K-T ISD: COMPRESSED SENSING WITH ITERATIVE SUPPORT DETECTION FOR DYNAMIC MRI

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ABSTRACT

In this paper, we propose a new *k-t* Iterative Support Detection (*k-t* ISD) method to improve the CS reconstruction for dynamic cardiac MRI by incorporating additional information on the support of the dynamic image in *x-f* space. The proposed method uses an iterative procedure for alternating image reconstruction and support detection in *x-f* space. Experimental results demonstrate that the proposed *k-t* ISD method improves the reconstruction quality of dynamic cardiac MRI over the basic CS method in which support information is not exploited.

Index Terms**—**Compressed sensing, dynamic MRI, *k-t* Iterative Support Detection (*k-t* ISD), partially known support, truncated ℓ_1 minimization.

1. INTRODUCTION

Dynamic cardiac cine magnetic resonance imaging (MRI) is a technique to acquire a time series of images from cardiac motion at a high frame rate. The acquisition time is an important factor to minimize so that cardiac function can be accurately evaluated in a clinical workflow. Undersampling *k*-space accelerates the acquisition speed but may compromise spatial resolution, temporal resolution, signal-to-noise ratio (SNR), or introduce image artifacts. The emerging theory of compressed sensing (CS) [1, 2] holds great potential for significant data reduction while preserving the quality of the image sequence. In most existing CS-based dynamic cardiac MRI methods, the basic CS formulation is used to exploit only the prior that the dynamic image series is sparse in the spatial and temporal-frequency domain (*x-f* space). Since additional prior information about the unknown MR images may be available for certain applications, it is advantageous to incorporate this information into the CS reconstruction.

In this paper, we study how to obtain and exploit the support information to improve CS reconstruction in dynamic cardiac MRI applications. A new method, named *k-t* Iterative Support Detection (*k-t* ISD), is proposed. The method alternates between image reconstruction and support detection in *x-f* space iteratively. Within each iteration, the dynamic image in *x-f* space is reconstructed through a truncated ℓ_1 minimization implemented using a FOCUSS algorithm. Specifically, the truncated ℓ_1 minimization excludes the signal at the known support (detected from the previous iteration) from the cost function of the ℓ_1 minimization. Once the image is reconstructed in this iteration, the support information is updated by thresholding the reconstruction and used in the next iteration of image reconstruction. Improvement of the proposed method over the basic CS approaches is demonstrated using *in vivo* cardiac cine MR experiments.

2. BACKGROUND

2.1. Summary on Dynamic MRI Using CS

In the CS framework, a dynamic image series is reconstructed in the spatial and temporal domain — the *x-t* domain by

$$\min_{\mathbf{m}} \|\mathbf{\Psi m}\|_1 \text{ s.t. } \mathbf{d} = \mathbf{F}_x \mathbf{m}, \tag{1}$$

where $\mathbf{F}_x$ is the Fourier transform matrix in the spatial domain, $\mathbf{m}$ is a vector representing the signal in the *x-t* domain, $\mathbf{d}$ is the vector for the reduced data in the spatial-frequency and temporal domain, the so-called *k-t* space, and $\mathbf{\Psi}$ denotes the sparsifying transform. The most popular sparsifying transform is Fourier transform applied along the temporal direction [3-6], and the image series is sparse in the *x-f* domain. Several CS reconstruction algorithms have been used to solve (1), such as orthogonal matching pursuit (OMP) [5], non-linear conjugate gradient algorithm [6], and FOCUSS algorithm [3, 4].

2.2. Theory on CS with Partially Known Support

Recently, an extension of CS — CS with partially known support has been studied in [7-10] to incorporate the support information of a sparse signal into the CS reconstruction. It is shown through theoretical analysis and numerical study that this new technique can effectively reduce the number of measurements required to achieve a given reconstruction quality or to improve performance for a given number of measurements.

Now we consider the CS reconstruction of an s-sparse signal $\mathbf{x}$ with partial known support from measurements $\mathbf{y} = \mathbf{\Phi x}$, where $\mathbf{\Phi}$ is sensing matrix. The support of $\mathbf{x}$, denoted as S, can be represented as $S = T \cup \Delta$, where T is the known part with size $|T|$ and Δ is the unknown part with size $|\Delta|$. With the partially known support T, the candidates for the s-sparse signal $\mathbf{x}$ will lie in a smaller signal space. It allows us to minimize the number of nonzeros outside the support T when searching for a sparse solution to $\mathbf{\Phi x} = \mathbf{y}$. Similar to the basic CS, the signal can be reconstructed by solving for the sparsest solution that contains the support T and satisfies the data consistency constraint. This procedure can be formulated as

$$\min_{\mathbf{x}} \|\mathbf{x}_\Delta\|_0 \text{ s.t. } \mathbf{\Phi x} = \mathbf{y}, \quad (2)$$

or, more practically

$$\min_{\mathbf{x}} \|\mathbf{x}_\Delta\|_1 \text{ s.t. } \mathbf{\Phi x} = \mathbf{y}, \quad (3)$$

where $\mathbf{x}_\Delta$ denotes the signal outside the known support. This problem is referred to as the truncated minimization because the cost function to be minimized is the ℓ_0 quasi-norm or ℓ_1 norm of a truncated version of the signal that leaves out the part with known support. This formulation favors a solution with more zeros outside T [7], and thus it may recover the signal more accurately than the basic CS does for signals whose support includes T.

The robustness of truncated ℓ_1 minimization under noisy measurements has been studied in [8-10]. An upper bound is given for the reconstruction error as

$$\| \mathbf{x}^* - \mathbf{x} \|_2 \le c_3 \| \mathbf{r} - \mathbf{r}^{|\Delta|} \|_1 \Big/ \sqrt{|\Delta|} + c_4 \varepsilon, \quad (4)$$

where $\mathbf{r} = \mathbf{x} - \mathbf{x}_T$ is the residual signal containing only the elements in Δ, $\mathbf{x}^*$ is the solution to truncated ℓ_1 minimization, c_3 and c_4 are the constants. This bound is linearly proportional to the noise level ε and the approximation error between the residual $\mathbf{r}$ and its closest $|\Delta|$-sparse signal $\mathbf{r}^{|\Delta|}$.

3. PROPOSED METHOD

Based on the above theory, we propose a *k-t* ISD method that adaptively and iteratively learns and utilizes the support information in compressed sensing for dynamic MRI, such that the images are reconstructed more accurately. The method alternates between CS reconstruction with partially known support and adaptive learning of support knowledge, which are elaborated below.

3.1. Compressed Sensing for Dynamic MRI with Partially Known Support

We consider the Fourier transform along the temporal direction as the sparsifying transform because dynamic cardiac MR images have been shown to be sparse in the *x-f* space [3-6], and the support information can be readily obtained with such a transform. The data consistency constraint becomes

$$\mathbf{F\rho} = \mathbf{d}, \quad (5)$$

where $\mathbf{F}$ denotes the two-dimensional Fourier transform in both *k-t* directions and $\boldsymbol{\rho}$ is the signal vector in *x-f* domain. Then, reconstruction of the signal $\boldsymbol{\rho}$ in *x-f* domain can be formulated as a truncated ℓ_1 minimization problem

$$\min_{\boldsymbol{\rho}} \|\boldsymbol{\rho}_\Delta\|_1 \text{ s.t. } \|\mathbf{d} - \mathbf{F\rho}\|_2 \le \varepsilon, \quad (6)$$

when $\boldsymbol{\rho}$ has a known part T and an unknown part Δ.

3.2. Adaptive Learning of Support Knowledge

The support information in *x-f* space has been studied and used in prior works outside the CS context. In order to exploit accurate *x-f* support information in the design of the time-sequential acquisition scheme, patient-adapted reconstruction and acquisition dynamic imaging method (PARADIGM) uses a pre-scan to obtain the support [11]. Brinegar et al [12] try to estimate the *x-f* support empirically to improve the reconstruction using the partially separable function model.

To avoid pre-scan or empirical estimate, we propose to learn the support in *x-f* space from the reconstructions iteratively. The idea of iterative support detection is based on a new CS algorithm by Wang and Yin [9]. The observation that the signals in *x-f* space for cardiac cine MRI rapidly decay in magnitude makes the development of *k-t* ISD possible. In other words, the nonzero components of such signals have relatively large magnitude. This property ensures that even when the initial reconstruction is poor (due to insufficient measurements), the signal magnitude at the nonzero locations within the true support is likely to remain large, and thus these locations can be detected correctly by thresholding. In the event that some of the zero locations outside the true support also present large magnitude in the initial reconstruction, resulting in false locations to be included in the detected support, the ISD still converges as long as the number of wrong locations is less than the number of correct locations by a certain amount [9].

In *k-t* ISD, we start with a basic CS reconstruction without any support information. Because fewer *k-t* data are acquired than those required for perfect recovery using the basic CS construction, the initial reconstruction is poor. We then learn the support in *x-f* space using the locations whose values are above a threshold. This support is held fixed and used in truncated ℓ_1 minimization for an updated reconstruction. The support detection and signal reconstruction steps are then repeated alternately until convergence.

The above two steps can be mathematically represented as follows. In the reconstruction step of the i-th iteration in k-t ISD, the intermediate reconstruction $\boldsymbol{\rho}^{(i)}$ is obtained by solving a truncated ℓ_1 minimization problem

$$\min_{\boldsymbol{\rho}} \left\| \boldsymbol{\rho}_{\Delta^{(i-1)}} \right\|_1 \text{ s.t. } \left\| \mathbf{d} - \mathbf{F}\boldsymbol{\rho} \right\|_2 \le \varepsilon \, . \tag{7}$$

with a known support $T^{(i-1)} \left(\Delta^{(i-1)} = \overline{T}^{(i-1)} \right)$, where the $\overline{T}$ denotes the complimentary set of T. Specifically, $\boldsymbol{\rho}^{(i)}$ can be reconstructed by rewriting (7) as the weighted ℓ_1 minimization problem

$$\min_{\boldsymbol{\rho}} \left\| \mathbf{W}^{(i-1)} \boldsymbol{\rho} \right\|_1 \text{ s.t. } \left\| \mathbf{d} - \mathbf{F}\boldsymbol{\rho} \right\|_2 \le \varepsilon \, , \tag{8}$$

and solved using FOCUSS algorithm. $\mathbf{W}^{(i-1)}$ is a diagonal weighting matrix whose diagonal element equals 1 if the corresponding element in (i-1)-th x-f reconstruction belongs to the unknown support Δ, or equals 0 otherwise. In the support detection step, the support is obtained by thresholding the above reconstructed signal $\boldsymbol{\rho}^{(i)}$ in x-f space

$$T^{(i)} := \left\{ z : \left| \boldsymbol{\rho}_z^{(i)} \right| > \tau^{(i)} \right\} \, .$$

(9)

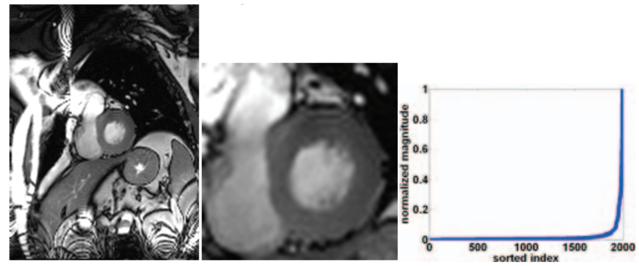
The threshold $\tau^{(i)}$ largely affects the performance of k-t ISD. It has been found that k-t ISD works well for fast-decaying signals $\boldsymbol{\rho}$ and by setting $\tau^{(i)} = \left\| \boldsymbol{\rho}^{(i)} \right\|_\infty \big/ \delta^{(i)}$ with an increasing sequence of $\delta^{(i)} > 0$ as an exponential function of the number of iteration i.

4. EXPERIMENTS AND RESULTS

The feasibility of k-t ISD was validated on a number of datasets. Sample results from a set of *in vivo* data acquired on a 3T Siemens scanner are shown here. The SSFP sequence was used with a flip angle of 44 degree and TE/TR = 1.5/3.0msec. The fully acquired k-t measurements had a size of 160×133×15×5 (#frequency encoding × #phase encoding × #frame × #coil). The FOV was 350mm × 262mm and the slice thickness was 7 mm. The heart rate was 54 bpm.

A random sampling pattern with a zero-mean Gaussian distribution was used for undersampling the k-space. The central 8 phase encoding lines were fully sampled. The matrix **W** was initialized to an identity matrix for the first reconstruction when no support knowledge was available. The threshold is determined by empirically setting $\delta^{(i)} = 8^{i+1}$ based on our experiments.

The proposed k-t ISD, k-t FOCUSS [3], and the two-step OMP methods [5] were used to reconstruct the image for each coil separately, which are then combined using root of sum-of-squares (SoS). Since strict convergence is not observed in this application, we calculate the normalized difference between the adjacent iterations and then terminate iterations once the difference is below a threshold.



(a) (b) (c)

Fig. 1. (a) A single frame of reconstruction from full data and (b) its zoomed-in heart region. (c) Plot of the magnitude of the normalized and sorted signals in x-f space for a given position in the frequency-encoding direction.

The image series reconstructed from the full k-t data were used as the reference for comparison. All images were zero-padded to display images with the right ratio representing the original FOV. Figure 1 (a) and (b) show one frame of reconstruction from full data and its zoomed-in heart region for reference and (c) shows the normalized and sorted signals in x-f space for a given position in the frequency-encoding direction. It is seen that the signal in x-f space has a fast-decaying rate.

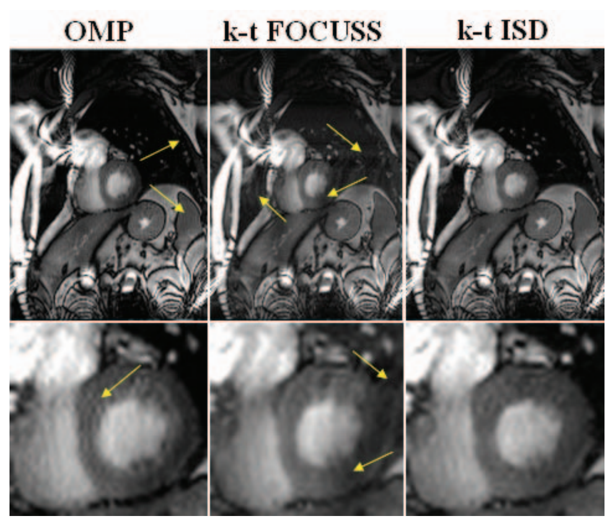


Fig. 2. Reconstructions of a single frame (top) from the reduced data with R=3 and the corresponding zoomed-in regions (bottom) using different methods.

Figure 2 shows the reconstructions (top) and corresponding zoomed-in images (bottom) at the 6th frame with a reduction factor of R = 3. The result of the k-t ISD method is after two iterations of support detection. It is shown that k-t ISD presents fewer undersampling artifacts than k-t FOCUSS and less noise than two-step OMP. The normalized mean-squared error (MSE) between the reconstruction and the reference were calculated and plotted as a function of time frame in Fig. 3. The *k-t* ISD is seen to have a lower MSE than the other methods for all frames examined with fixed R.

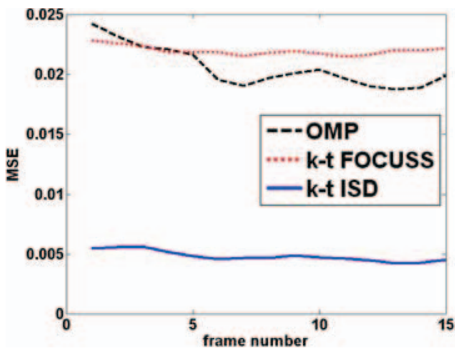


Fig. 3. Frame-by-frame plots of the normalized MSE for OMP, *k-t* FOCUSS, and *k-t* ISD with a reduction factor of 3.

In order to explore the effect of threshold τ on the performance of *k-t* ISD, we choose three different values $\delta^{(i)} = 15^{i+1}$, 8^{i+1} and 2^{i+1} where i is the iteration number. The MSE (averaged over all frames) versus the number of iterations are plotted in Fig. 4 for different choices of δ when R=3. It can be seen that when δ changes fast with iterations, the MSE initially decreases but then increases with more iterations. The initial decrease is because the detected support improves over iterations and so does the reconstruction quality. The subsequent increase with more iterations is due to the increased number of false locations in support detection as δ gets too large. For $\delta^{(i)} = 15^{i+1}$, *k-t* ISD stops after 4 iterations to an MSE value that is greater than the lowest MSE. On the other hand, slowly-changing δ will result in too few locations detected in support and thus slow convergence. For $\delta^{(i)} = 2^{i+1}$, *k-t* ISD does not terminate even after 5 iterations. A moderate changing rate with $\delta^{(i)} = 8^{i+1}$ needs only 3 iterations to stop which corresponds to the lowest MSE. In future work, adaptive thresholding scheme will be investigated to balance the reconstruction quality and computation complexity.

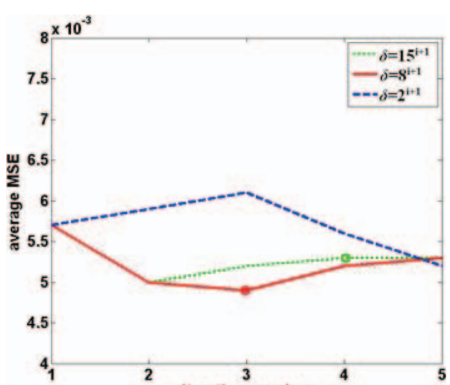


Fig. 4. The averaged MSE as a function of iterations for $\delta^{(i)} = 15^{i+1}$, 8^{i+1}, and 2^{i+1}. Each small circle indicates the terminating iteration.

5. CONCLUSION

In this paper, a new method for dynamic MRI, named *k-t* ISD, is developed. The method iteratively learns and exploits the support knowledge in *x-f* space to improve CS reconstruction. The dynamic cardiac MRI experiments show that the proposed method is able to suppress more artifacts and preserve more details of dynamic images than existing CS methods.

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7. REFERENCES

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