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A novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence☆

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ABSTRACT

This paper proposes a novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. First, the uncertain degrees of various parameters are determined via grey relational analysis, which is applied to calculate the grey mean relational degree. Second, suitable mass functions of different independent alternatives with different parameters are given according to the uncertain degree. Third, to aggregate the alternatives into a collective alternative, Dempster's rule of evidence combination is applied. Finally, the alternatives are ranked and the best alternatives are obtained. The effectiveness and feasibility of this approach are demonstrated by comparing with the mean potentiality approach because the measure of performance of this approach is the same as the mean potentiality approach's, the belief measure of the whole uncertainty falls from 0.4723 to 0.0782 (resp. 0.3821 to 0.0069) in the example of Section 5 (resp. Section 6).

1. Introduction

To solve complicated problems in economics, engineering, environmental science and social science, classical mathematical methods are not always successful because of various types of uncertainties present in these problems. There are several theories: probability theory, fuzzy set theory [30], rough set theory [22] and interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. To overcome these kinds of difficulties, Molodtsov [19] proposed soft set theory for modeling uncertainty.

Recently works on soft set theory are progressing rapidly. Maji et al. [20] defined fuzzy soft sets by combining soft sets with fuzzy sets, in other words, a degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. The study of hybrid models combining soft sets or fuzzy soft sets with other mathematical structures and new operations are emerging as an active research topic of soft set theory [10,29]. Aktas et al. [1] initiated soft groups. Jun applied soft sets to BCK/BCI-algebras [9]. Jiang et al. [10] extended soft sets with description logics. Li et al. [18] investigated relationships among soft sets, soft rough sets and topologies.

At the same time, there have been some progress concerning applications of soft set theory, especially the usage of soft sets in decision making. Using soft set theory to describe or set objects with traditional mathematics tools is very different. We can describe approximately the original objects in soft set theory. There is no limiting condition when objects are described. Researchers can choose parameters and their forms according needs. The fact that setting parameters is non-binding greatly simplifies decision-making process and then we can still do effective decisions under the circumstance of the absence of partial information.

Maji et al. [21] first applied soft sets to solve decision making problems by means of rough set theory. Chen et al. [3] defined the parameterization reduction of soft set and discussed its application of decision making problem. Çağman et al. [4] constructed an uni-int decision making method which selects a set of optimum elements from the alternatives by using uni-int decision functions. Roy et al. [23] discussed score value as the evaluation basis to finding an optimal choice object in fuzzy soft sets. Kong et al. [13] argued that the Roy's method was incorrect in general and they proposed a revised algorithm. Feng et al. [7] applied level soft sets to discuss fuzzy soft sets based decision making. Jiang et al. [11] generalized the adjustable approach to fuzzy soft sets based decision making and presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic

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fuzzy soft sets. Based on Feng' works, Basu et al. [2] further investigated the previous methods to fuzzy soft sets in decision making and introduced the mean potentiality approach, which was showed more efficient and more accurate than the previous methods.

The existing approaches have significant contributions to solve fuzzy soft sets in decision making. However, these approaches are mainly based on the level soft set, and the decision makers select any level soft set with much subjectivity and uncertainty [2]. Moreover, there exists no unique or uniform criterion for the selection, the same decision problem may induce many different results by using different evaluation criteria. As a result, it is difficult to judge that which result is adequate, and which method or level soft sets should be chosen for selecting the optimal choice object. The key to this problem is how to reduce subjectivity and uncertainty when we choose making decisions method. Then it is necessary to pay attention to this issue.

Grey relational analysis initiated by Deng [6] is utilized for generalizing estimates under small samples and uncertain conditions, and it can be regarded an effective method to solve decision making problems [12,26,33]. Dempster–Shafer theory of evidence is a new important reasoning method under uncertainty, which has an advantage to deal with subjective judgments and to synthesize the uncertainty of knowledge [32].

Compared to probability theory, Dempster–Shafer theory of evidence [5,24] can capture more information to support decision-making by identifying the uncertain and unknown evidence. It provides a mechanism to derive solutions from various vague evidences without knowing much prior information and has been successfully applied into many fields such as intelligent medical diagnosis [8], knowledge reduction [27], fault diagnosis [28], multi-class classification [17], supplier selection [25], etc. Moreover, applying both theories enables the ultimate decision makers to take advantage of both methods' merits and make evaluation experts to deal with uncertainty and risk confidently [16,25]. The hybrid model has been proved to have its usefulness and versatility in successfully solving a variety of problems in the information sciences, such as data mining, knowledge discovery, and decision making.

Therefore, it is very meaningful to explore an approach to fuzzy soft set in decision making by combining Dempster–Shafer theory of evidence with grey relational analysis.

The purpose of this paper is to give a novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. The novelty aspects or advantages of this approach is avoiding the selection of appropriate level soft sets and distribution of parameters' weight, reducing significantly the uncertainty of decision-making and the fuzziness of people's subjective understanding, improving the reliability of decision making and increasing the level of decision-making.

2. Preliminaries

Throughout this paper, U denotes an initial universe, E denotes the set of all possible parameters, I^U denotes the family of all fuzzy sets in U. We only consider the case where U and E are both nonempty finite sets.

In this section, we briefly recall some basic concepts about fuzzy soft sets, the measure of performance of methods and Dempster–Shafer theory of evidence.

2.1. Fuzzy soft sets

Definition 2.1 *([20])*. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F: A \to I^U$.

Table 1
Tabular representation of the fuzzy soft set (F, A).

	h_1	h_2	h_3	h_4	h_5	h_6
e_1	0.6	1	0.2	0.3	1	0.7
e_2	1	0.5	0.3	0.2	0.1	0.9
e_3	0.1	0.4	0.8	1	0	0.1
e_4	0.1	0.3	1	0.9	0	1

In other words, a fuzzy soft set (F, A) over U is a parametrized family of fuzzy sets in the universe U.

Example 2.2. Let $U=\{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $A=\{e_1, e_2, e_3, e_4\}$. Let (F, A) be a fuzzy soft set over U, defined as follows:

$$F(e_1) = \left\{ \frac{h_1}{0.6}, \frac{h_2}{1}, \frac{h_3}{0.2}, \frac{h_4}{0.3}, \frac{h_5}{1}, \frac{h_6}{0.7} \right\},$$

$$F(e_2) = \left\{ \frac{h_1}{1}, \frac{h_2}{0.5}, \frac{h_3}{0.3}, \frac{h_4}{0.2}, \frac{h_5}{0.1}, \frac{h_6}{0.9} \right\},$$

$$F(e_3) = \left\{ \frac{h_1}{0.1}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{1}, \frac{h_5}{0}, \frac{h_6}{0.1} \right\},$$

$$F(e_4) = \left\{ \frac{h_1}{0.1}, \frac{h_2}{0.3}, \frac{h_3}{1}, \frac{h_4}{0.9}, \frac{h_5}{0}, \frac{h_6}{1} \right\}.$$

Then (F, A) is described by Table 1.

It is easy to see that every soft set may be considered as a fuzzy soft set (see [7]).

Definition 2.3 *([20])*. Let $A, B \subseteq E$. Let $((F, A)$ and (G, B) be two fuzzy soft sets over U. Then "(F, A) AND (G, B)" is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for $\alpha \in A$ and $\beta \in B$.

2.2. Measure of performance of methods

Definition 2.4 *([2])*. The measure of performance of the method (M) which satisfies the optimality criteria to solve a fuzzy soft set in decision making is defined as follows

$$\Upsilon_M = \frac{1}{\sum_{i=1}^{m} \sum_{j=1, i \neq j}^{m} |F(e_i)(O_p) - F(e_j)(O_p)|} + \sum_{i=1}^{m} F(e_i)(O_p),$$

where m is the number of choice parameters and $F(e_i)(O_p)$ is the membership value of the optimal object O_p for the choice parameter e_i.

Suppose there are two methods M_1, M_2 which satisfy the optimality criteria and their measure of performances are respectively Υ_{M_1} and Υ_{M_2}. If $\Upsilon_{M_1} > \Upsilon_{M_2}$, then M_1 is better than M_2. If $\Upsilon_{M_1} < \Upsilon_{M_2}$, then M_2 is better than M_1. If $\Upsilon_{M_1} = \Upsilon_{M_2}$, then the performance of the both methods are the same.

2.3. Dempster–Shafer theory of evidence

Dempster–Shafer theory of evidence is a new important reasoning method under uncertainty. It has an advantage to deal with subjective judgments and to synthesize the uncertainty knowledge (see [32]). This theory discusses a frame of discernment, denoted by Θ, which is a finite nonempty set of mutually exclusive and exhaustive hypotheses (or all possible outcomes of an event), denoted by $\{A_1, A_2, \ldots, A_n\}$. 2^Θ denotes the set of all subsets of Θ.

The independence of evidences does not hold in many cases and people emphasize the independence of evidences in applications of Dempster–Shafer theory of evidence. Thus researchers ideally assume the independence of evidences in using Dempster–Shafer theory of evidence.

In this paper, we only consider that the sources of evidences are mutually exclusive ($i.e., A_i \cap A_j = \emptyset\ (i \neq j)$).

Definition 2.5 *([24])*. Let Θ be a frame of discernment. A basic probability assignment function (for short mass function) on Θ is defined a mapping $m : 2^{\Theta} \to [0, 1]$, m satisfies

$$m(\emptyset) = 0, \sum_{A \subseteq \Theta} m(A) = 1.$$

For each $A \subseteq \Theta$, $m(A)$ represents the belief measure that one is willing to commit exactly to A, given a certain piece of evidence.

Definition 2.6 *([24])*. Let Θ be the frame of discernment and $m : 2^{\Theta} \to [0, 1]$ be a mass function. Then a belief function $Bel : 2^{\Theta} \to [0, 1]$ on Θ is defined as follows:

$$Bel(\emptyset) = 0, Bel(A) = \sum_{B \subseteq A} m(B).$$

$Bel(A)$ represents the sum of the possibilities measurement of all subsets of A, namely, the total degree of support of A. Belief function represents the imprecision and uncertainty in the decision making process. In the case of singleton element, $Bel(A) = m(A)$.

In reality, a decision maker can often gain access to more than one information source in order to make decisions. The evidence theory constructs a set of hypotheses of known mass function values from these information sources and then computes a new set of combined evidences. This construction rule is called Dempster's rule of evidence combination for group aggregation.

Definition 2.7 *([24])*. Let Θ be the frame of discernment. Suppose there are two mass functions are m_1 and m_2 over Θ, induced by two independent items of evidences $A_1, A_2, \ldots, A_s$ and $B_1, B_2, \ldots, B_t$, respectively. Dempster's rule of evidence combination is as follows:

$$m(A) = m_1 \oplus m_2(A) = \begin{cases} \dfrac{1}{1-K} \displaystyle\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j), \forall A \subseteq \Theta, A \neq \emptyset, \\ 0, A = \emptyset, \end{cases}$$

where $K = \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j) < 1$.

K is called the conflict probability and reflects the extent of the conflict between the evidences. Coefficient $1/1 - K$ is called normalized factor, its role is to avoid the probability of assigning non-0 to empty set $\emptyset$ in the combination.

The synthesis of multiple evidences can be promoted according to Dempster's rule of evidence combination:

$$m_1 \oplus m_2 \cdots \oplus m_n(A) = \frac{1}{1-K} \sum_{\bigcap_{i=1}^{n} A_i = A, A_i \subseteq \Theta} m_1(A_1) m_2(A_2) \ldots m_n(A_n),$$

where $A \subseteq \Theta$, $A \neq \emptyset$, and $K = \sum_{\bigcap_{i=1}^{n} A_i = \emptyset, A_i \subseteq \Theta} m_1(A_1) m_2(A_2) \ldots m_n(A_n) < 1$.

Dempster's rule of evidence combination can increase belief measure and reduce the uncertain degree of the whole evidences to improve reliability.

Example 2.8. Let $\Theta = \{A_1, A_2\}$ be the frame of discernment. Suppose there are two mass functions m_1 and m_2 over Θ, induced by a independent piece of evidences A_1, A_2, given by

$$m_1(A_1) = 0.2, \quad m_1(A_2) = 0.5, \quad m_1(\Theta) = 0.3,$$
$$m_2(A_1) = 0.4, \quad m_2(A_2) = 0.3, \quad m_2(\Theta) = 0.3.$$

We apply Dempster's rule of evidence combination to combine the two evidences and then have:

$$m(A_1) = m_1 \oplus m_2(A_1) = \frac{m_1(A_1)m_2(A_1) + m_1(A_1)m_2(\Theta) + m_1(\Theta)m_2(A_1)}{1-K} = 0.3514,$$

$$m(A_2) = m_1 \oplus m_2(A_2) = \frac{m_1(A_2)m_2(A_2) + m_1(A_2)m_2(\Theta) + m_1(\Theta)m_2(A_2)}{1-K} = 0.5270,$$

$$m(\Theta) = m_1 \oplus m_2(\Theta) = \frac{m_1(\Theta)m_2(\Theta)}{1-K} = 0.1216,$$

where $K = m_1(A_1)m_2(A_2) + m_1(A_2)m_2(A_1) = 0.26$.

3. Mean potentiality approach

Like most of decision making problems, fuzzy soft sets based on decision making involve the evaluation of all decision alternatives. Recently, applications of fuzzy soft set based on decision making have attracted more and more attentions. The works of Roy et al. [7,13,23] are fundamental and significant. Later Kong et al. [14] applied grey relational analysis to solve fuzzy soft sets in decision making. Generally, there does not exist any unique or uniform criterion for the evaluation of decision alternatives under uncertain conditions. Thus, Basu et al. [2] further studied and proposed a mean potentiality approach to fuzzy soft sets in decision making, which is more deterministic and accurate than Feng's approach [7].

Below we introduce the mean potentiality approach.

Let $U = \{x_1, x_2, \ldots, x_m\}$ be a universe set and $A = \{e_1, e_2, \ldots, e_n\}$ be a parameter set. Given a fuzzy soft set (F, A), $F(e_j)(x_i)$ denotes the membership value that object x_i holds the parameter e_j and ρ denotes the maximum number of significant figures among all the membership values of the objects concerned with (F, A). Next, we mainly recall the mean potentiality approach to (F, A) based on decision making problem with equally weighted choice parameters:

Step 1. Find a normal parameter reduction B of A. If it exists, we construct the tabular representation of (F, B). Otherwise, we construct the tabular representation of (F, A) with the choice values of each object.

Step 2. Compute the mean potentiality $m_p = \sum_{i=1}^{m} \sum_{j=1}^{n} F(e_j)(x_i)/m \times n$ up to ρ significant figures, denoted by m'_p.

Step 3. Construct a m'_p-level soft set of (F, A) and represent it in a tabular form, then compute the choice value c_i for each x_i.

Step 4. Denote max $\{c_1, c_2, \ldots, c_m\} = c_k$. If c_k is unique, then the optimal choice object is x_k and the process will be stopped. Otherwise, go to Step 5.

Step 5. Compute the non-negative difference between the largest and the smallest membership value in each column (resp. each row) and denote it as a_j $(j = 1, 2, \ldots, n)$ (resp. β_i $(i = 1, 2, \ldots, m)$).

Step 6. Compute the average $\alpha = \sum_{j=1}^{n} a_j/n$ up to ρ significant figures, denoted by α'.

Step 7. Construct a α'-level soft set of (F, A) and represent it in tabular form, then compute the choice value c'_i for each x_i.

Step 8. Denote max $\{c'_1, c'_2, \ldots, c'_m\} = c'_l$. If c'_l is unique, then the optimal choice object is x_l and the process will be stopped. Otherwise, go to Step 9.

Step 9. Consider the object corresponding to the minimum value of β_i $(i = 1, 2, \ldots, m)$ as the optimal choice of decision makers.

We give the following example to illustrate mean potentiality approach to fuzzy soft sets in decision making.

Table 2
Tabular representation of the fuzzy soft set (F, A).

	e_1	e_2	e_3	e_4	e_5
x_1	0.85	0.71	0.38	0.32	0.75
x_2	0.56	0.82	0.76	0.64	0.43
x_3	0.84	0.51	0.82	0.53	0.47

Table 3
Tabular representation of the $L((F, A), 0.63)$ with choice values.

	e_1	e_2	e_3	e_4	e_5	Choice value
x_1	1	1	0	0	1	3
x_2	0	1	1	1	0	3
x_3	1	0	1	0	0	2

Table 4
Tabular representation of (F, A) with α_i and β_j values.

	e_1	e_2	e_3	e_4	e_5	β_i
x_1	0.85	0.71	0.38	0.32	0.75	0.53
x_2	0.56	0.82	0.76	0.64	0.43	0.39
x_3	0.84	0.51	0.82	0.53	0.47	0.37
α_j	0.29	0.31	0.44	0.32	0.32	

Table 5
Tabular representation of the $L((F, A), 0.34)$ with choice values.

	e_1	e_2	e_3	e_4	e_5	Choice value
x_1	1	1	1	0	1	4
x_2	1	1	1	1	1	5
x_3	1	1	1	1	1	5

Example 3.1. Let (F, A) be the fuzzy soft set given in Table 2.

(1) Since A is indispensable, there does not exist any normal parameter reduction of A.
(2) The mean potentiality of (F, A) is $m_p = \sum_{i=1}^{3}\sum_{j=1}^{5} F(e_j)(x_i)/3 \times 5 = 0.626$, thus $m'_p = 0.63$.
(3) m'_p-level soft set of (F, A) with choice values is given by Table 3.
(4) Since x_1 and x_2 have the same maximum choice values (3), we have to calculate the α_j and β_i values of (F, A).
(5) See Table 4
(6) Now $\alpha = 0.29 + 0.31 + 0.44 + 0.32 + 0.32/5 = 0.336$, thus $\alpha' = 0.34$.
(7) So the α'-level soft set of (F, A) with choice values c'_i $(i = 1, 2, 3)$ is given by Table 5.
(8) Here max $\{c'_1, c'_2, c'_3\} = \{c'_2, c'_3\}$, i.e., not unique, we have to consider the β_2 and β_3.
(9) Since $\min\{\beta_2, \beta_3\} = \beta_3(= 0.37)$, x_3 is the optimal choice object.

4. A novel fuzzy soft set approach in decision making

The existing approaches to fuzzy soft sets in decision making are mainly based on the level soft set to obtain useful information such as choice values and score values. However, it is very difficult for decision makers to select a suitable level soft set. Inspired by the work of Wu et al. [16,25], we introduce a new approach to fuzzy soft sets in decision making based on grey relational analysis and Dempster–Shafer theory of evidence. It not only allows us to avoid the problem of selecting the suitable level soft set, but also helps reducing uncertainty caused by people's subjective cognition so as to raise the choice decision level.

Our approach includes three phases: first, grey relational analysis is applied to calculate the grey mean relational degree between each independent alternative and the mean of all alternatives with each parameter, and the uncertain degree of each parameter is obtained. Second, the suitable mass function with respect to each parameter (or evidence) is constructed by the uncertain degree of each parameter. Third, we apply Dempster's rule of evidence combination to aggregate independent evidences into a collective evidence, by which the candidate's alternatives are ranked and the best alternative(s) are obtained.

In the following, we consider a decision making problem concerned with m mutually exclusive alternatives x_i and n evaluation parameters (or evidences) e_j. x_{ij} denotes the value that the choice decision x_i is supported by the evidence e_j. Put

$$\Theta = \{x_1, x_2, \ldots, x_m\} \quad \text{and} \quad A = \{e_1, e_2, \ldots, e_n\}.$$

Define $F : A \to I^{\Theta}$ by $F(e_j)(x_i) = x_{ij}$. Then (F, A) is a fuzzy soft set over Θ and $D = (x_{ij})_{m\times n}$ is called a fuzzy soft decision matrix induced by (F, A).

In this paper, we will consider the parameter set of the decision making as a set of evidence.

Compared to probability theory, Dempster–Shafer theory of evidence captures more information to support decision making, by identifying the uncertain and unknown evidences. It provides a mechanism to derive solutions from various vague evidences (or parameter) without knowing much prior information. We must get mass functions of alternatives with each evidence (or parameter) beforehand if we apply Dempster–Shafer theory of evidence to make decisions. However, how to find uncertain degree of evidences (or parameters) is a critical problem. Grey relational analysis is employed as a means to reflect uncertainty between experts in multiple parameter models through the membership value. Next, we apply grey relational analysis to obtain uncertain degree of evidences (or parameters).

We first present some basic notions.

Definition 4.1 *([15])*. Let $\Theta = \{x_1, x_2, \ldots, x_m\}$, $A = \{e_1, e_2, \ldots, e_n\}$ and let (F, A) be a fuzzy soft set on Θ. Suppose that $D = (x_{ij})_{m\times n}$ is a fuzzy soft decision matrix induced by (F, A). For each i, j, denote

$$\widetilde{x_i} = \frac{1}{n}\sum_{j=1}^{n} x_{ij}, \quad \Delta x_{ij} = |x_{ij} - \widetilde{x_i}|,$$

$$r_{ij} = \frac{\min_{1\leqslant i\leqslant m}\Delta x_{ij} + \rho \max_{1\leqslant i\leqslant m}\Delta x_{ij}}{\Delta x_{ij} + \rho \ \max_{1\leqslant i\leqslant m}\Delta x_{ij}}, \quad \text{where} \quad \rho \in (0, 1),$$

$$DOI(e_j) = \frac{1}{m}\left(\sum_{i=1}^{m}(r_{ij})^q\right)^{\frac{1}{q}} \quad (j = 1, 2, \ldots, n).$$

Then

(1) $\widetilde{x_i}$ is called the mean of all parameters with respect to x_i,
(2) Δx_{ij} is called the difference information between x_{ij} and $\widetilde{x_i}$,
(3) ρ is called the distinguishing coefficient and r_{ij} is called the grey mean relational degree between x_{ij} and $\widetilde{x_i}$,
(4) $DOI(e_j)$ is called q order uncertain degree of the parameter e_j.

The distinguishing coefficient ρ is an important factor to affect the correlation analysis resolution and the value distribution of the grey mean relational degrees r_{ij}. The purpose of ρ is to weaken the Max value's affect when this value is so large that loses really and to improve the difference between the grey mean relational degrees r_{ij}. ρ and the correlation analysis resolution is inversely proportional, i.e., the smaller ρ is, the greater the correlation analysis resolution is; the greater ρ is, the smaller the correlation analysis resolution is. According to the experience, we pick $\rho = 0.5$ to obtain strong distinguishing effectiveness, which can more accurately reflect the related degree between x_{ij} and $\widetilde{x_i}$.

Similarly, to obtain strong distinguishing effectiveness, we use the Euclidean distance rather than the Hamming distance, namely $q=2$.

It is worthy to notice that the approach to obtain the uncertain degree in Definition 4.1 varies in different situation. Since a parameter is specially more matching with the mean of the parameter set than other parameters, the parameter contains more satisfying information for decision making and the uncertain degree of the parameter with respect to alternatives is lower. Then, in this paper we just consider grey mean relational degree between x_{ij} and $\widetilde{x_i}$.

Definition 4.2 *([31])*. Let $X=(x_1, x_2, \ldots, x_m)$ be a finite difference information sequence, where there exists $x_{i_k} \neq 0$ for $k=1, 2, \ldots, m$ and $1 \leqslant i_k \leqslant m$. Then the information structure image sequence $Y=(y_1, y_2, \ldots, y_m)$ is given by $y_i = x_i/\sum_{i=1}^{m} x_i$.

In a fuzzy soft decision matrix $D=(x_{ij})_{m\times n}$ concerned with m mutually exclusive alternatives x_i and n evaluation parameters e_j, where x_{ij} is the membership value of x_i with e_j. The information structure image sequence with respect to e_j is denoted by $x_j = \{\widetilde{x_{1j}}, \widetilde{x_{2j}}, \widetilde{x_{3j}}, \ldots, \widetilde{x_{mj}}\}$, where $\widetilde{x_{ij}} = x_{ij}/\sum_{i=1}^{m} x_{ij}$. Then we obtain an information structure image matrix by $x_j (j=1, 2, \ldots, n)$.

Dempster–Shafer theory of evidence is a powerful method for combining accumulative evidences of changing prior opinions in the light of new evidences [24]. The primary procedure about combining the known evidences or information with other evidences is to construct suitable mass functions of evidences. It is flexible to obtain mass function and people's experience, knowledge or thinking will affect the selection of mass function.

Now, by the uncertain degree of each parameter, we can obtain mass function of each alternative with respect to each parameter.

Theorem 4.3. *Let $\Theta=\{x_1, x_2, \ldots, x_m\}$, $A=\{e_1, e_2, \ldots, e_n\}$ and let (F, A) be a fuzzy soft set on Θ. Suppose that $D=(x_{ij})_{m\times n}$ is a fuzzy soft decision matrix induced by (F, A), where x_{ij} denotes the membership value that the alternative x_i holds the parameter e_j. Denote $\widetilde{x_{ij}} = x_{ij}/\sum_{i=1}^{m} x_{ij}$. We define functions $m_{e_j}(j=1, 2, \ldots, n)$ with respect to the parameter e_j, it satisfies:*

$$m_{e_j}(x_i) = \widetilde{x_{ij}}(1 - DOI(e_j)) \quad (i=1, 2, \ldots, m, j=1, 2, \ldots, n),$$

$$m_{e_j}(\Theta) = 1 - \sum_{i=1}^{m} m_j(i) \quad (j=1, 2, \ldots, n).$$

Then $m_{e_j}(j=1, 2, \ldots, n)$ are mass functions.

In a fuzzy soft decision matrix $D=(x_{ij})_{m\times n}$, denote $m_{e_j}(x_i)$, $m_{e_j}(\Theta)$ by $m_j(i)$ and $m_j(m+1)$, respectively. $m_j(i)$ implies the belief measure of the alternative x_i with the parameter e_j. $m_j(m+1)$ implies the belief measure of the whole uncertainty with respect to the parameter e_j.

Since a single evidence does not fully reflect the characteristics of the operating rules of things, then multiple evidences are used when we make decisions. How to promote the organic integration of different evidences, this needs Dempster's rule of evidence because using this rule can make the fusion of evidences, the fusion of evidences is to reduce the uncertainty and to manage the conflicting information, and the results of the fusion of evidences characterizes the nature of complex phenomena. Thus, we use Dempster's rule of evidence in this paper.

Next, using Dempster's rule of evidence combination to compose $m_1, m_2, \ldots, m_n$ with respect to each alternative x_i, we get the belief measure of each alternative x_i. Thus we obtain decision making results.

Based on the above analysis, given a fuzzy soft set (F, E) concerned with m mutually exclusive alternatives x_i and n evaluation parameters e_j, the decision procedure of our approach for (F, E) can be summarized as follows:

Step 1. Construct a fuzzy soft decision matrix $D=(x_{ij})_{m\times n}$ induced by (F, A), where x_{ij} is the membership value of x_i with e_j.

Step 2. Calculate the mean of all parameters with respect to each alternative by

$$\widetilde{x_i} = \frac{1}{n}\sum_{j=1}^{n} x_{ij} (i=1, 2, \ldots, m).$$

Step 3. Calculate the difference information between x_{ij} and $\widetilde{x_i}$ and construct the difference matrix by

$$\Delta x_{ij} = |x_{ij} - \widetilde{x_i}|, \quad \Delta D = (\Delta x_{ij})_{m\times n}$$
$$(i=1, 2, \ldots, m, j=1, 2, \ldots, n).$$

Step 4. Calculate the grey mean relational degree between x_{ij} and $\widetilde{x_i}$ by

$$r_{ij} = \frac{\min_{1\leqslant i\leqslant m} \Delta x_{ij} + 0.5 \quad \max_{1\leqslant i\leqslant m} \Delta x_{ij}}{\Delta x_{ij} + 0.5 \quad \max_{1\leqslant i\leqslant m} \Delta x_{ij}}$$
$$(i=1, 2, \ldots, m, j=1, 2, \ldots, n).$$

Step 5. Calculate the uncertain degree of each parameter e_j by

$$DOI(e_j) = \frac{1}{m}\left(\sum_{i=1}^{m} (r_{ij})^2\right)^{\frac{1}{2}} \quad (j=1, 2, \ldots, n).$$

Step 6. Calculate the information structure image sequence x_j with respect to each parameter e_j and construct the matrix by Definition 4.2.

Step 7. Calculate mass function values of each alternative x_i and Θ with respect to each parameter e_j by Theorem 4.3.

Step 8. Calculate belief measure of each alternative x_i by Definition 2.10.

Step 9. Obtain decision making. The decision is x_k if $c_k = maxBel(\{x_i\})$.

Optimal choices have more than one object if there are more alternatives corresponding to the maximum.

5. An illustrative example

In this section, we give the following example to illustrate our approach.

Example 5.1. Using our approach, we reconsider the fuzzy soft set (F, A) given in Example 3.1.

Now, we suppose that the three mutually exclusive and exhaustive alternatives construct a frame of discernment, denoted by $\Theta=\{x_1, x_2, x_3\}$. We consider the set of parameters $A=\{e_1, e_2, e_3, e_4, e_5\}$ as a set of evidences.

(1) Construct a fuzzy soft decision matrix induced by (F, A) as follows:

$$D = (x_{ij})_{3\times 5} = \begin{pmatrix} 0.85 & 0.71 & 0.38 & 0.32 & 0.75 \\ 0.56 & 0.82 & 0.76 & 0.64 & 0.43 \\ 0.84 & 0.51 & 0.82 & 0.53 & 0.47 \end{pmatrix}$$

(2) Calculate the mean of all parameters with respect to each alternative x_i as follows:

$$\widetilde{x_1} = 0.6020, \widetilde{x_2} = 0.6420, \widetilde{x_3} = 0.6340,$$

(3) Calculate the difference information between x_{ij} and $\widetilde{x_i}$, and construct the difference matrix as follows:

$$\Delta D = \begin{pmatrix} 0.2480 & 0.1080 & 0.2220 & 0.2820 & 0.1480 \\ 0.0820 & 0.1780 & 0.1180 & 0.0020 & 0.2120 \\ 0.2060 & 0.1240 & 0.1860 & 0.1040 & 0.1640 \end{pmatrix}$$

(4) Calculate the grey mean relational degree between x_{ij} and $\widetilde{x_i}$ based on ΔD as follows:

$$(r_{ij})_{3\times 5} = \begin{pmatrix} 0.5538 & 1.0000 & 0.6877 & 0.3381 & 1.0000 \\ 1.0000 & 0.7378 & 1.0000 & 1.0000 & 0.7987 \\ 0.6242 & 0.9249 & 0.7710 & 0.5837 & 0.9407 \end{pmatrix}$$

(5) Calculate the uncertain degree of each parameter e_j as follows:

$$DOI(e_1) = 0.4341, \quad DOI(e_2) = 0.5164, \quad DOI(e_3) = 0.4793,$$
$$DOI(e_4) = 0.4021, \quad DOI(e_5) = 0.5295.$$

(6) Calculate the information structure image sequence with respect to each parameter e_j and construct the matrix as follows:

$$\widetilde{D} = (\widetilde{x_{ij}})_{3\times 5} = \begin{pmatrix} 0.3778 & 0.3480 & 0.1939 & 0.2148 & 0.4545 \\ 0.2489 & 0.4020 & 0.3878 & 0.4295 & 0.2606 \\ 0.3733 & 0.2500 & 0.4184 & 0.3557 & 0.2848 \end{pmatrix}$$

(7) For each $A \in 2^{\Theta}$ with $|A| = 0$ or 2, put $m(A) = 0$. We calculate mass function values of each alternative x_i and Θ with respect to parameter e_j by Theorem 4.3:

$$(m_j(i))_{3\times 5} = \begin{pmatrix} 0.2138 & 0.1683 & 0.1010 & 0.1284 & 0.2139 \\ 0.1408 & 0.1944 & 0.2019 & 0.2568 & 0.1226 \\ 0.2113 & 0.1209 & 0.2179 & 0.2127 & 0.1340 \\ \end{pmatrix} \tag{1}$$

and

$$m_1(4) = 0.4341, \quad m_2(4) = 0.5164, \quad m_3(4) = 0.4793,$$
$$m_4(4) = 0.4021, \quad m_5(4) = 0.5295.$$

The mean of belief measure of the whole uncertainty is

$$(0.4341 + 0.5164 + 0.4793 + 0.4021 + 0.5295)/5 = 0.4723$$

Table 6
Tabular representation of the soft set (G, B).

Decision approach	Reference	Ranking result	$m(\Theta)$	Υ
Mean potentiality approach	[2]	$x_3 \succ x_2 \succ x_1$	0.4723	3.6462
Our approach		$x_2 \succ x_3 \succ x_1$	0.0782	3.7202

(8) Calculate belief measure of each alternative x_i by combining these evidences respectively by Definition 2.7:

$$Bel(\{x_1\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_5(\{x_1\}) = 0.2690,$$
$$Bel(\{x_2\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_5(\{x_2\}) = 0.3309,$$
$$Bel(\{x_3\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_5(\{x_3\}) = 0.3218,$$
$$Bel(\Theta) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_5(\Theta) = 0.0782.$$

Then the final rang order is $x_2 \succ x_3 \succ x_1$.

(9) According to the maximum belief measure principle, the optimally choice decision is x_2, which is different from the choice decision by using the mean potentiality approach in Example 3.1.

Now, by Definition 2.7, we can calculate the measure of performance of our approach $\Upsilon = 3.7202$, and the measure of performance of the mean potentiality approach $\Upsilon = 3.6462$.

In summary, the result of the comparison of the above two approaches can be shown in Table 6.

From the above results, the belief measure of the whole uncertainty falls from the initial mean 0.4723–0.0782 after evidence combination. This implies that our approach combined grey relational analysis with Dempster–Shafer theory of evidence can help reducing uncertainty caused by people's subjective cognition so as to raise the choice decision level. Moreover, judged by the measures Υ of performance, our approach is more accurate and effective than the mean potentiality approach under uncertain information.

6. An application for medical diagnosis

A major task of medical science is to diagnose diseases. Generally a patient suffering from a disease may have multiple symptoms and the information available to physician about his patient is inherently uncertain. Again it is also observed that there are certain symptoms which may be common to more than one diseases leading to diagnostic dilemma. Doctors always detect clinical manifestations by the comparison with predefined classes to find the most similar disease. Only one comprehensive result can be gotten from existing methods for medical diagnosis, which cannot provide the certainty or uncertainty of the result. One of the toughest challenges in medical diagnosis is handling uncertainty. Therefore, it is necessary to find another method to deal with the unknown factors in the process of medical diagnosis and improve level of medical diagnosis, then we apply our approach to solve medical diagnosis problems.

Now we consider a medical diagnosis problem with seven symptoms such as fever, running nose, weakness, oro-facial pain, nausea vomiting, swelling, trismus which have more or less contribution

Table 7
Tabular representation of the soft set (F, A).

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
x_1	0.6	0	0.6	0.9	0	0.7	0.8
x_2	0.2	0	0.1	0.9	0.8	0	0
x_3	0.3	0.7	0.3	0.8	0.3	0.4	0
x_4	0.4	0	0.2	0.7	0.1	0.6	0.5

Table 8
Tabular representation of the soft set (G, B).

	s_1	s_2	s_3
x_1	0.6	0.8	0.4
x_2	0.8	0.3	0.6
x_3	0.8	0.4	0.7
x_4	0.6	0.8	0.3

in four diseases such as acute dental abscess, migraine, acute sinusitis, peritonsillar abscess. Now, from medical statistics, the degree of availability of these seven symptoms in these four diseases is observed as follows. The degree of belonging of all the symptoms fever, running nose, weakness, oro-facial pain, nausea vomiting, swelling, trismus for the diseases acute dental abscess, migraine, acute sinusitis, peritonsillar abscess respectively are {0.6, 0, 0.6, 0.9, 0, 0.7, 0.8}, {0.2, 0, 0.1, 0.9, 0.8, 0, 0}, {0.3, 0.7, 0.3, 0.8, 0.3, 0.4, 0}, {0.4, 0, 0.2, 0.7, 0.1, 0.6, 0.5}. The degree of belonging of all the detecting tools history, physical examination, laboratory investigation for these diseases are {0.6, 0.8, 0.4}, {0.8, 0.3, 0.6}, {0.8, 0.4, 0.7}, {0.6, 0.8, 0.3}, respectively. Suppose a patient who is suffering a disease, have the symptoms fever, running nose, oro-facial pain and is diagnosed by the three tools. Now the problem is how a doctor detects the actual disease with effective symptoms and diagnosed tools among these four diseases for that patient. To solve this problem first we detect the disease which is most suited with the observed symptoms of patients and then secondly we find the actual symptoms which is optimal for that disease. These can be solved by means of fuzzy soft sets.

We use the following notations:

(*i*) {fever, running nose, weakness, oro-facial pain, nausea vomiting, swelling, trismus, history, physical examination, laboratory investigation}

$=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, s_1, s_2, s_3\} = E$. $A = \{e_1, e_2, e_4\}$, $B = \{s_1, s_2, s_3\}$.

(*ii*) {acute dental abscess, migraine, acute sinusitis, peritonsillar abscess} $= \{x_1, x_2, x_3, x_4\} = U$

Now, (F, A) and (G, B) are given in Tables 7 and 8, respectively. They are describing "symptoms of the diseases" and "decision making tools of the diseases", respectively. $(F, A) \wedge (G, B)$ is given in Table 9.

The key problem is how a doctor reaches to the most suitable diagnosis according to these symptoms, history, physical examination and laboratory investigation of patients. To solve this problem, we consider "(F, A) AND (G, B)", given by Table 10. There are four diseases x_1, x_2, x_3, x_4, and nine pairs of parameters $a_1 = (e_1, s_1)$, $a_2 = (e_1, s_2)$, $a_3 = (e_1, s_3)$, $a_4 = (e_2, s_1)$, $a_5 = (e_2, s_2)$, $a_6 = (e_2, s_3)$, $a_7 = (e_4, s_1)$, $a_8 = (e_4, s_2)$, $a_9 = (e_4, s_3)$, which is a pair of one symptom and one decision making tool, respectively.

Next we will apply our approach to detect which disease is most suited with the symptoms and these investigative procedures. Then, in the making decision, we consider that the four diseases construct a frame of discernment, denoted by $\Theta = \{x_1, x_2, x_3, x_4\}$. We consider the nine pairs of parameters as a set of evidences, which contains a diagnosis parameter system, denoted by $P = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$.

Step 1. We construct a fuzzy soft decision matrix induced by "(F, A) AND (G, B)", which completely presents the degree that a patient is suffering from a disease x_i with one symptom and one decision making tool a_j.

$$D = (x_{ij})_{4\times 9} = \begin{pmatrix} 0.6 & 0.6 & 0.4 & 0 & 0 & 0 & 0.6 & 0.8 & 0.4 \\ 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0.8 & 0.3 & 0.6 \\ 0.3 & 0.3 & 0.3 & 0.7 & 0.4 & 0.7 & 0.8 & 0.4 & 0.7 \\ 0.4 & 0.4 & 0.3 & 0 & 0 & 0 & 0.6 & 0.7 & 0.3 \end{pmatrix}$$

Step 2. Since a_j is specially more matching the mean of the parameter set than other parameters, a_j contains the satisfying information for decision making and the uncertain degree of a_j is low. Now, we consider the mean possible of the diagnosis parameter system with respect to the four diseases, calculated by $\widetilde{x_i} = \frac{1}{9}\sum_{j=1}^{9} x_{ij}$ as follows:

$$\widetilde{x_1} = 0.3778, \widetilde{x_2} = 0.2556, \widetilde{x_3} = 0.5111, \widetilde{x_4} = 0.3000.$$

Step 3. To obtain the grey mean relational degree, we need to calculate the difference information between x_{ij} and $\widetilde{x_i}$ and construct the difference matrix as follows:

$$\Delta D = \begin{pmatrix} 0.2222 & 0.2222 & 0.0222 & 0.3778 & 0.3778 & 0.3778 & 0.2222 & 0.4222 & 0.0222 \\ 0.0556 & 0.0556 & 0.0556 & 0.2556 & 0.2556 & 0.2556 & 0.5444 & 0.0444 & 0.3444 \\ 0.2111 & 0.2111 & 0.2111 & 0.1889 & 0.1111 & 0.1889 & 0.2889 & 0.1111 & 0.1889 \\ 0.1000 & 0.1000 & 0.0000 & 0.3000 & 0.3000 & 0.3000 & 0.3000 & 0.4000 & 0.0000 \end{pmatrix}$$

Table 9
Tabular representation of the soft set $(F, A) \wedge (G, B)$

	(e_1, s_1)	(e_1, s_2)	(e_1, s_3)	(e_2, s_1)	(e_2, s_2)	(e_2, s_3)	(e_4, s_1)	(e_4, s_2)	(e_4, s_3)
x_1	0.6	0.6	0.4	0	0	0	0.6	0.8	0.4
x_2	0.2	0.2	0.2	0	0	0	0.8	0.3	0.6
x_3	0.3	0.3	0.3	0.7	0.4	0.7	0.8	0.4	0.7
x_4	0.4	0.4	0.3	0	0	0	0.6	0.7	0.3

Table 10
Tabular representation of the soft set (G, B).

Decision approach	Reference	Ranking result	$m(\Theta)$	Υ
Mean potentiality approach	[2]	$x_3 \succ x_1 = x_4 \succ x_2$	0.3821	4.716
this approach		$x_3 \succ x_1 \succ x_4 \succ x_2$	0.0069	4.716

Step 4. Based on the difference matrix ΔD, the gray mean relational degree between x_{ij} and $\widetilde{x_i}$ is calculated as follows:

$$(r_{ij})_{4\times 9} = \begin{pmatrix} 0.5000 & 0.5000 & 0.8261 & 0.6667 & 0.5294 & 0.6667 & 1.0000 & 0.4035 & 0.8857 \\ 1.0000 & 1.0000 & 0.6552 & 0.8500 & 0.6750 & 0.8500 & 0.6054 & 1.0000 & 0.3333 \\ 0.5172 & 0.5172 & 0.3333 & 1.0000 & 1.0000 & 1.0000 & 0.8812 & 0.7931 & 0.4769 \\ 0.7895 & 0.7895 & 1.0000 & 0.7727 & 0.6136 & 0.7727 & 0.8641 & 0.4182 & 1.0000 \end{pmatrix}$$

Step 5. In order to obtain mass functions of x_i and Θ with respect to a_j, now we need to calculate the uncertain degree of a_j as follows:

$$DOI(a_1) = 0.3658, \quad DOI(a_2) = 0.3658, \quad DOI(a_3) = 0.3727,$$
$$DOI(a_4) = 0.4156, \quad DOI(a_5) = 0.3634, \quad DOI(a_6) = 0.4156,$$
$$DOI(a_7) = 0.4250, \quad DOI(a_8) = 0.3506, \quad DOI(a_9) = 0.3643.$$

Step 6. We calculate the information structure image sequence with respect to a_j and construct the matrix as follows:

$$\widetilde{D} = (\widetilde{x_{ij}})_{4\times 9} = \begin{pmatrix} 0.4000 & 0.4000 & 0.3333 & 0 & 0 & 0 & 0.2143 & 0.3636 & 0.2000 \\ 0.1333 & 0.1333 & 0.1667 & 0 & 0 & 0 & 0.2857 & 0.1364 & 0.3000 \\ 0.2000 & 0.2000 & 0.2500 & 1.0000 & 1.0000 & 1.0000 & 0.2857 & 0.1818 & 0.3500 \\ 0.2667 & 0.2667 & 0.2500 & 0 & 0 & 0 & 0.2143 & 0.3182 & 0.1500 \end{pmatrix}$$

Step 7. For each $A \in 2^{\Theta}$ with $|A| = 0, 2, 3$, put $m(A) = 0$. We calculate mass function values of x_i and Θ with respect to a_j by Theorem 4.3 as follows:

$$(m_j(i))_{4\times 9} = \begin{pmatrix} 0.2537 & 0.2537 & 0.2091 & 0 & 0 & 0 & 0.1232 & 0.2361 & 0.1271 \\ 0.0846 & 0.0846 & 0.1045 & 0 & 0 & 0 & 0.1643 & 0.0886 & 0.1907 \\ 0.1268 & 0.1268 & 0.1568 & 0.5844 & 0.6366 & 0.5844 & 0.1643 & 0.1181 & 0.2225 \\ 0.1691 & 0.1691 & 0.1568 & 0 & 0 & 0 & 0.1232 & 0.2066 & 0.0954 \end{pmatrix}$$

and

$$m_1(5) = 0.3658, \quad m_2(5) = 0.3658, \quad m_3(5) = 0.3727,$$
$$m_4(5) = 0.4156, \quad m_5(5) = 0.3634, \quad m_6(5) = 0.4156,$$
$$m_7(5) = 0.4250, \quad m_8(5) = 0.3506, \quad m_9(5) = 0.3643.$$

The mean of belief measure of the whole uncertainty is $1/9\sum_{j=1}^{9} m_j(5) = 0.3821$.

Step 8. The combination of parameters (or evidences) is used to provide the strongest evidence for medical diagnosis problem. By Definition 2.10, we can get the following results:

$$Bel(\{x_1\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_9(\{x_1\}) = 0.0827,$$
$$Bel(\{x_2\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_9(\{x_2\}) = 0.0284,$$
$$Bel(\{x_3\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_9(\{x_3\}) = 0.8349,$$
$$Bel(\{x_4\}) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_9(\{x_4\}) = 0.0471,$$
$$Bel(\Theta) = m_1 \oplus m_2 \oplus m_3 \cdots \oplus m_9(\Theta) = 0.0069.$$

Then the final range order is $x_3 \succ x_1 \succ x_4 \succ x_2$.

Step 9. According to the maximum belief measure principle, the patient is suffering from acute sinusitis x_3, which is the same choice decision based on the mean potentiality approach of Example 6.2 in [2].

By Definition 2.7, the measure of performance of our approach is the same $\Upsilon = 4.1726$ as the mean potentiality approach's.

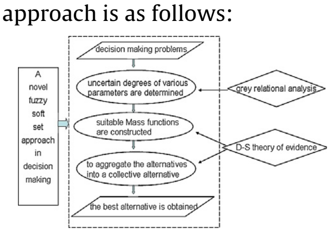
In summary, the results of this comparison study can be shown in Table 10.

From above results, the initial mean of the belief measure of the whole uncertainty is $1/9\sum_{j=1}^{9} m_j(5) = 0.3821$, but the belief measure of the whole uncertainty falls the originally mean value 0.3821–0.0069 after evidence combination. It implies that our approach combined Dempster–Shafer theory of evidence with grey relational analysis declines the uncertainty to a great extent and improves accuracy and reliability of medical diagnosis effectively.

It is worthy noting that the result from our approach also gives the second and the third possible disease, which is clearer than that of Basu's approach.

7. Conclusions

In this paper, we have introduced a novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence and given an application for medical diagnosis. Detailed flow chart and schematic block diagram of the proposed approach is as follows:



Since the measure of performance of this approach is the same as the mean potentiality approach's, the belief measure of the whole uncertainty falls from 0.4723 to 0.0782 (resp. 0.3821 to 0.0069) in the example of Section 5 (resp. Section 6), the effectiveness and feasibility of this approach are demonstrated by comparing with the mean potentiality approach. The disadvantages of this approach are also reflected in here because it do not compare with more approaches. This approach can help reducing uncertainty caused by people's subjective cognition so as to raise the choice decision level, allows us to avoid this problem of selecting the suitable level soft set and are more feasible and practical for dealing with applications under uncertainty. Moreover, this approach sets up decision making models and thus broadens application fields of grey system theory. In future work, we will test the accuracy and efficiency of the proposed system.

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