under random disruptions considering inventory control decisions. They assumed that when a retailer is disrupted, any inventory on hand at the retailer is destroyed and also the unmet demands of customers assigned to a retailer are backlogged with a penalty cost. The resulting model was a concave minimization problem and the Lagrangian relaxation algorithm was implemented to solve it.

Li and Ouyang (2010) studied the SCND under random disruption risks, in which the disruption probabilities are assumed to be site-dependent and also geographically correlated. They also used CA to formulate the model. Lim et al. (2010) proposed the SCND under random disruptions with the option of strengthening selected DCs. The disruption probabilities are assumed to be site-dependent. By considering two types of DCs (namely *unreliable* and *reliable*), they used the reliable backup DCs assumption to formulate their model. The disruption occurs in unreliable DCs. Reliable DCs are those that are strengthened against disruptions by a financial investment and disruptions does not affect them, the so-called *hardening strategy*. Like previous works in this area, they assumed that when a disruption occurs, an unreliable DC completely fails. In their model the customers in disruption situation are assigned to the nearest reliable DCs. Same as many studies in the literature, the Lagrangian relaxation was used for solving the problem. Peng et al. (2011) developed a capacitated version of SCND under random disruptions with stochastic p-robustness criteria and site dependent disruption probabilities. Similar to Snyder and Daskin (2006), they used the scenario approach to formulate the problem. A hybrid metaheuristic algorithm which is based on genetics algorithm, local improvement search, and the shortest augmenting path method was used to solve the model.

While the above studies can be motivated from a modeling standpoint, there are three clear and related critiques. (1) Why does the literature ignore capacity issue[1] (although uncapacitated models provide important features for SCND)? (2) Why the literature is interested on dealing with a fully disrupted DC? (3) What if disruptions were not limited to DCs and they could cut off the DCs/customers' transportation connections? These issues might be too limiting to capture preferences adequately in disruption situation, because in scenarios in which the network crashes, we would likely desire guarantees that are inherently different than those in situations in which the network performs well. One popular assumption among previous models in the literature is that they assume that a disrupted DC cannot fulfill a part of their assigned demand with its available resources. This assumption is not applicable in real world situations since each DC might fail partially and still be able to serve below its expected capacity. Therefore we consider a *capacity failure fraction* (CFF) for DCs. The CFF has not been considered in the literature so far, because DCs have been assumed to be uncapacitated as mentioned.

[1] Except Peng et al. (2011).

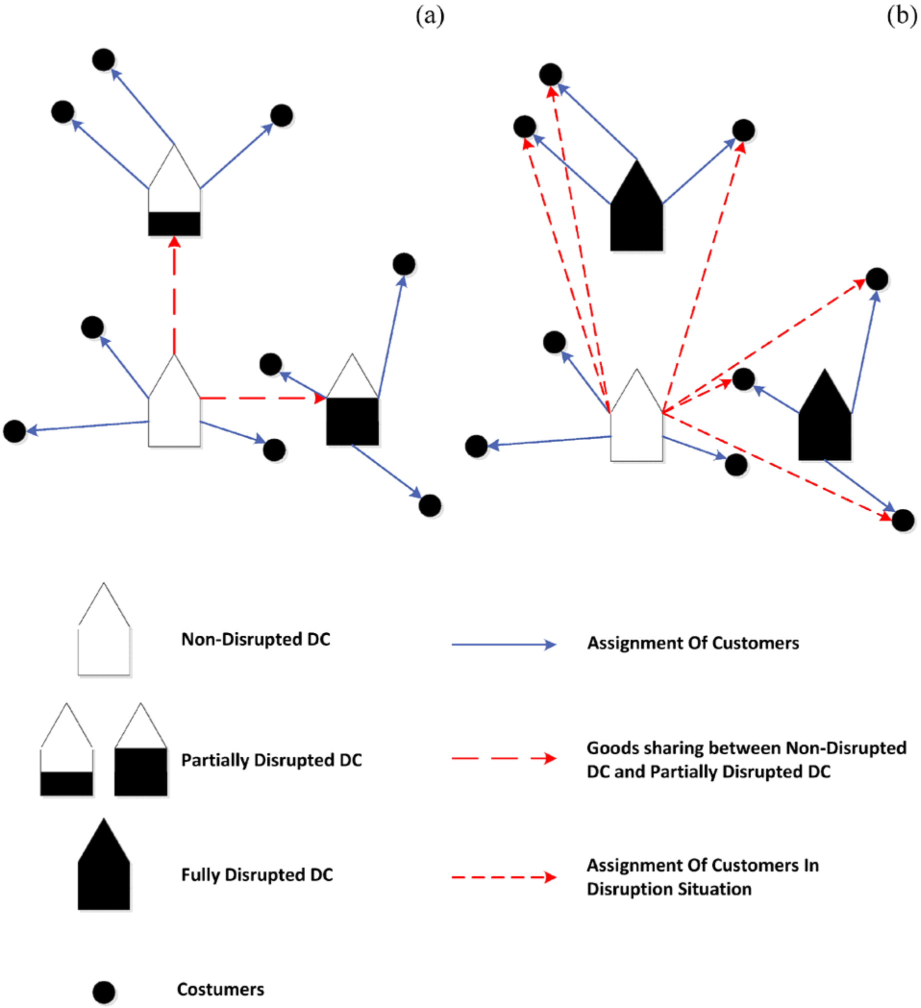

Fig. 1 SCND considering random disruption risks based on (**a**) CFF and goods sharing strategy (**b**) Re-assignment of customers strategy

In this regard, literature assume that the solution to deal with disruption is the reassignment of customers to non-disrupted DCs, although this approach does charge the network with excessive costs. In other words, by CFF we postulate that customers of a disrupted DC are not assigned to other DCs necessarily, since the capacity lost in the disrupted DC will be amended from a non-disrupted DC. We call this approach *Goods sharing* strategy. Thus, CFF can be supplied by non-disrupted DCs, and customers reassignment patterns will not be changed accordingly. Figure 1 shows the SCND considering random disruption risks based on reassignment of customers strategy (previous approaches) and *goods sharing* strategy (the proposed strategy).

Another concern of SCND is the DCs/customers connection failures during disruption event, which has been disregarded in the literature. However, discovering a way to solve this problem is important, *transportation mode*, finds how to bridge this break (see Table 1).

Table 1 Summary of characteristics in SCND under random disruption models

Paper	DC disruption	Transportation disruption	Site-dependent disruption probability	Capacitated model	Correlated disruption probability	Strategy of SCND in disruption	Formulation approach[a]	Solution approach[b]
Drezner (1987)	✓					CR	SB	He
Lee (2001)	✓					CR	SB	He
Snyder and Daskin (2005)	✓					CR	NP	LR
Snyder and Daskin (2006)	✓		✓			CR	SB	LR
Berman et al. (2007)	✓		✓			CR	NP	He
Berman et al. (2009)	✓					CR	NP	He
Cui et al. (2010)	✓		✓			CR	A	LR
Li and Ouyang (2010)	✓		✓		✓	CR	A	CA
Lim et al. (2010)	✓		✓			CR + HS	RB	LR
Peng et al. (2011)	✓		✓	✓		CR	SB	He
This paper	✓	✓	✓	✓ + CFF		G.Sh + S-HS	RB	BD

[a] SB: Scenario-based, NP: Non-linear probability, A: Approximation, RB: Reliable Backups

[b] CR: customer reassignment, HS: *hardening strategy*, S-HS: *soft-hardening strategy*, G.Sh: *Goods sharing*, He: Heuristic, LR: Lagrangian Relaxation, BD: Benders Decomposition, CA: Continuum approximation

To guard against the risks associated with potential DC failures, it is likely to construct more reliable and robust networks with decreased CFFs. One straightforward way of accomplishing this is through the use of redundant or backup serving models which Lim et al. (2010) considered recently. They proposed an approach, namely *hardening strategy*, in which fully reliable backup DCs are taken into account so as to DCs be operative during disruptions by a means of an initial financial investment in the network. In this paper we extend this view and assume that the CFF of a non-disrupted DC depends on the primary robustness investment. We call this strategy the *soft-hardening strategy.* To achieve this goal, a *soft-hardening strategic* model including investment level concept is developed in order to design more robust DCs.

The advantages of these strategies are stated in Sect. 5. In summary, we explicitly consider the following four contributions of a distribution network with the mentioned characteristics:

i. The *CFF* is presented to determine the lost amount of capacity as a result of disruption.
ii. The *goods sharing* strategy is presented to make a better use of customers' demands availability.
iii. The *transportation mode* is presented to avoid incurring excessive cost to the network in disruption event.
iv. The *soft-hardening strategy* is presented to make the network more robust by financial investment.

The formal form of our formulation is a mixed-integer linear program. In some cases, standard commercial solvers can be used to solve it directly. However, due to the large number of variables and constraints involved in this problem, direct usage of standard solvers is inefficient. On the other hand, the special structure of the problem naturally leads to consideration of a decomposition method for efficient solution schemes. In regards of these considerations, we developed an accelerated algorithm based on Benders decomposition (BD) method. To our knowledge this is the first time that BD is employed to solve the SCND under random disruption risk problems.

3 Problem description

The SCND is a traditional problem which companies are dealing with. SCND problems are concerned with determining logistics infrastructure over a long-term strategic planning period. The strategic decisions include determining the location and the capacity of DCs along with the suitable assignment of customers to them. The objective is to provide the most effective solution that minimizes total costs while providing customers with the highest possible service level. Moreover, acquiring this objective highly depends on disruption and reliability topics. To prepare a setting for our modeling framework, we consider a general supply chain network which aims to satisfy the demands of several customers. The distribution network may comprise two types of facilities[2] (i.e. *reliable* DCs, *unreliable* DCs). No restrictions are imposed on the number of different DC types and on the transportation links used by the company for shipping its goods. In other words, goods can be transported between any types of DC both for inbound link (from reliable DC to unreliable DC) and

[2] Lim et al. (2010) defined that disruptions occur at unreliable DCs, and reliable DCs are the outsourcing ones which are secured against disruption and assumed to be uncapacitated.

outbound link (from DC to customer). The reliable outbound links (reliable DCs to customers) are protected against disruption, since they have been outsourced beforehand. For unreliable outbound links (unreliable DCs to customers), two types of transportation modes are assumed: (i) *safe transportation modes*, and (ii) *unsafe transportation modes*. Disruptions occur on unsafe modes, while safe modes are protected against disruptions since they have been outsourced in advance.[3] Therefore, customers are assigned in two ways: (i) *primary assignment*, and (ii) *secondary assignment*. The former, is about the usual situation when no disruptions are present and could be either safe or unsafe. Conversely, the latter is used during the disruption situation and could be dealt with only by safe transportation mode. Naturally, cost of using safe mode is more than unsafe mode, also cost of using safe mode in the secondary assignment is more than primary assignment, since in disruption situations, emergency services will be required and therefore the cost of transportation modes increase.[4]

On the other hand, supply chain operations in such a network are mainly dedicated to distribution events with the purpose of satisfying known demands. Thus, due to the strategic nature of the problem, any description of the underlying production system in each DC is not considered.

In this paper, associated costs are explained in two categories: the *reliability costs*, which result from fixed opening/operating costs of the network and the other, are *transportation costs* specified for DCs' locations. It is clear that the fixed cost of an operating reliable DC is more than that of an unreliable one. The final objective is to find the optimal network design with minimum sum of the mentioned costs under given side constraints.

Other *strategic assumptions* are as follow:

i. *Capacity failure fraction* (CFF): The amount of the capacity of a disrupted unreliable DC. An unreliable DC does not fully fail in disruption situation and it loses only a portion of its capacity i.e. it could support some demands.
ii. *Soft-hardening strategy:* The amount of the capacity, the unreliable DC loses during disruption depends on the amount of initial investment for opening and operating, i.e. the CFF of an unreliable DC can remain robust by a means of additional investments.
iii. *Goods sharing*: The reliable DC can share its goods with unreliable DCs to compensate their lost capacity in disruption condition.

3.1 Notations

3.1.1 Indices and sets

k index of customers; $k \in K$
j Index of potential locations of DCs; $j \in J$
n Index of available investment levels for opening and operating unreliable DCs; $n \in N$
r Index of available unsafe transportation modes between DCs and customers; $r \in R$

[3]For example, consider a company which intends to supply its customers located in three different area zones. This must be planned in three particular modes: truck, rail and air in which the experience shows that many accidents (disruptions) have happened so far. Thus, this transportation strategy seems excessively risky. One proper safe strategy is to get assistance from outsources (similar to reliable DCs in Lim et al. 2010) in order to prevent the massive cost that company may be charged with; this is what we call safe transportation mode.

[4]If safe transportation mode in the primary assignment is used, there will be no need to use the secondary assignment, because the safe mode is also safe in disruption situation.

3.1.2 Parameters

D_k	Demand of customer k
fU_{jn}	Fixed cost of opening and operating unreliable DC at j with investment level n
fR_j	Fixed cost of opening and operating reliable DC at j
e_{jkr}	Transportation cost from unreliable DC at j to customer k with unsafe transportation mode r
dp_{jk}	Transportation cost in the primary assignment from unreliable DC at j to customer k with safe transportation mode
db_{jk}	Transportation cost in the secondary assignment from unreliable DC at j to customer k with safe transportation mode
dr_{jk}	Transportation cost from reliable DC at j to customer k
C_{ij}	Transportation cost from reliable DC at i to unreliable DC at j $(i \neq j)$
κ_j	Capacity of unreliable DC at j
τ_{jn}	Percentage of total capacity of disrupted unreliable DC at j opened with investment level n
q_j	Disruption probability in unreliable DC at j
π_{jkr}	Disruption probability between DC at j and customer k when the unsafe transportation mode r is used

3.1.3 Decision variables

XU_{jn}	1, if unreliable DC is opened at j with investment level n; 0, otherwise
XR_j	1, if reliable DC is opened at j; 0, otherwise
YR_{jk}	1, if customer k is assigned to reliable DC at j; 0, otherwise
YM_{jkr}	1, if customer k is assigned to unreliable DC at j with unsafe transportation mode r in the primary assignment; 0, otherwise
YS_{jk}	1, if customer k is assigned to unreliable DC at j with safe transportation mode in the primary assignment; 0, otherwise
T_{ij}	Amount of shipped goods from reliable DC at i to unreliable DC at j at disruption situation $(i \neq j)$

3.2 Problem formulation

$$\begin{aligned} \text{Min } & \sum_j \sum_n fU_{jn} XU_{jn} + \sum_j fR_j XR_j + \sum_j \sum_k dr_{jk} D_k YR_{jk} \\ & + \sum_j \sum_k \sum_r (1 - \pi_{jkr}) e_{jkr} D_k YM_{jkr} + \sum_j \sum_k dp_{jk} D_k YS_{jk} \\ & + \sum_j \sum_k \sum_r \pi_{jkr} db_{jk} D_k YM_{jkr} + \sum_i \sum_j \sum_n q_j C_{ij} XU_{jn} T_{ij} \end{aligned} \tag{1}$$

st.

$$\sum_j \left(\sum_r YM_{jkr} + YS_{jk} + YR_{jk} \right) = 1 \quad \forall k \tag{2}$$

$$\sum_j XR_j \geq 1 \tag{3}$$

$$XR_j + \sum_n XU_{jn} \leq 1 \quad \forall j \tag{4}$$

$$\sum_k D_k\left(\sum_r YM_{jkr} + YS_{jk}\right) \le \sum_n \kappa_j XU_{jn} \quad \forall j \tag{5}$$

$$YR_{jk} \le XR_j \quad \forall j, k \tag{6}$$

$$T_{ij} \le \sum_k D_k \cdot XR_i \quad \forall i, j \tag{7}$$

$$T_{ij} \le \kappa_j \sum_n XU_{jn} \quad \forall i, j \tag{8}$$

$$\sum_i T_{ij} + \kappa_j\left(\sum_n (1 - \tau_{jn}) XU_{jn}\right) \ge \sum_k D_k\left(\sum_r YM_{jkr} + YS_{jk}\right) \quad \forall j \tag{9}$$

$$XR_j, XU_{jn}, YM_{jkr}, YS_{jk}, YR_{jk} \in \{0, 1\} \quad \forall j, n, k, r \tag{10}$$

$$T_{ij} \ge 0 \quad \forall i, j \tag{11}$$

Objective function (1) aims to minimize the total fixed cost of opening DCs, cost of transportation from DCs to customers, and expected cost of disruption situation. The 1st and 2nd terms represent the fixed cost of locating unreliable and reliable DCs, respectively. The 3rd term indicates the cost of assigning customers to reliable DCs. The 4th and 5th terms state the expected cost of assigning customers to unreliable DCs in primary assignment if unsafe and safe transportation modes are used, respectively. The 6th term interprets the expected cost of assigning customers to unreliable DCs in disruption situation (secondary assignment) if unsafe mode in primary assignment is adapted. Finally the 7th term depicts the expected costs of shipping goods from reliable to unreliable DCs during disruption in unreliable DCs.

Constraint (2) ensures that each customer is assigned exactly to one DC and one transportation mode. Constraint (3) guarantees that networks cannot be designed without a reliable DC, since at least one reliable DC is required to enforce *goods sharing* strategy in the disruption situation. Constraint (4) states that reliable and unreliable DCs cannot be located at same potential node j, simultaneously. Constraint (5) ensures that the demands of customers assigned to an opened unreliable DC cannot exceed its capacity. Constraint (6) links the location and allocation variables of reliable DCs, i.e. customers cannot be assigned to a reliable DC at potential node j, unless a reliable DC is opened there. Constraint (7) with respect to *goods sharing* strategy, ensures that in a disruption situation, goods cannot be shipped from potential node i, unless a reliable DC is opened at it. Constraint (8) with respect to *goods sharing* strategy, ensures that in a disruption situation, goods cannot be shipped to potential node j, unless an unreliable DC is opened at it. Constraint (9) addresses both *goods sharing* and *soft-hardening strategies*, which states that for each unreliable DC at j, sum of the shipped goods from reliable DCs and its total available capacity after disruption, should not be lower than the total demands of its assigned customers. Constraint (10) enforces the integrality restrictions on the binary variables and finally constraint (11) enforces the non-negativity restrictions on the corresponding decision variables. Note that in constraint (4) the value of $\sum_n XU_{jn}$ is either 0 or 1, therefore when it takes value 0, then by constraint (8) the value of T_{ij} will be zero, so we can drop the factor $\sum_n XU_{jn}$ from the last term of the objective function (1), subsequently the model turns into a linear program with $\sum_i \sum_j q_j C_{ij} T_{ij}$.

4 Solution approach

Due to the large number of variables and constraints involved in the proposed model, direct usage of standard solvers is inefficient. On the other hand, Benders decomposition (BD) approach has proven to be a powerful technique for solving such problems, i.e. large scale mixed-integer problems. In this regard, we develop an accelerated BD method to solve the model. In this section, first we introduce the BD method, and then briefly review the background of the method. Finally, the proposed solution approach is presented to solve the model.

4.1 Benders decomposition algorithm

BD is an appropriate decomposition approach when the problem under study is considered as a problem with complicating variables (Conejo et al. 2006). Complicating decision variables could be variables which make the problem non-convex, such as integer decision variables and/or a group of decision variables which appear in all or most of the constraints. We briefly recall the idea of the BD algorithm (Benders 1962) considering, without loss of generality the following linear problem:

$$\begin{aligned}
&\text{Initial Problem (IP)}\\
&\text{Min}\, c^T x + d^T y\\
&\text{st.}\\
&Ax + By \le b\\
&x \in \Re^n_+,\ \ y \in Z^q_+
\end{aligned}$$

Where $c \in \Re^n$, $d \in \Re^q$, $b \in \Re^{\text{m}}$, A and B are $m \times n$ and $m \times q$ matrices, respectively. The decision variables are partitioned into two sets x and y. For a fixed y $(y = \bar{y})$, IP takes the following form:

$$\begin{aligned}
&\text{Primal Slave Problem (PSP)}\\
&f(x) = \text{Min}\, c^{\text{T}} x + d^T \bar{y}\\
&\text{st.}\\
&Ax \le b - B\bar{y}\\
&x \in \Re^n_+
\end{aligned}$$

In BD we decompose the IP into sub-problem (SP) and master problem (MP). The former is a restriction of IP and provides an upper bound (UB) in the case of minimization, while the latter is a relaxation of IP and provides a lower bound (LB):

$$\begin{aligned}
&\text{Master Problem (MP)}\\
&F(y, z) = \text{Min}\, z\\
&\text{st.}\\
&\left.\begin{aligned}
&{v^i}^T (b - By) \le 0\\
&{u^j}^T (b - By) + d^T y - z \le 0
\end{aligned}\right\}\text{Benders Cuts}\\
&z \ge 0,\ \ y \in Z^q_+
\end{aligned}$$

Where v^i is the vector that corresponds to the extreme ray i and u^j is the vector that corresponds to the extreme point j of the dual of SP. In each iteration of the BD algorithm, the SP is solved for a different value of y $(y = \bar{y})$ which is updated by the optimal solution of MP obtained in the previous iteration. Notice that in the first iteration of the algorithm an arbitrary value is given to y. In practice instead of solving SP the dual of SP is solved in each iteration which has the following form:

Dual Slave Problem (DSP)

$$f'(u) = \text{Min}\, u^T(b - B\bar{y})$$

st.

$$A^T u \geq c$$

$$u \in \Re^m_-$$

In each iteration the objective function of DSP is updated using the optimal solution of MP obtained in the previous iteration. Notice that the solution space of DSP remains the same. In each iteration the BD algorithm produces a cut called the Benders cut which is added to MP. The cut is produced from the optimal extreme point (or extreme ray) of DSP solution space. Two different types of cuts can be produced in BD algorithm:

- Case 1: If the optimal value of DSP is unbounded then the following feasibility cut is added to MP: ${v^i}^T(b - By) \leq 0$ where v^i is the vector that corresponds to extreme ray i.
- Case 2: If the optimal value of DSP is bounded and the optimality condition is not satisfied then the following optimality cut is added to MP: ${u^j}^T(b - By) + d^T y - z \leq 0$ where u^j is the vector that corresponds to extreme point j.

The convergence criterion is satisfied when the difference between the UB obtained by the best optimal solution of SP and the LB obtained by the solution of the last MP is less than or equal to the parameter ε ($UB - LB \leq \varepsilon$), where ε is 0.01 %.

In our case, the SP is essentially a minimization problem that determines the optimum values of the goods sharing variables for a fixed location and assignment of DCs and customers, respectively, and it can be stated as:

$$\text{Min} \sum_i \sum_j q_j C_{ij} T_{ij}$$

st.

$$T_{ij} \leq \sum_k D_k \overline{XR}_i$$

$$T_{ij} \leq \kappa_j \sum_n \overline{XU}_{jn} \quad \forall i, j$$

$$\sum_i T_{ij} \geq \sum_k D_k \left(\sum_r \overline{YM}_{jkr} + \overline{YS}_{jk} \right) - \kappa_j \left(\sum_n (1 - \tau_{jn}) \overline{XU}_{jn} \right) \quad \forall j$$

$$T_{ij} \geq 0 \quad \forall i, j$$

As mentioned before, the MP includes the integer variables of the original model in addition to an auxiliary continuous variable introduced to incorporate Benders cut via the solution of DSP. The MP of the proposed model can be stated as:

$$\begin{aligned} \text{Min} & \sum_j \sum_n fU_{jn} XU_{jn} + \sum_j fR_j XR_j + \sum_j \sum_k dr_{jk} D_k YR_{jk} \\ & + \sum_j \sum_k \sum_r (1 - \pi_{jkr}) e_{jkr} D_k YM_{jkr} + \sum_j \sum_k dp_{jk} D_k YS_{jk} \end{aligned}$$

$$+\sum_j\sum_k\sum_r \pi_{jkr} db_{jk} D_k Y M_{jkr} + \psi$$

st.

(2)–(6), (10)

Where ψ is a continuous variable that plays the role of continuous objective (1), in the main problem.

According to the nature of our model, the SP is always feasible for given $y = \bar{y}$. In this case the DSP's polyhedron is bounded and one may consider only the extreme points of this polyhedron. Therefore we have only optimality cuts in each iteration of BD.

4.2 Background of acceleration methods in BD

In some cases a direct application of the classical BD does not lead to fast convergence (Saharidis et al. 2009). The literature mostly debates about the BD that it has not been uniformly successful in all the applications and has some deficiencies, e.g. Magnanti and Wong (1981) observed that the straightforward implementation of BD in network design problems often converged very slowly.

In fact, the main issues associated with the slow convergence of BD are (i) the solution of MP and SP, and (ii) the quality of the BD cuts produced in each iteration. As discussed in the previous subsection, during the application of classical BD, two problems are solved in each iteration, i.e. the MP and the DSP. McDaniel and Devine (1977) and Cote and Laughton (1984) proposed a new strategy when the MP is hard to be solved optimally. McDaniel and Devine (1977) proposed the generation of cuts using the solution of the MP relaxing the integrality constraint. Cote and Laughton (1984) presented an algorithm where the MP is not solved to optimality and for the development of SP the first integer feasible solution is used. Zakeri et al. (1998) proposed the generation of inexact cuts for multi-stage stochastic linear programs based on deriving suboptimal solutions to SP, while maintaining the convergence property of the algorithm. The suboptimal solution is obtained by applying a primal-dual interior-point method.

The weak lower bounds (in case of minimization) obtained by the MP is another reason leading to slow convergence of BD algorithm. The development and introduction of valid inequalities to MP, eliminating a priori a number of infeasible solutions, is a successful strategy to speed-up BD algorithm producing stronger lower bounds. Cordeau et al. (2006) introduced a new formulation of the logistics network design problem encountered in deterministic, single-country and single-period contexts where BD algorithm is applied and a series of valid inequalities are presented as the part of the solution methodology. Cordeau et al. (2000) also used BD algorithm to solve the locomotive and car assignment problem where three sets of valid inequalities are developed based on the special characteristic of the problem and accelerated the convergence when added to MP. Andreas and Smith (2009) presented a series of valid inequalities for the sub-tour elimination in order to ensure the existence of feasible solutions, each time the SP is resolved. Saharidis et al. (2009) have developed a series of valid inequalities for the optimal scheduling of crude oil in a refinery network where BD was applied in order to solve large instances of the problem. Finally, Saharidis et al. (2011) presented a series of valid inequalities which are the first attempt to generalize, the previously developed valid inequalities to be applicable to the general fixed-charged network design problem.

The other important issue regarding the efficiency of the BD algorithm is the quality of Benders cuts. The classical BD algorithm implementation is based on the generation of cuts using the optimal solution of current DSP. BD algorithm could converge in one iteration if all Benders cuts were added to MP in the beginning of the algorithm. Therefore producing multi-generation of good quality cuts can accelerate the BD algorithm. Magnanti and Wong (1981) proposed a multi-generation of cuts procedure to accelerate the BD algorithm, using what they refer to as Pareto-optimal cuts. A cut is defined as Pareto-optimal if no other cut dominates it. Later on Papadakos (2008) proved that it is not necessary to use a core point of MP solution space in order to produce a Pareto cut improving the convergence rate of the algorithm. Rei et al. (2006) investigated how local branching can be used in order to improve both the lower and upper bounds of BD algorithm. The authors showed that how Benders feasibility cuts can be strengthened or replaced by local branching constraints. A new multi-generation of cuts strategy referred to as maximum feasible subsystem (MFS) cut generation was proposed by Saharidis and Ierapetritou (2010) based on a maximal feasibility of subsystem of Benders SP when a Benders feasibility cut is generated and optimal Benders cuts are difficult to be achieved within the iterations of the algorithm. Sherali and Lunday (2011) proposed the generation of maximal non-dominated cuts utilizing a preemptively small perturbation of the right-hand-side of the Benders SP, as well as a complimentary strategy that generates an additional cut in each iteration via an alternative emphasis on decision variable weights. Recently, Saharidis et al. (2010) presented a new method referred to as covering cut bundle (CCB) generation which implements in a novel way of multiple constraints generation idea.

4.3 Proposed solution method

As discussed in the previous subsection, one of the important issues regarding the efficiency of BD algorithm is the quality of Benders cuts. The question is what characteristics a cut should possess in order to have faster convergence of the algorithm. One of the main reasons that make BD algorithm convergence slow is the form of the produced Benders cuts. If it was possible to define a priori the extreme points and extreme rays of the solution space of DSP, that correspond to active constraints in the optimal solution, then we could produce all these cuts simultaneously, and add them to MP and solve the augmented MP only once. The solution of this MP would be the optimal solution of IP. Thus the goal must be producing additional "good" cuts which can help the convergence of the algorithm. A good cut is defined as a cut which significantly restricts the solution space of MP.

As previously noted, we have the possibility from the first iteration of the BD algorithm to produce all the possible cuts, because the solution space of the DSP is not affected by the solution of MP. Using all extreme points and rays, we obtain an equivalent version of IP which is likely more complicated to be solved optimally. A suitable method in order to converge to optimal solution faster than the classical BD algorithm is to maintain a tradeoff between the number of iterations and the number of cuts produced in each iteration. This balance is based on the idea that increasing the number of cuts decreases the number of iterations, but MP becomes more complicated to be solved optimally and extra time is needed to generate the additional cuts.

The CCB method is based on this idea (Saharidis et al. 2010). Its main objective is to cover decision variables of MP by generating additional bundle of cuts in each iteration. In this paper, we present a method referred to as maximum density cut (MDC) generation which is based on the idea that it is computationally expensive to cover all decision variables in the bundle of cuts and for this reason we suggest the generation of a cuts bundle where a certain number of decision variables are covered and the rest are covered by MDC method. MDC generation also can be used as a standalone procedure especially in the case where the solution of SP is computationally expensive. In a combined CCB and MDC method, we observe that the feasible solution space of the MP is more restricted than CCB method, and the algorithm converges with a faster rate to the optimal solution (the comparison is presented in Sect. 5). In the following, we describe the CCB and MDC methods.

4.3.1 Covering cut bundle generation

Saharidis et al. (2010) proposed the CCB method which is based on the multiple constraints generation idea. In CCB method, in each iteration the decision variables of MP which are not "α-covered" by the classical BD cut are determined. Next, an additional cut to cover at least one of the non-covered variables is generated, and analyzed to update the set of non-covered variables, and the procedure continues until a predefined number of cuts are generated. The definition of "α-covered" and "α-covering cut bundle" are explained as follows:

Definition 1 In an optimality cut,[5] a variable y_k is said to be α-covered in the cut of the form $\sum_k (u^T B)_k y_k \geq u^T b + d^T y - z$ if the kth row of the matrix $u^T B$ is greater than or equal to α percent of the coefficient with the maximum absolute value in the current cut: $|(u^T B)_k| \geq 10^{-2}\alpha \, \mathrm{Max}_{\forall k}\{|(u^T B)_k|\}$, where α is a given parameter chosen in $[0, 100]$.

Definition 2 We call α-covering cut bundle, a set of (optimality or feasibility) cuts such that each variable y_k, is α-covered in one or some cuts of the bundle.

The CCB generation proceeds by generating a bundle of cuts instead of a single cut in each iteration. This bundle of cuts is generated by an auxiliary problem which is based on Benders SP developed in current iteration. The produced bundle of cuts is intended to involve decision variables of MP. Due to this feature, one can expect that the solution space of MP is significantly restricted towards all its direction due to the addition of this bundle of cuts, and the algorithm converges faster to an optimal solution.

To generate an α-covering cut bundle at each iteration of classic BD algorithm given the current optimal solution $(\bar{y})$ of the MP, initially we solve the corresponding SP and we produce the following cut:

$$u^{j^T}(b - By) + d^T y - z \leq 0$$

Where the extreme point u is the current optimal dual solution. The coefficient of y_j in the produced cut is equal to $(u^T B)_j$. In order to generate the α-covering cut bundle, we will consider bounds inducing constraints on u and add them to DSP. These constraints have the following form:

$$LB_j \leq \left(u^T B\right)_j \leq UB_j$$

[5]The CCB method which is presented in this subsection is for the case of optimally cut, for feasibility cut refer to Saharidis et al. (2010).

where the parameter LB_j is the lower bound on the coefficient of the variable decision y_j and UB_j is the upper bound on this coefficient. After including the above additional constraints in DSP, we obtain the following Auxiliary Dual Problem (ADP):

$$\text{ADP:}\quad \begin{cases} \Gamma = \text{Min}\, u^{\text{T}}(b - B\overline{y}) \\ \text{st.} \\ A^T u \geq c \\ -(u^T B)_j \geq UB_j \\ (u^T B)_j \geq LB_j \\ u \leq 0 \end{cases}$$

As shown below, this problem is solved for different values of LB_j and UB_j in order to cover the decision variable y_j of MP. Introducing two new sets of variables λ_j and μ_j for the additional inequalities, the corresponding auxiliary primal problem (APP) which defines an extreme point covering y_j takes the following form:

$$\text{APP:}\quad \begin{cases} \Gamma' = \text{Max}\, \text{c}^{\text{T}}x - \sum_j UB_j\lambda_j + \sum_j LB_j\mu_j \\ \text{st.} \\ Ax - \sum_j B^j\lambda_j + \sum_j B^j\mu_j \leq b - B\bar{y} \\ x, \lambda, \mu, \geq 0 \end{cases}$$

In order to generate a cut where a decision variable y_{j_0} of MP is α-covered, we setup the current APP, update the right hand side of APP using the current optimal solution $\bar{y}$ of the MP, and fix for $j = j_0$, the coefficients of λ_j and μ_j in the objective function: $LB_j = UB_j = +\eta(\text{or} - \eta)$. After solving the APP, we generate a cut which has exactly the same form as the optimal Benders cut using the optimal value of dual decision variables. The parameter η is the average of the coefficients of α-covered decision variables in the classical Benders cut and in each iteration takes the following value:

$$\eta = (1/k)\sum_j \left|\left(u^T B\right)_j\right|, \quad j \in \left\{j\colon \left|\left(u^T B\right)_j\right| \geq 10^{-2}\alpha \operatorname*{Max}_{\forall j}\left\{\left|\left(u^T B\right)_j\right|\right\}\right\}$$

Notice that k represents the total number of α-covered decision variables in the classical Benders cut: $k = \sum_j j,\ j \in \{j\colon |(u^T B)_j| \geq 10^{-2}\alpha\,\text{Max}_{\forall j}\{|(u^T B)_j|\}\}$. Fixing the coefficients $LB_j = UB_j = +\eta(\text{or} - \eta)$, for $j = j_0$ of APP's objective function, to a non-zero value equal to $+\eta$ (or $-\eta$), we make sure that at least the decision variable y_{j_0} will be α-covered in the next cut generated. Note that this procedure does not restrict the values of the other coefficients.

In the general form of the CCB method the values of LB_j and UB_j in APP are:

$$LB_j = -\frac{\eta}{\alpha} \quad \text{and} \quad UB_j = \frac{\eta}{\alpha}, \forall j \neq j_0 \tag{12}$$

$$LB_{j_0} = UB_{j_0} = +\eta \quad \text{or} \quad -\eta \tag{13}$$

The sign of the parameter η depends on the value of the dual variable of SP($\bar{y}$) and the corresponding column of matrix B (e.g. B^j):

- If $u_j B^j \geq 0$ then $LB_j = UB_j = +\eta$ and if $u_j B^j \leq 0$ then $LB_j = UB_j = -\eta$ for $j = j_0$
- If $u_j B^j \geq 0$ then $LB_j = 0$ and $UB_j = \frac{\eta}{\alpha}$, if $u_j B^j \leq 0$ then $LB_j = -\frac{\eta}{\alpha}$ and $UB_j = 0$ for $\forall j \neq j_0$

In CCB method, we produce not only one extra cut by APP but a number of cuts (α-covering cut bundle) where we guarantee that a number of variables are α-covered. In each solution of APP, the parameters LB_j and UB_j are changed and fixed to a certain value for the generation of a new cut. Before resolving the APP the cut produced is added to MP and for another not yet α-covered variable $y_{j_0'}$, the parameters $LB_{j_0'}$ and $UB_{j_0'}$ are fixed equal to $+\eta$ (or $-\eta$). At the same time the bounds LB_{j_0} and UB_{j_0} of the coefficient of the variable y_{j_0} are re-initialized. A second cut is produced and the iterations continue.

4.3.2 Maximum density cut generation

The basic idea explored in this subsection is the generation of a cut in each iteration where the maximum numbers of non-covered variables of MP are covered. In this case, the generated cut restricts significantly the solution space of MP. By maximizing the number of dual variables with non-zero values, we guarantee that the resulting cut has the maximum number of non-zero coefficients, covering maximum number MP decision variables.

Without loss of generality we consider a bounded DSP with $y = \bar{y}'$. In order to find the extreme point of DSP's solution space, where the maximum of number of decision variables take a non-zero value, two groups of decision variables and constraints are added. Revising also the objective function of DSP, we obtain the following Auxiliary DSP (ADSP):

Auxiliary Dual Slave Problem (ADSP)

$$fc = \text{Min} \sum_{\text{m}} \left(k_m^1 + k_m^2\right)$$

st.

$$A^T u \geq c$$

$$-\eta\left(1 - k_m^1\right) - \text{M}k_m^1 \leq u_m \sum_q B_{m,q} \leq -\eta\left(1 - k_m^1\right) + \text{M}k_m^1$$

$$\eta\left(1 - k_m^2\right) - \text{M}k_m^2 \leq u_m \sum_q B_{m,q} \leq \eta\left(1 - k_m^2\right) + \text{M}k_m^2$$

$$u \in \Re_-^m,\ k_m^1, k_m^2 \in Z^m = \{0, 1\}^m$$

Where A is $m \times n$ matrix, B is $m \times q$ matrix, u is an m-vector, k_m^1, k_m^2 are m-vectors and η is a positive number defined by the last generated Benders cut as in the CCB method. We solve the ADSP to optimality and we produce a cut in the same way as in the classic BD. This cut has the maximum number of non-zero coefficients giving rise to a cut with the highest density (i.e. a large number of decision variables of MP are covered). It should be noted that the form of ADSP does not change the structure that Benders SP may have (i.e. blocked decomposed), because the introduced auxiliary integer variables do not link the problem constraints. Finally, a modified version of ADSP could be used in order to speed-up the generation of MDC. The integrality constraint of k_m^1, k_m^2 can be relaxed and an additional constraint can be added resulting a continuous linear problem:

Relaxed-Auxiliary Dual Slave Problem (RADSP)

$$fc = \text{Min} \sum_{\text{m}} \left(k_m^1 + k_m^2\right)$$

st.

$$A^T u \geq c$$

$$
\begin{aligned}
&-\eta\left(1-k_m^1\right)-\mathrm{M}k_m^1 \leq u_m \sum_q B_{m,q} \leq -\eta\left(1-k_m^1\right)+\mathrm{M}k_m^1\\
&\eta\left(1-k_m^2\right)-\mathrm{M}k_m^2 \leq u_m \sum_q B_{m,q} \leq \eta\left(1-k_m^2\right)+\mathrm{M}k_m^2\\
&k_m^1+k_m^2 \leq 1\\
&u \in \Re_-^m,\ k_m^1, k_m^2 \in (0,1)^m
\end{aligned}
$$

The two constraints of RADSP which include η, are defined for the non-covered decision variables of MP, and the α-covered decision variables, are not included in these constraints. In this case the decision variables which are not α-covered are bounded with these constraints and we expect that these decision variables will be covered. In the context of this paper and for the computational results presented in Sect. 5, we use the RADSP to address the above procedure.

4.3.3 Combination of CCB and MDC methods

The role of MDC generation implemented after CCB method is to cover the rest of the decision variables which are not covered by the classical Benders cut and the bundle of cut produced by CCB generation.

In each iteration of BD, if the classical BD cut has low density then we run CCB and MDC. A low density cut is defined as follows:

Definition 3 Low density cut is a cut which includes less than b % of α-covered MP decision variables.

In each iteration, if the classical Benders cut is a low density cut, then the following steps are executed.

Step 1: We run the CCB generation and then add the bundle of cuts to MP.
Step 2: The classical Benders cut and CCB generation are checked to find which decision variables of MP are still not α-covered.
Step 3: We build the RADSP and then run it to cover the maximum decision variables found in Step 2. Finally we add the MDC cut to MP.

The flowchart of CCB and MDC methods combination for the case of optimally cut is shown in Fig. 2.

MDC generation can also be used as a standalone procedure especially in the case where solving SP is computationally expensive. In this case, in each iteration of the algorithm after solving the SP, the generated Benders cut is examined. If this cut is a low density cut then the classical Benders cut is checked to find which decision variables of MP are not α-covered. Finally the corresponding RADSP is solved, and the MDC cut is added to MP. After the solution of MP the optimality criterion is examined. If it is not satisfied then the SP is updated by the optimal solution of MP and the algorithm continues.

5 Computational result

We performed series of numerical experiments to evaluate the performance of our proposed solution method and model. We coded the algorithm in C++ and used CPLEX as the solver,

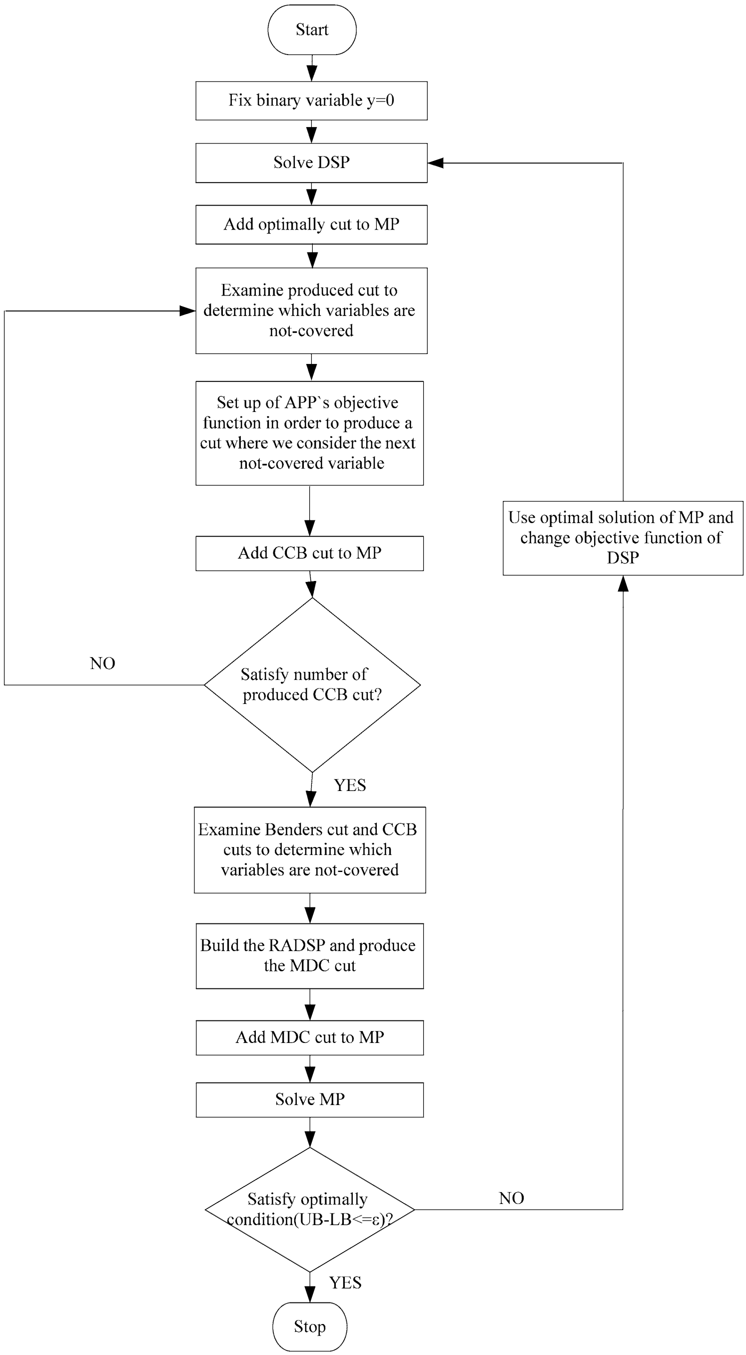

Fig. 2 The flowchart of combination CCB and MDC methods

executed it on a computer with 2.4 GH processor and 2 GB of RAM, operating under Microsoft Windows Vista. We benchmark our results using the branch-and-bound algorithm in CPLEX 11.0, which we ran on the same hardware. Computation run times are reported in seconds.

We generated 30 random datasets of different problem sizes with the number of customers (hereafter *size*) ranging from 10 to 150, the number of DCs ranging from 2 to 22 and the number of considered investment levels and transportation modes are 3 and 5.

The customers' demands $\sim U[100, 250]$. The DC disruption probability $(q_j) \sim U[0.025, 0.15]$ and is assumed to occur independently. To obtain the fixed cost of reliable and unreliable DCs, a strategy similar to Lim et al. (2010) is used. When we use three investment levels for opening unreliable DCs, the fixed cost of unreliable DCs are computed as follows: $fU_{j1} \sim U[2000, 3000]$, also $fU_{j2} - fU_{j1} = 4000q_j$ and $fU_{j3} - fU_{j1} = 10000q_j$. Thus, DCs facing higher disruption probabilities will be more costly to harden. In this setting, the hardening cost from level 1 to level 2 is roughly 10 % of the fixed cost of the unreliable DC (i.e. fixed cost of opening DC in level 1) and the hardening cost from level 1 to 3 is approximately 25 % of the fixed cost of the unreliable DC. Also in this case (three investment levels) corresponding CFFs are computed as follows: $\tau_{j1} \sim U[0.2, 0.6]$, $\tau_{j2} = 0.75\tau_{j1}$ and $\tau_{j3} = 0.5\tau_{j1}$. In case of using five investment levels for opening DCs, the fixed cost of unreliable DCs are calculated as follows: $fU_{j1} \sim U[2000, 3000]$, also $fU_{j2} - fU_{j1} = 2000q_j$, $fU_{j3} - fU_{j1} = 4000q_j$, $fU_{j4} - fU_{j1} = 7000q_j$ and $fU_{j5} - fU_{j1} = 10000q_j$. In this case (five investment levels) corresponding CFFs assumed to take the values as follows: $\tau_{j1} \sim U[0.2, 0.6]$, $\tau_{j2} = 0.875\tau_{j1}$, $\tau_{j3} = 0.75\tau_{j1}$, $\tau_{j4} = 0.625\tau_{j1}$ and $\tau_{j5} = 0.5\tau_{j1}$. To obtain the opening cost of reliable DCs, the difference of fixed costs between reliable and unreliable DC for each DC is determined as a linear function of disruption probability; i.e., $fR_j - (\sum_n fU_{jn}/|N|) = 23000q_j$. In this setting, the cost of using a reliable DC instead of an unreliable one is estimated roughly 50 % more than the average fixed cost of opening an unreliable DC. The safe transportation cost in the primary assignment $(dp_{jk}) \sim U[30, 35]$ and the safe transportation cost in the secondary assignment is set to $db_{jk} = 1.25dp_{jk}$. We consider three or five unsafe transportation modes between customers and DCs in different sizes. The unsafe transportation mode disruption probability $(\pi_{jkr}) \sim U[0.04, 0.20]$.

5.1 Algorithm performance

In this subsection, we investigate the performance of the proposed solution method (CCB-MDC method). First, the proposed solution method is compared with CPLEX and BD, and then it is compared with CCB and MDC standalone methods separately. In Table 2, we describe the sizes of solved instances.

5.1.1 Comparison between CPLEX, BD and CBB-MDC methods

In this subsection we present results based on instances of Table 2, and show that the use of CCB-MDC method significantly improves the efficiency of CPLEX and BD algorithm.

In order to accelerate BD algorithm the priority is to reduce the total number of iterations the MP problems are solved. A decrease in the number of main iterations usually results in a reduction of the resolution time. Resolution time consists of time the algorithm spends in order to solve the MP and a series of APP problems in CCB method and RADSP problems in MDC methods. Reducing the number of iterations results in a significant reduction due to a smaller number of MPs that have to be solved. However, significant amount of time is required to generate the additional cuts. Therefore, in order to reduce the total resolution

Table 2 Size of instances

No.	$\|K\|$	$\|J\|$	$\|N\|$	$\|R\|$	# Int. var.	# Cont. var.	# Total var.	# Cons.
P1	10	2	3	3	108	4	112	45
P2	10	2	5	5	152	4	156	45
P3	15	2	3	3	158	4	162	60
P4	15	2	5	5	222	4	226	60
P5	20	3	3	3	312	9	321	108
P6	20	3	5	5	438	9	447	108
P7	20	4	3	3	416	16	432	145
P8	20	4	5	5	584	16	600	145
P9	30	5	3	3	770	25	795	246
P10	30	5	5	5	1080	25	1105	246
P11	30	6	3	3	924	36	960	301
P12	30	6	5	5	1296	36	1332	301
P13	40	6	3	3	1224	36	1260	371
P14	40	6	5	5	1716	36	1752	371
P15	40	8	3	3	1632	64	1696	513
P16	40	8	5	5	2288	64	2352	513
P17	50	8	3	3	2032	64	2096	603
P18	50	8	5	5	2848	64	2912	603
P19	60	10	3	3	3040	100	3140	891
P20	60	10	5	5	4260	100	4360	891
P21	70	12	3	3	4248	144	4392	1235
P22	70	12	5	5	5952	144	6096	1235
P23	80	14	3	3	5656	196	5852	1635
P24	80	14	5	5	7924	196	8120	1635
P25	100	17	3	3	8568	289	8857	2430
P26	100	17	5	5	12002	289	12291	2430
P27	120	19	3	3	11476	361	11837	3180
P28	120	19	5	5	16074	361	16435	3180
P29	150	22	3	3	16588	484	17072	4485
P30	150	22	5	5	23232	484	23716	4485

time, the time reduction achieved by the reduction of iterations must be greater than the time spent to produce the additional cuts.

In Table 3, we compare the CCB-MDC method with CPLEX and classical BD algorithm. This table shows the optimal cost of each instance. For the classical BD and CCB-MDC methods, the run time (in seconds), as well as the total number of iterations to solve the problem optimally, are reported. Also the average density of cuts produced by the classical BD algorithm is shown, and finally in the last two columns of table, the relative difference between classical BD and CCB-MDC methods are gathered. The algorithm is terminated for each instance if it failed to attain an 0.01 %–optimal solution within 18000 CPU seconds, and Table 3 records such problems with '*' instead of the CPU time. As we observe, the CCB-MDC method outperforms the CPLEX and classical BD approaches and it solves all of the instances to optimality with the tolerance of 0.01 %.

Table 3 Comparison between CPLEX, classical BD and CCB-MDC methods

No.	Cost	CPLEX	BD			CCB-MDC		Relative difference between CCB-MDC and BD (%)	
		CPU	CPU	# Iter.	Average density of cut (%)	CPU	# Iter.	CPU	# Iter.
P1	47117	0.02	0.04	2	48.86	0.05	2	−25	0
P2	46152	0.02	0.04	3	49.24	0.04	3	0	0
P3	66098	0.06	0.1	3	24.61	0.08	2	20	33
P4	63161	0.09	0.06	2	49.48	0.06	2	0	0
P5	82988	0.1	0.14	2	65.87	0.14	2	0	0
P6	81598	0.19	0.31	4	55.29	0.32	4	−3	0
P7	81131	0.3	0.61	7	29.17	0.53	6	13	14
P8	79832	0.56	0.39	4	33.2	0.35	3	10	25
P9	128155	5.06	16.47	17	41.41	8.99	10	45	41
P10	124216	6.83	4.67	11	58.02	3.9	8	16	27
P11	126849	8.71	9.81	13	41.76	7.29	9	26	31
P12	123650	10.95	5.59	11	41.75	4.33	7	23	36
P13	165852	11.26	5.31	10	44.53	4.01	7	24	30
P14	161829	13.48	4.59	9	50.05	4.11	7	10	22
P15	166953	18.39	30.05	17	47.01	18.37	11	39	35
P16	163699	51.49	53.2	25	42.81	26.17	15	51	40
P17	201151	167.29	23.37	13	43.85	15.32	8	34	38
P18	197462	429.37	96.42	34	42.52	52.02	18	46	47
P19	232298	1570.83	131.55	26	43.34	89.87	18	32	31
P20	226836	5248.95	67.23	19	41.21	48.02	14	29	26
P21	273790	*	348.28	34	42.8	139.73	15	60	56
P22	265939	*	684.45	53	37.92	241.12	21	65	60
P23	308689	*	3692.67	99	41.9	1512.68	37	59	63
P24	301915	*	789.25	39	42.57	428.32	20	46	49
P25	379226	*	11952.34	122	45.11	3899.48	36	67	70
P26	371966	*	14311.7	138	44.48	4386.67	45	69	67
P27	414243	*	*	–	–	5683.18	50	–	–
P28	436809	*	*	–	–	7686.72	60	–	–
P29	500118	*	*	–	–	9463.49	60	–	–
P30	518783	*	*	–	–	12224.19	75	–	–

* Algorithm terminated at 18000 seconds without attaining optimal solution

This implies that by applying CCB-MDC method, the produced cut involves the maximum number of decision variables of MP and thus it improves the convergence of BD algorithm. Also CCB-MDC method requires relatively less computational effort to solve the instances.

Moreover, when the size increases, the relative difference between CCB-MDC and BD methods grows (e.g. see instances P25 and P26), since BD method exhibits a great average computational effort, primarily due to its poor performance on the instances.

Table 4 Comparison between CCB, MDC and CCB-MDC methods

No.	CCB		MDC		CCB-MDC		Relative difference between CCB-MDC and CCB (%)		Relative difference between CCB-MDC and MDC (%)	
	CPU	# Iter.	CPU	# Iter.	CPU	# Iter.	CPU	# Iter.	CPU	# Iter.
P1	0.05	2	0.05	2	0.05	2	0	0	0	0
P2	0.04	3	0.04	3	0.04	3	0	0	0	0
P3	0.08	3	0.08	2	0.08	2	0	33	0	0
P4	0.06	2	0.06	2	0.06	2	0	0	0	0
P5	0.14	2	0.14	2	0.14	2	0	0	0	0
P6	0.32	4	0.32	4	0.32	4	0	0	0	0
P7	0.53	6	0.52	6	0.53	6	0	0	−2	0
P8	0.35	3	0.33	3	0.35	3	0	0	−6	0
P9	10.65	12	11.08	14	8.99	10	16	17	19	29
P10	3.59	8	3.98	10	3.9	8	−9	0	2	20
P11	7.89	10	8.29	11	7.29	9	8	10	12	18
P12	4.61	8	5.16	10	4.33	7	6	13	16	30
P13	4.33	8	4.58	9	4.01	7	7	13	12	22
P14	4.31	8	4.24	8	4.11	7	5	13	3	13
P15	22.05	13	22.44	14	18.37	11	17	15	18	21
P16	36.02	18	38.58	21	26.17	15	27	17	32	29
P17	18.84	9	19.47	11	15.32	8	19	11	21	27
P18	70.43	23	74.74	26	52.02	18	26	22	30	31
P19	108.44	21	112.75	22	89.87	18	17	14	20	18
P20	53.19	15	58.92	17	48.02	14	10	7	18	18
P21	197.49	19	228.81	23	139.73	15	29	21	39	35
P22	363.98	28	418.24	37	241.12	21	34	25	42	43
P23	1971.76	47	2243.81	58	1512.68	37	23	21	33	36
P24	576.29	24	625.19	29	428.32	20	26	17	31	31
P25	5148.27	47	5987.53	60	3899.48	36	24	23	35	40
P26	5548.52	55	6635.66	73	4386.67	45	21	18	34	38
P27	7066.49	61	7873.18	72	5683.18	50	20	18	28	31
P28	9553.85	76	11515.24	87	7686.72	60	20	21	33	31
P29	12084.47	76	13359.51	90	9463.49	60	22	21	29	33
P30	15291.34	93	16824.79	108	12224.19	75	20	19	27	31

This important reduction in solution times, justifies that for BD method the generation of low density cuts is a significant drawback. Therefore regardless of the method applied to cover a maximum number of MP decision variables, the improvement would be significant.

5.1.2 Comparison between CCB, MDC and CCB-MDC methods

In this subsection, to validate the CCB-MDC method, we compare it with the CCB and MDC methods individually and the results are shown in Table 4. In this table, we display the total run time and the number of iteration to achieve the optimal solution in these methods. Also the relative differences between CCB-MDC method and the other two approaches are reported.

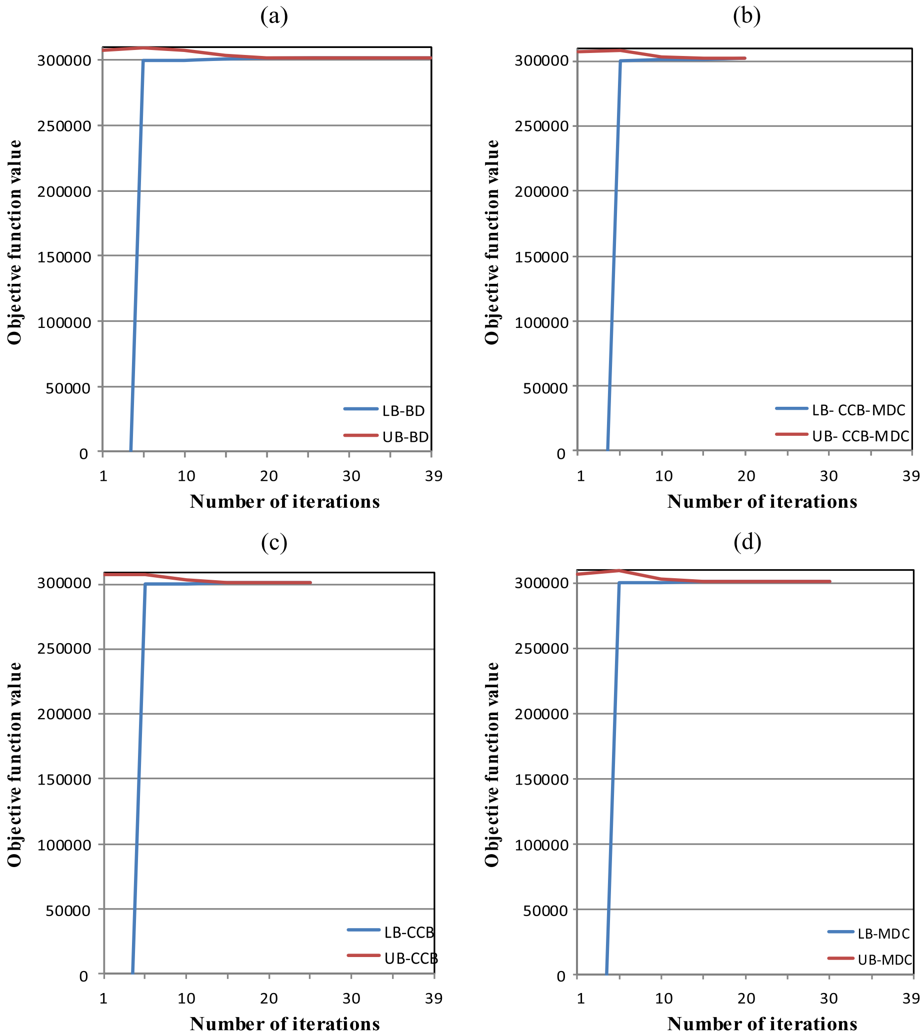

Fig. 3 Behavior of lower bound and upper bound of objective function at size 80 (**a**) classical BD method (**b**) CCB-MDC method (**c**) CCB method (**d**) MDC method

As the size increases, the CCB-MDC method outperforms the other approaches significantly both in the run time and the number of iterations. This indicates that the combination of the two methods significantly restricts the solution space of MP and yields a quick convergence compared to when each method is utilized alone.

Also Table 4 shows that the performance of CCB method is better than MDC method. In fact, our numerical results show that in general it is better to cover the decision variable of MP by producing a bundle of low density cuts (as CCB method does) than only a high density cut (as MDC method does). Considering that the time spent for the generation of the bundle of low density cuts is less than the time spent for the generation of the high density cut.

Figure 3 shows the behavior of the lower and upper bounds of the objective function at size 80 for classical BD, CCB, MDC and CCB-MDC methods. Obviously the upper bounds decrease and the lower bounds increase as the number of iterations increases. However, as the classical BD algorithm, CCB and MDC methods require respectively 39, 24, and 29 iterations to reach the optimality tolerance, the CCB-MDC method requires 20 iterations. The results in Fig. 3 clearly show that in this case the CCB-MDC method converges much faster does the other mentioned methods.

Table 5 verifies our attempt to improve the CCB-MDC method. An improvement of the proposed solution approach could be aiming to cover more than one MP decision variable in each APP, thus reducing the consumed time to make the whole bundle. By considering decision variables of MP that are not α-covered by the classical BD cut in an iteration, we generate the next cut covering for example 2 or 5 non-covered variables, update the set of remaining non-covered variables and continue this procedure until a predetermined maximum number of cuts in a bundle attained. This will be provided by fixing 2 or 5 of *LBj*'s and *UBj*'s equal to η (or $-\eta$), instead of 1, each time APP is solved. The α-covering cut bundle is generated by the same approach discussed in the Sect. 4, but in the improved approach instead of producing say 100 cuts each aim to cover a variable (as in original CCB approach), we produce 50 cuts each aim to cover 2 variables at the same time, or 20 cuts each aim to cover 5 decision variables.

Saharidis et al. (2010) investigated that a bundle of low-density cuts is more desirable for the acceleration of the algorithm than a cut corresponding to the sum of these low-density cuts (which creates a high-density cut), we further examine our comparison with the case that in CCB strategy only one additional cut is used where all predetermined maximum number of non-covered variables of MP (e.g. 100 variables in the above example) are covered in it (see Table 5).

5.2 Benefit of considering random disruption risk

In this subsection, we study the benefits of considering random disruption risks in SCND. For this purpose, we attempt to consider the problem from two perspectives: (1) when there is no consideration of disruption in DCs and transportation modes in the model. In this case we consider the model stated in Sect. 3.2 with the probabilities q_j and π_{jkr} set to 0 and also the variable T_{ij} and constraints 7–9 are removed from the model. (2) When disruption is considered in the modeling with its corresponding probabilities and the proposed model in Sect. 3.2 is considered. We refer to the cost of networks designed without disruption in both cases as *nominal cost*. In fact nominal cost in the first case is equal to the objective value of the optimal solution and in the second case after solving the problem, we set q_j and π_{jkr} to 0 in the objective function and save the remaining costs as its nominal cost.

In both cases first the corresponding problems are solved and nominal cost is saved. Then we assume a scenario takes place in the model (i.e. disruption is occurred at some DCs as well as transportation links), in this case we calculate the disruption cost of both cases as follows: The capacity losses in disrupted DCs will be provided from their nearest reliable opened DCs and the disrupted transportation modes are outsourced with the safe transportation. Total cost incurred in the network in each model is considered as the disruption cost.

There are numerous disruption scenarios which could take place in the network in fact if $|J|$ and $|K|$ denote the number of DCs and customers respectively, then disruption scenarios in DCs and transportation links would be $2^{|J|}$ and $2^{|J|\times|K|}$, respectively. Therefore in order to compare both cases efficiently, 500 random disruptions scenarios were generated by the Monte Carlo procedure. It should be noted that this evaluation technique is related to what is called "*in-sample stability*" in stochastic programming's tradition (for further information, please refer to Kaut et al. 2007).

Table 5 Comparisons of types of CCB-MDC methods

No.	CCB-MDC2		CCB-MDC5		CCB-MDC (one cut)		Relative difference between CCB-MDC1 and CCB-MDC2 (%)		Relative difference between CCB-MDC1 and CCB-MDC5 (%)		Relative difference between CCB-MDC1 and CCB-MDC (one cut) (%)	
	CPU	#Iter.	CPU	#Iter.	CPU	#Iter.	CPU	# Iter.	CPU	#Iter.	CPU	#Iter.
P2	0.04	3	0.04	3	0.04	3	0	0	0	0	0	0
P4	0.06	2	0.06	2	0.06	2	0	0	0	0	0	0
P8	0.35	3	0.35	3	0.35	3	0	0	0	0	0	0
P12	4.33	7	4.33	7	5.01	8	0	0	0	0	14	13
P16	24.71	14	24.69	14	29.58	17	6	7	6	7	12	12
P18	49.48	17	49.42	17	57.43	20	5	6	5	6	9	10
P20	47.64	14	47.41	14	51.47	15	1	0	1	0	7	7
P22	229.07	20	227.15	20	256.58	22	5	5	6	5	6	5
P24	413.18	19	408.54	19	464.71	22	4	5	5	5	8	9
P26	4297.16	44	4242.39	43	4695.82	48	2	2	3	4	7	6
P28	7484.53	59	7352.47	58	8376.59	65	3	2	4	3	8	8
P30	11893.72	73	11715.09	72	13597.18	82	3	3	4	4	10	9

CCB-MDC1: CCB-MDC method where at least one variable of MP is covered in each cut of bundle (proposed solution approach)

CCB-MDC2: CCB-MDC method where at least two variables of MP are covered in each cut of bundle

CCB-MDC5: CCB-MDC method where at least five variables of MP are covered in each cut of bundle

CCB-MDC (one cut): CCB-MDC method where all predetermined maximum number of variables of MP are covered in only one cut

Table 6 presents both cases. The following table presents nominal costs of both models as well as the percentage of increase in nominal cost in the second case (designed network considering disruptions) compared to the first case (designed network without considering disruptions). Also as this table uses 500 scenarios[6] to present the mentioned information, the expected as well as minimum and maximum reduction of disruption cost in the second case compared to the first case in all scenarios is collected. It can be explicitly observed that the reliability of the network has improved when the proposed model is implemented. From Table 6, it can be observed that the reliability of the network has notably improved whereas the nominal cost does not have a trivial change. For example, at size 60 with 7.3 % increase in nominal cost, an expected decrease of 25.2 % in disruption cost appears, which denotes that the reliability of network grows by a slight improvement of facility cost.

Figure 4 displays that as the size raises, the existing gap between the increase in nominal cost unlike the decrease in disruption cost grows. At size 10, for instance, the gap between the increase in nominal cost and expected decrease in disruption cost becomes 3.8 % whereas for size 150 it shows a significant increase plotting to nearly 21.7 %.

5.3 Benefits of considering soft-hardening strategy

In this subsection, we compare the advantage of considering *soft-hardening strategy*, with *hardening-strategy* presented by Lim et al. (2010). They also elaborated two types of DCs: reliable and unreliable, and assumed that by an investment a fully reliable DC is emerged and the partial DC disruptions in unreliable DCs are not considered.

Figure 5 shows the comparison of the costs of both strategies. To apply the *hardening strategy*, we omit the investment level index (n) from the model, therefore the capacity lost in an unreliable DC j, and fixed cost of opening and operating of Lim et al.'s model are computed from the capacity lost and fixed cost of opening of all levels of the *soft-hardening strategy* as follows:

$$\tau_j = \frac{\sum_n \tau_{jn}}{|N|} \quad \text{and} \quad fU_j = \frac{\sum_n fU_{jn}}{|N|}$$

From Fig. 5, we see that the expected cost of the model based on the *soft-hardening strategy* is lower than that of *hardening strategy*. The percentages of improvements are 9 % at size 30, 11 % for 50 customers, 14 % for 80 customers, 17 % for 100 customers and 21 % for 120 customers. In fact, in *soft-hardening strategy*, partial DC disruption considered in unreliable DCs, leads to amend the network to be more robust and flexible in a disruption situation.

We further study how changes in the critical parameters such as the DC disruption probability affects the relative difference between the costs of our model based on *soft-hardening strategy* and *hardening strategy*. For simplicity, we suppose an identical disruption probability (q) for all DCs, and vary q from 0.1 to 0.5. Figure 6 shows how the relative difference between the costs of *soft-hardening strategy* and *hardening strategy* decreases as q grows. Then as the size grows, we see that the cost variation of two strategies highly depends on the values of q. For example, in size 120 at $q = 0.1$, the gap is 27.68 % while at $q = 0.5$, this gap drops to 14.96 %, implying that once the network is disrupted, the model intensifies to open more number of reliable DCs rather than unreliable ones, this leads to reduction of the cost deference in two strategies.

[6] It should be noted that the literature of SCND under random disruption risks which utilize scenario-based approach (see Table 1, e.g. Peng et al. 2011 and Snyder and Daskin 2006), generate up to 65 scenarios for their instances.

Table 6 Benefit of considering random disruption risk

No.	# Customers	Nominal cost of capacitated SCND without disruption	Nominal cost of capacitated SCND with disruption	Increasing in nominal cost (%)	Decreasing in disruption cost with 500 scenarios (%)		
					Minimum value	Expected value	Maximum value
P2	10	36152	36441	0.8	4.2	4.6	4.9
P4	15	48537	49174	1.3	6.7	7.4	8.0
P8	20	59758	61399	2.7	9.4	10.6	11.8
P12	30	89658	93456	4.2	11.9	13.6	15.2
P16	40	115893	122541	5.7	16.9	19.7	22.7
P20	50	134216	142879	6.5	16.8	20.4	23.9
P24	60	158344	169963	7.3	20.1	25.2	33.1
P28	70	177625	193468	8.9	19.6	27.1	32.2
P32	80	199326	218163	9.5	22.9	27.8	34.2
P26	100	241572	268892	11.3	22.8	30.5	41.1
P28	120	277245	310003	11.8	24.1	33.1	42.6
P30	150	316978	357860	12.9	27.5	34.6	45.3

Fig. 4 Increasing in nominal cost and decreasing in disruption cost curves

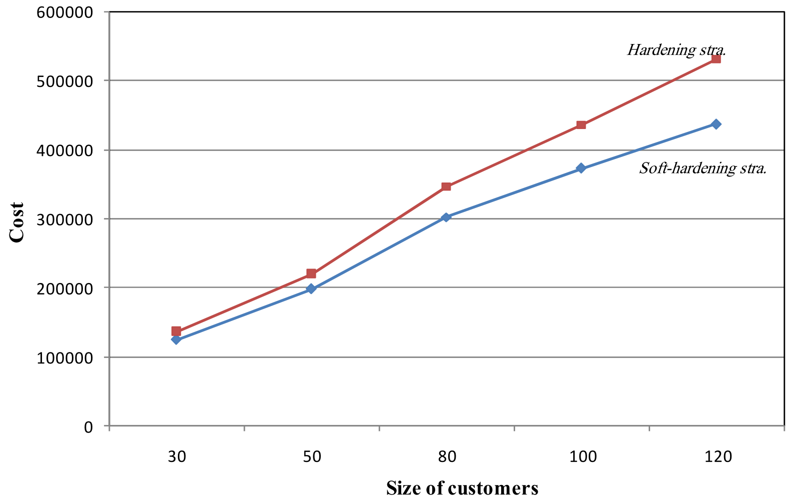

Fig. 5 Comparing soft-hardening strategy and hardening strategy

5.4 Impact of disruption probability and CFF on unreliable DCs

In this subsection, we explore how changes in the disruption probability and CFF of unreliable DCs (τ_{jn}) affect the optimal number of opened reliable and unreliable DCs. Consider the following four scenarios:[7]

- *CFF scenario* 1: τ_{jn} in this scenario ($\tau_{jn}^{(1)}$), is considered as mentioned in Sect. 5

[7] Hereafter we compile our investigation for medium and large sizes 80 and 120 by means of instances P24 and P28 in Table 2, respectively.

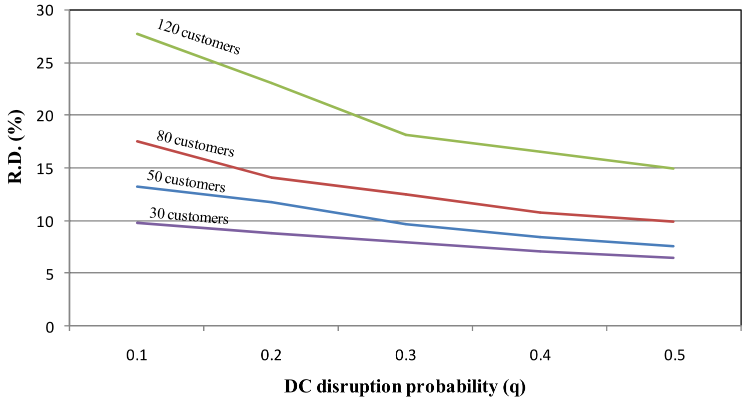

Fig. 6 Sensitivity analysis of q on comparing soft-hardening strategy and hardening strategy. *R.D. (%)*: Relative difference between the costs of model based on *soft-hardening strategy* and *hardening strategy*

- *CFF scenario* 2: τ_{jn} in this scenario ($\tau_{jn}^{(2)}$), is considered as $\tau_{jn}^{(2)} = 1.2 \times \tau_{jn}^{(1)}$
- *CFF scenario* 3: τ_{jn} in this scenario ($\tau_{jn}^{(3)}$), is considered as $\tau_{jn}^{(3)} = 1.4 \times \tau_{jn}^{(1)}$
- *CFF scenario* 4: τ_{jn} in this scenario ($\tau_{jn}^{(4)}$), is considered as $\tau_{jn}^{(4)} = 1.6 \times \tau_{jn}^{(1)}$

The results at sizes 80 and 120 are shown in Fig. 7 and Fig. 8, respectively. These figures show that the optimal number of opened reliable DCs increases (while the optimal number of opened unreliable DCs decreases) as CFF and q increases (i.e. unreliable DCs lose more capacity in disruption situation). For example Fig. 7 shows that at size 80 and $q = 0.1$, in scenario 1 there are 3 and 7 (scenario 4, 5 and 3) opened reliable and unreliable DCs, respectively, whereas at $q = 0.5$, in scenario 1 there are 5 and 3 (scenario 4, 7 and 1) opened reliable and unreliable DCs. Similar results are also found in Fig. 8 at size 120.

As q decreases, the number of opened reliable and unreliable DCs increase, for example, at size 120 and $q = 0.1$, scenario 1 presents 3 and 10 opened reliable and unreliable DCs, respectively, on the other hand, scenario 4 offers 8 and 3 opened reliable and unreliable ones which emphasizes our intuition (the difference between number of opened reliable and unreliable DCs in these two scenarios are 5 and 7, respectively), e.g. when $q = 0.5$ (see Fig. 8).

5.5 Impact of transportation mode disruption probability

In this subsection, we investigate how changes in disruption probability of unsafe transportation modes (π) affect the optimal percentage of used safe and unsafe transportation modes in the primary assignment which is computed as follows:

- Percentage of used safe transportation modes in primary assignment (%) = 100 × (Optimal number of used safe transportation modes in primary assignment)/Total number of used transportation modes in primary assignment.
- Percentage of used unsafe transportation modes in primary assignment (%) = 100 − Percentage of used safe transportation modes in primary assignment.

Similar to the previous subsection, we employ four scenarios for π as follows:

- *Tran. scenario 1:* π in this scenario ($\pi_{jkr}^{(1)}$), is considered as mentioned in Sect. 5
- *Tran. scenario 2:* π in this scenario ($\pi_{jkr}^{(2)}$), is considered as follows $\pi_{jkr}^{(2)} = 1.2 \times \pi_{jkr}^{(1)}$

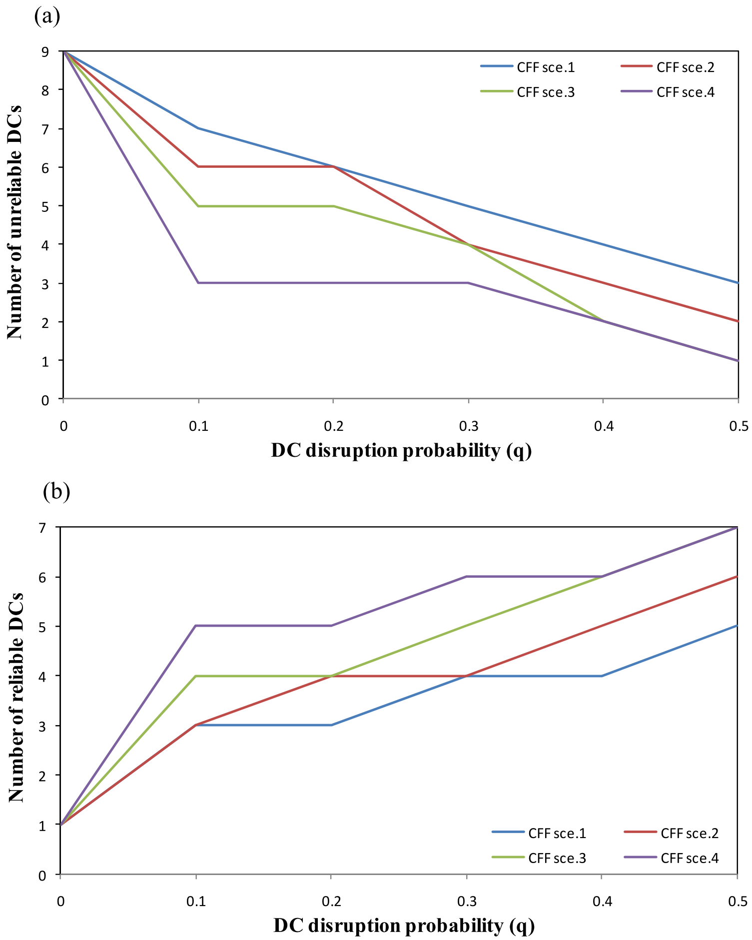

Fig. 7 Sensitivity analysis of q and CFF for size of 80 (**a**) optimal number of opened unreliable DCs (**b**) optimal number of opened reliable DCs

- *Tran. scenario 3:* π in this scenario ($\pi_{jkr}^{(3)}$), is considered as follows $\pi_{jkr}^{(3)} = 1.4 \times \pi_{jkr}^{(1)}$
- *Tran. scenario 4:* π in this scenario ($\pi_{jkr}^{(4)}$), is considered as follows $\pi_{jkr}^{(4)} = 1.6 \times \pi_{jkr}^{(1)}$

The results at size 80 and 120 are shown in Fig. 9 and Fig. 10, respectively. We see that when π increases, the optimal percentage of used safe transportation modes in primary assignment increases resulting in the decrease of unsafe transportation mode usage.

Intuitively safe transportation modes are more suitable to achieve the minimum cost when there is a high probability of disruption as they tend to react more robust than unsafe modes. The computational results also confirm this insight. For instance, scenario 1 at size 80 employs 18.3 % of safe modes and significantly is outnumbered by the unsafe modes (81.7 %), while in scenario 4, at the same size, 59.7 % and 40.3 % safe and unsafe modes are present, respectively (follow this observation at size 120).

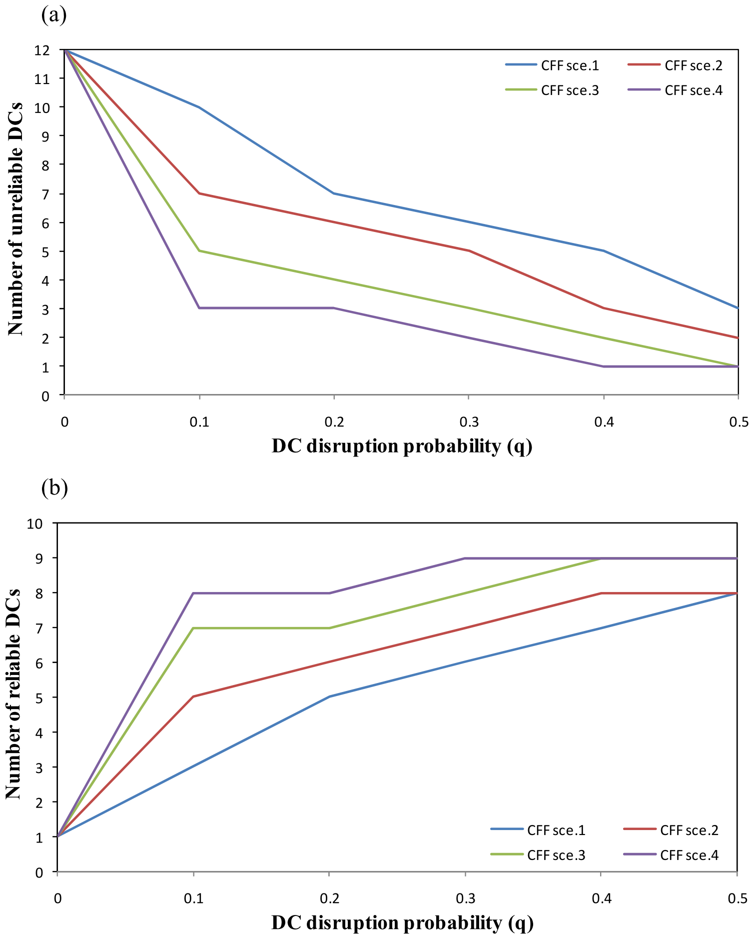

Fig. 8 Sensitivity analysis of q and CFF for the size of 120 (**a**) optimal number of opened unreliable DCs (**b**) optimal number of opened reliable DCs

5.6 Impact of disruption probability and fixed opening cost of DCs

In this subsection, we study how the changes in q and fixed opening cost of DCs affect the optimal number of opened reliable and unreliable DCs. To analyze the fixed opening cost of DCs, we define the hardening fixed cost factor as follows:

$$hf_j = \frac{fR_j}{\frac{\sum_n fU_{jn}}{|N|}} \quad \forall j$$

Again we vary q from 0.1 to 0.5 under four scenarios of hardening fixed cost factor in DCs and consider $hf_j = 1.5$, $hf_j = 1.8$, $hf_j = 2.1$ and $hf_j = 2.4$ for all the DCs in these scenarios. The results at size 80 and 120 are shown in Fig. 11 and Fig. 12, respectively. These figures show that the optimal number of opened reliable DCs increases (while the optimal number of opened unreliable DCs decreases) as q increases and hardening fixed cost factor decreases (reliable DCs become cheaper to build).

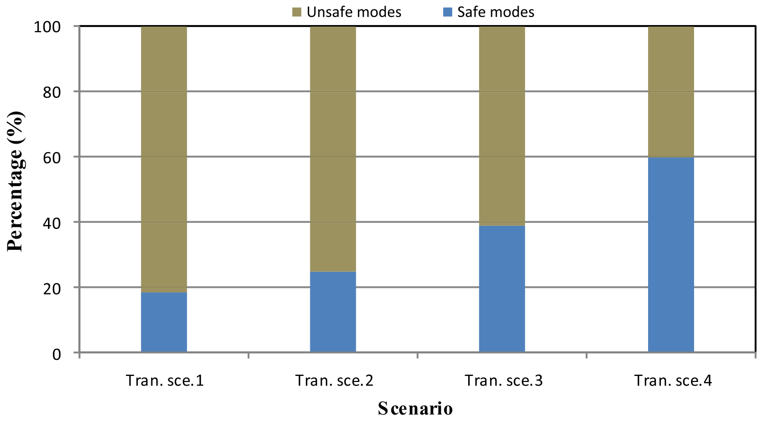

Fig. 9 Sensitivity analysis of π at size 80

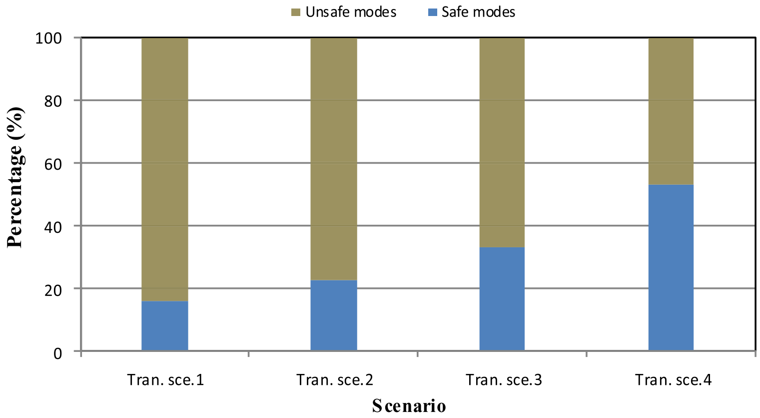

Fig. 10 Sensitivity analysis of π at size 120

Note that since in scenario $hf = 2.4$, each reliable DC incurs more cost for opening, the model tends to use fewer number of reliable DCs. Figure 12 shows a similar behavior of problem at size 120. Another interesting point is when q and hf increase simultaneously, the overall number of DCs reduces, e.g. at size 120 (Fig. 12) and $q = 0.1$, total number of DCs (for all scenarios of *hf*) is about 11 and 13, while at $q = 0.5$ when $hf = 2.4$, resulting number of DCs is 9. In fact, number of reliable DCs dominates the number of unreliable DCs as q raises, and when *hf* also grows, network is charged immensely to open reliable DCs; hence, the model performs with less number of reliable DCs to satisfy more customers' demands, consequently, by an increase in q, number of unreliable DCs decreases and by an increase in *hf*, number of reliable DCs decreases. Therefore, a simultaneous growth in q and *hf*, will reduce the number of reliable and unreliable DCs.

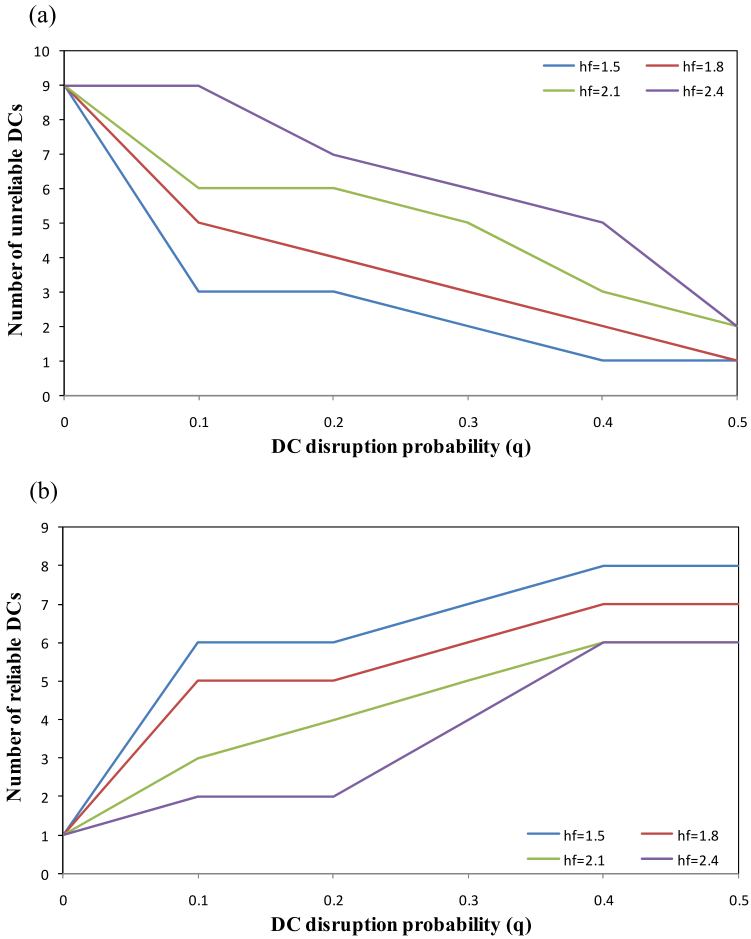

Fig. 11 Sensitivity analysis of q and hardening cost factor at size 80 (**a**) optimal number of opened unreliable DCs (**b**) optimal number of opened reliable DCs

6 Conclusions

Facility disruption can have a serious impact on the service quality of a network. In light of this, we have developed a new capacitated SCND model under random disruption to find a robust arrangement of DCs that is flexible in situations with failed DCs. This could be an effective response to the managerial needs to control the logistics costs and maintain high customer service levels in a network system. In particular an original mixed-integer linear programming model taking into account disruptions in DCs and outbound links is proposed. The results are illustrated and discussed in order to identify the most significant factors affecting the system's performance and to suggest effective guidelines. Scenarios are drawn for instances of up to 150 customers. Also we specifically hold the following strategic attributes considered in the model:

i. The *CFF* is presented for assessing how much capacity is lost as a result of a disruption in a DC.

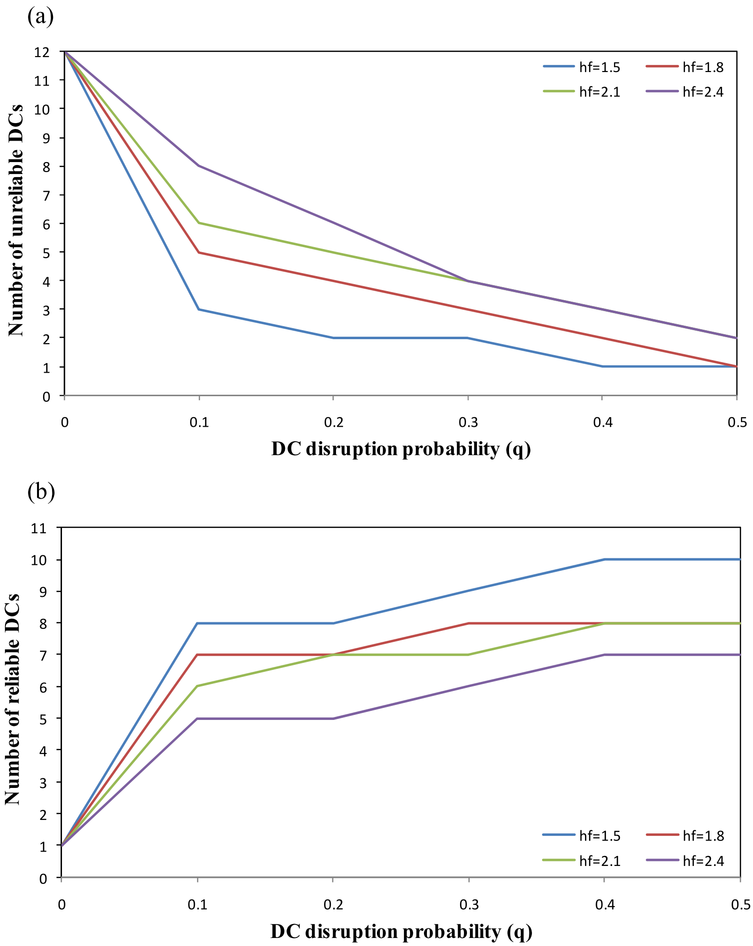

Fig. 12 Sensitivity analysis of q and hardening cost factor at size 120 (**a**) optimal number of opened unreliable DCs (**b**) optimal number of opened reliable DCs

ii. The *goods sharing* strategy is presented to maintain service quality level under disruption.
iii. The *soft-hardening* strategy is presented to make the network more robust by financial investment.

The above topics have not been considered simultaneously so far and this paper is the first to consider such contributions. From managerial view points, we have shown that reliability of DCs and transportations can be improved without large increases in operating costs. In our problem, we also have shown that using *soft-hardening strategy* outperforms using *hardening strategy* and improves the optimal cost for different sizes. However for larger values of disruption probability of DCs, the relative difference cost of *soft-hardening strategy* and *hardening strategy* decreases, because in this case the optimal number of opened reliable DCs increases.

The sensitivity analysis of applying mentioned strategies revealed some key insights: (1) the optimal number of opened reliable DCs in contrast to unreliable DCs, increases as

CFF and disruption probability of DCs grows. (2) The optimal number of opened reliable DCs compared to unreliable DCs increases as hardening fixed cost factor decreases (reliable DCs become cheaper to build) and as DC's disruption probability increases. (3) The optimal percentage of used safe transportation modes, contrary to unsafe modes, increases in the primary assignment as the unsafe transportation mode's disruption probability increases. One may explore this improvement to check whether the proposed approach is beneficial in practice. This may require extending our dataset to capture other complex features arising from the necessities of automotive companies that work in logistics areas, e.g. Toyota and General Motors that both were inflicted with impacts from the latest Japan's Tsunami. The proposed strategies are also applicable within the context of natural disasters when the probability that a disruption occurs at a given DC is difficult to estimate accurately and/or the consequences of an extreme event may be so severe to justify a highly risk-averse decision making approach. Probabilistic models, however, should be devised for modeling disruptions which are random in nature if the disruption probabilities can be either forecasted or estimated from reliable historical data. In this case protection measures may be undertaken to reduce the probability of disruption.

A new BD accelerating approach, namely Maximum Density Cut (MDC), has been proposed in order to solve the model optimally. The inspiration to develop MDC method was that it was found that the number of cuts generated in each iteration of CCB method (see Sect. 4.3.1) affects the performance of the algorithm. Combining MDC and CCB methods was the solution to this difficulty. Using MDC cut generation independently or in combination with CCB method, the number of iterations always decreases, which results in significant solution time decrease since the solution space of MP has been restricted.

For future works, one possible extension is to consider a duration and frequency for random disruptions. Incorporating these features in the model will enable the decision maker to examine optimal decisions in a dynamic setting. It is also possible to integrate the model with other decisions in SCND like inventory management, production management, routing decisions etc., which suffer from the lack of disruption risks management.

References

Altner, D. S., Ergun, O., & Uhan, N. A. (2010). The maximum flow network interdiction problem: valid inequalities, integrality gaps, and approximability. *Operations Research Letters*, *38*(1), 33–38.

Andreas, A. K., & Smith, J. C. (2009). Decomposition algorithms for the design of a non simultaneous capacitated evacuation tree network. *Networks*, *53*(2), 91–103.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, *4*(1), 238–252.

Berman, O., Krass, D., & Menezes, M. (2007). Facility reliability issues in network p-median problems: strategic centralization and co-location effects. *Operations Research*, *55*(2), 332–350.

Berman, O., Krass, D., & Menezes, M. (2009). Locating facilities in the presence of disruptions and incomplete information. *Decision Sciences*, *40*(4), 845–868.

Chopra, S., & Sodhi, M. S. (2004). Managing risk to avoid supply-chain breakdown. *MIT Sloan Management Review*, *46*(1), 53–61.

Chopra, S., Reinhardt, G., & Mohan, U. (2007). The importance of decoupling recurrent and disruption risks in a supply chain. *Naval Research Logistics*, *54*(5), 544–555.

Church, R. L., & Scaparra, M. P. (2006). Protecting critical assets: the R-interdiction median problem with fortification. *Geographical Analysis*, *39*(2), 129–146.

Church, R. L., Scaparra, M. P., & Middleton, R. S. (2004). Identifying critical infrastructure: the median and covering facility interdiction problem. *Annals of the Association of American Geographers*, *94*(3), 491–502.

Colbourn, C. (1987). *The combinatorics of network reliability*. New York: Oxford University Press.

Conejo, A. J., Castillo, E., Minguez, R., & Garcia-Bertrand, R. (2006). Decomposition techniques in mathematical programming. In *Engineering and science applications*. Berlin: Springer.

Cordeau, J. F., Soumis, F., & Desrosiers, J. (2000). A Benders decomposition approach for the locomotive and car assignment problem. *Transportation Science*, *34*(2), 133–149.

Cordeau, J. F., Pasin, F., & Solomon, M. M. (2006). An integrated model for logistics network design. *Annals of Operations Research*, *144*(1), 59–82.

Cote, G., & Laughton, M. (1984). Large-scale mixed integer programming: Benders-type heuristics. *European Journal of Operational Research*, *16*, 327–333.

Craighead, C. W., Blackhurst, J., Rungtusanatham, M. J., & Handfield, R. B. (2007). The severity of supply chain disruptions: Design characteristics and mitigation capabilities. *Decision Sciences*, *38*(1), 131–156.

Cui, T., Ouyang, Y., & Shen, Z.-J. (2010). Reliable facility location design under the risk of disruptions. *Operations Research*, *58*(4), 998–1011.

Drezner, Z. (1987). Heuristic solution methods for two location problems with unreliable facilities. *Journal of Operations Research Society*, *38*(6), 509–514.

Elkins, D., Handfield, R. B., Blackhurst, J., & Craighead, C. W. (2005). 18 ways to guard against disruption. *Supply Chain Management Review*, *9*(1), 46–53.

Kleindorfer, P. R., & Saad, G. (2005). Managing disruption risks in supply chains. *Production and Operations Management*, *14*(1), 53–68.

Klibi, W., Martel, A., & Guitouni, A. (2010). The design of robust value-creating supply chain networks: a critical review. *European Journal of Operational Research*, *203*(2), 283–293.

Kraiselburd, S., Narayanan, V. G., & Raman, A. (2004). Contracting in a supply chain with stochastic demand and substitute products. *Production and Operations Management*, *13*(1), 46–62.

Kaut, M., Wallace, S. W., Vladimirou, H., & Zenios, S. (2007). Stability analysis of portfolio management with conditional value-at-risk. *Quantitative Finance*, *7*(4), 397–409.

Kulkarni, S. S., Magazine, M. J., & Raturi, A. S. (2004). Risk pooling advantages of manufacturing network configuration. *Production and Operations Management*, *13*(2), 186–199.

Lee, S. D. (2001). On solving unreliable planar location problems. *Computers & Operations Research*, *28*(4), 329–344.

Li, X., & Ouyang, Y. (2010). A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions. *Transportation Research. Part B*, *44*(4), 535–548.

Liberatore, F., Scaparra, M. P., & Daskin, M. S. (2011). Analysis of facility protection strategies against an uncertain number of attacks: the stochastic R-interdiction median problem with fortification. *Computers & Operations Research*, *38*(1), 357–366.

Liberatore, F., Scaparra, M. P., & Daskin, M. S. (2012). Hedging against disruptions with ripple effects in location analysis. *Omega*, *40*, 21–30.

Lim, M., Daskin, M., Bassamboo, A., & Chopra, S. (2010). A facility reliability problem: formulation, properties and algorithm. *Naval Research Logistics*, *57*(1), 58–70.

Magnanti, T., & Wong, R. (1981). Accelerating Benders decomposition algorithmic enhancement and model selection criteria. *Operations Research*, *29*, 464–484.

Martinez-de-Albeniz, V., & Simchi-Levi, D. (2005). A portfolio approach to procurement contracts. *Production and Operations Management*, *14*(1), 90–114.

McDaniel, D., & Devine, M. (1977). A modified Benders partitioning algorithm for mixed integer programming. *Management Science*, *24*, 312–319.

Mohebbi, E. (2004). A replenishment model for the supply-uncertainty problem. *International Journal of Production Economics*, *87*, 25–37.

Oke, A., & Gopalakrishnan, M. (2009). Managing disruptions in supply chains: a case study of a retail supply chain. *International Journal of Production Economics*, *181*, 168–174.

Papadakos, N. (2008). Practical enhancements to the Magnanti-Wong method. *Operations Research Letters*, *36*(4), 444–449.

Parlar, M. (1997). Continuous review inventory problem with random supply interruptions. *European Journal of Operational Research*, *99*, 366–385.

Parlar, M., & Berkin, D. (1991). Future supply uncertainty in EOQ models. *Naval Research Logistics*, *38*, 107–121.

Peng, P., Snyder, L. V., Lim, A., & Liu, Z. (2011). Reliable logistics networks design with facility disruptions. *Transportation Research. Part B*. doi:10.1016/j.trb.2011.05.022.

Qi, L., Shen, Z.-J., & Snyder, L. V. (2010). The effect of supply disruptions on supply chain design decisions. *Transportation Science*, *44*(2), 274–289.

Rei, W., Gendreau, M., Cordeau, J. F., & Soriano, P. (2006). Accelerating Benders decomposition by local branching. In: *Hybrid methods and branching rules in combinatorial optimization*, Montreal, 2006.

Saharidis, G. K. D., & Ierapetritou, M. G. (2010). Improving Benders decomposition using maximum feasible subsystem (MFS) cut generation strategy. *Computers & Chemical Engineering*, *8*(34), 1237–1245.

Saharidis, G. K. D., Minoux, M., & Dallery, Y. (2009). Scheduling of loading and unloading of crude oil in a refinery using event-based discrete time formulation. *Computers & Chemical Engineering*, *33*(8), 1413–1426.
Saharidis, G. K. D., Minoux, M., & Ierapetritou, M. G. (2010). Accelerating Benders method using covering cut bundle generation. *International Transactions in Operational Research*, *17*(2), 221–237.
Saharidis, G. K. D., Boile, M., & Theofanis, S. (2011). Initialization of the Benders master problem using valid inequalities applied to fixed-charge network problems. *Expert Systems with Applications*. doi:10.1016/j.eswa.2010.11.075.
Scaparra, M. P., & Church, R. L. (2008). A bi level mixed-integer program for critical infrastructure protection planning. *Computers & Operations Research*, *35*(6), 1905–1923.
Sherali, H. F., & Lunday, B. J. (2011). On generating maximal non dominated Benders cuts. *Annals of Operations Research*. doi:10.1007/s10479-011-0883-6.
Shier, D. R. (1991). *Network reliability and algebraic structures*. Oxford: Clarendon Press.
Shooman, M. L. (2002). *Reliability of computer systems and networks: fault tolerance, analysis, and design*. New York: Wiley.
Snyder, L. V., & Daskin, M. S. (2005). Reliability models for facility location: the expected failure cost case. *Transportation Science*, *39*(3), 400–416.
Snyder, L. V., & Daskin, M. S. (2006). Stochastic p-robust location problems. *IIE Transactions*, *38*(11), 971–985.
Snyder, L. V., Scaparra, M. P., Daskin, M. S., & Church, R. L. (2006). Planning for disruptions in supply chain networks. In: *Tutorials in operations research INFORMS 2006* (pp. 234–257).
Tang, C. S. (2006). Perspectives in supply chain risk management. *International Journal of Production Economics*, *103*, 451–488.
Tomlin, B. (2006). On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, *52*(5), 639–657.
Tomlin, B., & Wang, Y. (2005). On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*, *7*(1), 37–57.
Wagner, S. M., & Bode, C. (2006). An empirical investigation into supply chain vulnerability. *Journal of Purchasing and Supply Management*, *12*(6), 301–312.
Zakeri, G., Philpott, A. B., & Ryan, D. M. (1998). Inexact cuts in benders decomposition. *SIAM Journal on Optimization*, *10*(3), 643–657.