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# Multivariate dependence risk and portfolio optimization: An application to mining stock portfolios

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ABSTRACT

This study proposes an integrated framework to model and estimate relatively large dependence matrices using pair vine copulas and minimum risk optimal portfolios with respect to five risk measures within the context of the global financial crisis. We apply this methodology to two 20-asset mining (gold and iron ore-nickel) sector portfolios from the Australian Securities Exchange. The pair vine copulas prove to be powerful tools for the modeling of changing dependence risk under three different period scenarios combined with the optimization of portfolios that have complex patterns of dependence. The portfolio optimization results converge, on average, in some stocks.


## 1. Introduction

The recent global financial crisis (GFC) had contributed to global dependence shifts and portfolio losses, mainly due to changes in joint dynamics and network-dependence relationships, and the large increases in volatility spillovers between international financial markets (Brunnermeier, 2009; Moshirian, 2011; Florackis et al., 2014). Doubtlessly, the severe financial fluctuations, which resulted from this global crisis, have put into question the reliability and the estimation abilities of the *status quo* mathematical and statistical models used in dependence estimation and portfolio optimization. This in turn has led to the quest for finding techniques that more accurately approximate the underlying interactions of the variables and better optimize portfolios, while considering important risk factors other than the traditional ones.

The copula approach in the form of bivariate copula and pair vine copula models (e.g., c-vine, d-vine and r-vine) has recently been proposed to more accurately estimate the dependence matrix of financial variables (e.g., Chollete et al., 2009; Aloui et al., 2011; Low et al., 2013).[2] It overcomes the restrictive and deterministic features of the bilateral correlation coefficient approach, traditionally used in portfolio optimization algorithms, due to its suitability to capture the distributional characteristics of asset returns such as volatility clustering, fat tails, tail dependence and asymmetric correlation.[3] In the context of multivariate dependence modeling, the pair vine copulas, which are built on the theory of graphical models, provide greater flexibility than the bivariate copulas because they allow for dissections and decompositions, while capturing, in a more localized and specialized manner, the distributional characteristics of different forms (see,

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[2] C-vine, d-vine and r-vine refer to the canonical, the drawable and the regular vine copulas. These copula models have been found to outperform alternative models in terms of dependence structure estimation (e.g., Low et al., 2013).

[3] Asymmetric correlations between financial markets in bear and bull periods have been documented by, among others, Erb et al. (1994), Ang and Bekaert (2002), and Patton (2004). They refer to the fact that negative returns are more correlated than positive returns, suggesting that financial markets tend to be more dependent in crisis times characterized for low or no confidence in the financial markets. Aloui et al. (2011) find evidence of asymmetric dependence of stock returns between the BRIC and U.S. markets.

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e.g., Aas et al., 2009; Czado et al., 2012,2013).

This article deals with dependence structure estimation and optimal asset allocation with respect to the variance, the mean absolute deviation (*MAD*), the minimizing regret (*Minimax*), the conditional Value-at-Risk (*CVaR*) and the conditional Drawdown-at-Risk (*CDaR*) measures. The returns of the assets in the portfolios are random variables generated under various market conditions. We first contribute to the related literature by adopting a flexible pair vine copula approach to model the dependence risk and the risk features of Australian gold and iron ore-nickel stock portfolios. This task is carried out within the context of three financial period scenarios: the pre-GFC, the GFC and the post-GFC periods. The first distinction of this specific type of methodology and analysis is that it allows one to find out the market conditions under which one sector portfolio may be more volatile and risky than the other. Besides, it enables one to identify the type of vine copula model which better accounts for dependence in the tails.

Our second contribution stems from the integration of the pair vine copula and portfolio optimization models. This combined modeling approach is applied to investigate the portfolio allocation characteristics of the gold and iron ore-nickel stock portfolios under consideration. In practice, we introduce into the portfolio optimization models the pair vine copula estimates of the dependence structure which captures the linear and nonlinear dependence relationships between the gold stocks as well as between the iron ore-nickel stocks. We expect the use of pair vine copulas in portfolio optimization to provide an estimation edge over the rest of the models.

Our third contribution arises from the multi-angled portfolio optimization approach implemented. We specifically propose an alternative avenue to address the problem of investment confidence that the mining portfolio investors face when a variety of optimal weight allocations is presented to them for selection.[4] The proposed approach handles the multiple weight allocation investment possibilities in terms of “average model convergence”. This means that while we are still interested in finding the best risk measure to be used for the optimization of portfolios, we particularly focus on identifying the stocks to which most of the alternative optimization models converge without a large deviation from a mean of weights. While this approach represents a shift of perspective in the analysis, evaluation and interpretation of multiple optimal weight allocations, it is also thought to be an effective way to deal with the problem of investment confidence.

Empirically, we illustrate the relevance of our analytical approach for dependence modeling and dependence shift detection by considering two mining portfolios (gold versus iron ore – nickel) of 20 stocks, which trade on the Australian Securities Exchange (ASX).[5] We select these two mining portfolios because they include highly important mineral commodities which are extracted, processed, and traded in and exported from Australia. During the 2008–2009 global financial crisis, for instance, the production and export of gold had contributed to softening the effect of the financial crisis on the Australian economy. On the other hand, the 2015 sharp decline in prices of iron ore has proven, in terms of the harsh domestic budgetary adjustments that took place, how important the iron-ore production and exports are to the Australian economy. In this context, the identification of stocks for investment and the stock portfolios' risk profile is important to portfolio managers, investors and policymakers since the obtained results could be used to manage the resource allocation risk, market downturn risk and market sector risk. Another important reason for selecting those mining portfolios is that they are different in terms of structure, volatility, uses and their importance in asset investment than other sectors, which enables one to test and analyze the behavior of different components in our integrated modeling framework (i.e., vine copulas, risk measures and portfolio optimization).[6] The gold stock sector, for instance, does not have a dominant company that has exceptionally high correlations with the rest of the stocks in the portfolio. Instead, there are a handful of companies having relatively high correlations with each other but none of them is dominant. This is not the case for the iron ore – nickel stock portfolio, which has BHP BILLITON (BHPX) as the dominant stock in the sense that it has high correlation values with the rest of the stocks in this portfolio. Through the modeling of the dependence structure of the portfolios, we are also able to address the question of whether the Australian gold stocks can serve as a hedge and safe haven during financial crisis periods.

With respect to the literature that uses copulas in portfolio optimization, our research is broadly linked to the contributions of Kakouris and Rustem (2014), Low et al. (2013) and Brechmann and Czado (2013) and Ye et al. (2012). The latter provided a measurement methodology for the subprime crisis contagion based on copula change point analysis, whilst Kakouris, Rustem (2014) employ a mixture of copulas to derive *CVaR* and the worse-case *CVaR* used for the optimization of a convex portfolio of stock indices. Low et al. (2013) make use of the bivariate Clayton and the Clayton canonical vine copulas to address the asset allocation for loss-averse investors through the minimization of *CVaR* in portfolios of up to 12 constituents. Brechmann and Czado (2013) develop a regular vine copula-based factor model which is applied to the asset returns of the Euro Stoxx 50 index constituents in order to investigate the Value-at-Risk forecasting and asset allocation. Compared to these studies, we use both the c-vine, d-vine and r-vine copulas to draw information about the dependence risk and the low and high risk features of the gold and iron ore-nickel stocks in specific market conditions. Our study also is differentiated from those studies by deliberately searching for average model convergence in the weight allocations resulting from the fit of various different portfolio optimization models with respect to multiple risk measures.

Other applications of vine copulas, which relate to our study in terms of dependence modeling, have explored the dynamic dependence behavior between financial markets (e.g., Chollete et al., 2009; Min and Czado, 2010; Mendes et al., 2010; Czado et al.,

[4] Investment confidence is the issue underlying any type of portfolio optimization approach. The abandonment of the single risk measure-based portfolio optimization is partially driven by the inability of that approach to adequately address the problem of investment confidence. Although the optimization of portfolios with respect to multiple risk measures to some extent successfully addresses the problem by providing more information and investment choices, which could cater to a wider variety of investors, it does not offer a generalized and quantitatively objective approach for the selection of stocks. Instead it adopts a relativistic and subjective perspective for the selection of stocks. On the contrary, the “average model convergence” approach proposed for the analysis, evaluation and interpretation of the multiple weight allocation scenarios does provide the investment confidence investors require by finding the models' points (i.e. stocks in our case) of convergence.

[5] As of December 2012, the mining stocks (including coal and uranium stocks in this category) which are listed on the ASX accounted for approximately 39% of the total market capitalization of the total market, with the gold and the iron-ore nickel sectors playing an important role in the functioning and development of the Australian economy. While the pair vine copula approach can handle a larger portfolio, we only illustrate its use by a 20-stock portfolio as the higher number of stocks will make the estimation of the dependence matrix truly complex, particularly due to the consideration of almost all existing bivariate copula families.

[6] Our empirical approach could be extended to modeling the mining sectors in other countries, including those from African countries such as South Africa which is a major producer of gold, platinum, diamond and coal. The mining sector in South Africa makes up about 60% of the country's exports where eight of the 10 largest individual export categories are commodities.

2012), the forecasting of portfolio Value-at-Risk (e.g. Weiß and Supper, 2013), the modeling of systematic dependence risk (e.g., Brechmann et al., 2013), and the optimization and management of portfolios (e.g., Low et al., 2013; Brechmann and Czado, 2013).[7]

Our main results show that the gold stock portfolio is less volatile and less risky than the iron ore-nickel in financial crisis periods, commonly characterized by low or no confidence in the financial markets. A possible reason is that gold is not only an industrial commodity but also an asset with a strong monetary value. Also, while all three pair vine copula models enable one to detect the shifts of dependence concentration across the financial period scenarios, the r-vine copula model is suitable for capturing in the gold portfolio the existence of higher concentration of dependence in the center of the joint distributions within the gold portfolio. The c-vine copula model, on the other hand, best captures the presence of larger concentration of dependence in the tails of the iron ore-nickel portfolio. The combination of pair vine copulas and portfolio optimization is found to produce an estimation edge, indicating the suitability of pair vine copulas to model the multivariate dependence and investment risk of sector-specific portfolios. Finally, the optimal weight allocations produced by the multiple risk measure-based portfolio optimizations are found to converge without a large variation from a mean of weights in some stocks which could be seen as good candidates for investment.

The remainder of this article is organized as follows. Section 2 introduces the pair vine copulas, the portfolio optimization problem and the risk measures. Section 3 presents the data. Section 4 reports and discusses the estimation results. Section 5 concludes the article.

## 2. Empirical models

The bivariate copulas have been proven to be very successful statistical tools for flexibly modeling the cross-sectional dependence structure between random variables (Smith et al., 2010). Bivariate copulas are designed to split the marginal distribution from the joint dependence, while maintaining the original distribution of the marginals (Patton, 2012). They have a comparative advantage over traditional methods of correlations because they include various families that are capable of modeling joint distributions of different characteristics. They also can be used as the building blocks of a vine tree structure to model the dependence structure of high dimensional distributions. On the other hand, the pair vine copulas are graphical tree models consisting of marginal distributions and bivariate copulas at the nodes that enable a localized and specialized measurement of complex multivariate distributions. They are more flexible and advantageous than the bivariate copulas because they account for the distributional differences in the pairs of variables' joint distributions (Brechmann and Schepsmeier, 2011).

### 2.1. Pair vine copulae

#### 2.1.1. Vines and Sklar's theorem

Graphical vine trees were initially employed by Bedford and Cooke (2001, 2002) as a means to organize and specify multivariate statistical models known as regular vines. A vine $V$ is a graphical structure of $n$ elements so that in $V=(T_1,\cdots,T_{n-1})$ every $T_i$ is a connected tree with nodes $N_i=E_{i-1}$ and edge set $E_i$, implying that the edges of the tree $T_i$ are the nodes of the tree $T_{i+1}$ (Kurowicka and Cooke, 2006). The theorem of Sklar (1959) laid the statistical framework, which led to the development of analytical-inferential models such as Eqs. (1) and (3) for the separation of a multivariate density function into factors replaceable by the bivariate copulas and marginals. The three necessary elements for the estimation and selection tasks of the pair vine copula modeling are: the vine trees which identify the pairs of variables to be modeled; the pair copula families used to capture the characteristics of the bivariate joint distributions; and the parameters of the selected bivariate copula families.

#### 2.1.2. Canonical, drawable and regular vines

Regular vines encompass a large number of tree structures which include but are not limited to the c-vine (*canonical vine*) and the d-vines (*drawable vine*). Bedford and Cooke (2001, 2002) utilize them to orderly display and organize multivariate distributions. A regular vine is called a canonical vine if its trees are comprised of nodes (the node with the maximal degree in $T_1$ of a canonical vine is the root) and edges and, each tree $T_i$ has a unique node of degree $n-i$. On the other hand, a regular vine is called a drawable vine if each node in $T_i$ has a degree of at most 2. Both types of vines are subject to the proximity condition which states that for $i=2,\cdots,n-1$, if $\{a,b\}\epsilon E_i$, then $\#a\Delta b=2$, ( $\Delta$ denotes a union without the intersection). In other words, if $a$ and $b$ are nodes of a tree $T_i$ connected by an edge, where $a=\{a_1,a_2\}$ and $\mathrm{b}=\{b_1,b_2\}$, then exactly one of the $a_i$ equals to one of the $b_i$.

Canonical vines have a star-like shape and for every tree $T_i$, $i\in\{1,\cdots,n-1\}$, a root node is selected based on the criterion of having the highest correlation with the rest. They are suggested in applications where, among the variables involved in the modeling, there is one (i.e. the root node) having the highest correlation with the rest (Czado et al., 2013). The d-vines are in general more suitable to model datasets where no dominant variables exist (i.e., none of them has exceptionally high correlation values with the rest). They are represented through line trees, and every node of any $T_i$ cannot be linked to more than two edges. Here instead of a single node, the first tree of the vine and the order of the variables in it play a defining role in subsequent trees and in the structure of the entire vine (Min and Czado, 2010).

The following models for the separation of multivariate densities and inference of c-vine and d-vine pair-copula structures are proposed by Aas et al. (2009)

$$f(\boldsymbol{x})=\prod_{k=1}^{n}f_k(x_k)\prod_{i=1}^{n-1}\prod_{j=1}^{n-i}c_{i,i+j|1:(i-1)}\left(\mathrm{F}(x_i|x_1,\cdots x_{i-1}),\mathrm{F}(x_{i+j}|x_1,\cdots,x_{i-1})|\boldsymbol{\theta}_{i,i+j|1:(i-1)}\right) \quad (1)$$

and

$$f(\boldsymbol{x})=\prod_{k=1}^{n}f_k(x_k)\cdot\prod_{i=1}^{n-1}\prod_{j=1}^{n-i}c_{j,j+i|(j+1):(j+i-1)}\left(\mathrm{F}(x_j|x_{j+1},\cdots x_{j+i-1}),\mathrm{F}(x_{j+i}|x_{j+1},\cdots,x_{j+i-1})|\boldsymbol{\theta}_{j,j+i|(j+1):(j+i-1)}\right) \quad (2)$$

In Eqs. (1) and (2), $i$ identifies the trees and $j$ runs over the edges in each tree. $F(x_i)$ represents the distribution function of $x_i$, the return distribution. On the other hand, $f(\boldsymbol{x})$ represents the return distribution density function. The term $c_{i,i+j|1:(i-1)}$ accounts for the bivariate copulas.

An r-vine copula structure on $n$ variables is the one in which two edges in tree $j$ are joined by an edge in tree $j+1$ only if these edges share a common node. This is due to the proximity condition described above. The shape of the r-vine, unlike those of the c-vine and d-vine, can vary significantly according to the statistical features of the multivariate distribution being modeled. An exact and generalized analytical model has not yet been proposed for

[7] Low et al. (2013) specifically use the multi-dimensional elliptical and asymmetric copula models to forecast the returns on portfolios of financial assets and find that the c-vine copulas are 'worth it' when managing relatively large portfolios with 3–12 constituents.

the decomposition of multivariate densities and the inference of r-vine structures, most likely because the set of possible r-vine structures is vast, diverse and complex to be captured by an equation. Despite this obstacle, Kurowicka and Cooke (2006) build the following analytical model to decompose the multivariate densities and approximate the inference of the r-vine structures:

$$f(x_1, \cdots, x_n) = \left[\prod_{k=1}^{n} f_k(x_k)\right] \times \left[\prod_{i=1}^{n-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}\left(\mathrm{F}\left(x_{j(e)}|x_{D(e)}\right), \mathrm{F}\left(x_{k(e)}|x_{D(e)}\right)\right)\right] \quad (3)$$

where $X = (X_1,\cdots,X_n)$ is a vector of random variables, $f(x_1,\cdots,x_n)$ stands for a multivariate density, $c_{j(e),k(e)|D(e)}$ represents a bivariate conditional density copula with $j(e)$ and $k(e)$ as the conditioned nodes, and $D(e)$ as the conditioning set. The parameter $e = j(e), k(e)| D(e)$ is an edge that belongs to the edge set $\varepsilon = \{E_1,\cdots,E_{n-1}\}$. The vector $\boldsymbol{X}_{D(e)}$ is a vector of variables conditioned by the components of the conditioning set $D(e)$. Eq. (3) is uniquely determined since there is not a common-based tree structure shared among the r-vine statistical models (Kurowicka and Cooke, 2006).

The use of multiple risk measures is important because the portfolio optimization problem is addressed through different angles. Besides, it exposes the difficulties involved in the optimization of portfolios and the limitations of mathematical models to account for all the information and sources of risk. Each of the risk measures applied in this study differs from the others by stressing a particular portfolio optimization component, as opposed to various methodologies already presented in the recent relevant literature, such as in Hassanzadeh et al. (2014), Levy and Levy (2014), Utz et al. (2014) or Ponomareva et al., (2015). For instance, in our work the *CVaR* and *CDaR* measures stress the modeling of returns values falling below a threshold value commonly expressed as a horizontal line (i.e. boundary line). The five competing risk measures include the variance, the mean absolute deviation (*MAD*), the minimizing regret (*Minimax*), the conditional Value-at-Risk (*CVaR*), and the conditional Drawdown-at-Risk (*CDaR*). Since these risk measures may have different theoretical and practical advantages, their simultaneous use in our study enables the comparison of multiple and diverse optimal portfolio holdings.

The application of the *variance* risk measure to the quadratic programming (*QP*) portfolio optimization problem given in Eqs. (4)–(7) defined below rests on the assumptions of normally distributed returns and investors' preferences being expressed through a quadratic utility function.[8] The optimization problem to be solved is

$$\min_{w} \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{m} w_j\left(r_{i,j} - \mu_j\right)\right)^2 \quad (4)$$

subject to

$$\sum_{j=1}^{m} w_j \mu_j = \mu_p \quad (5)$$

$$\sum_{j=1}^{m} w_j = 1 \quad (6)$$

$$w_j \geq 0, \; for = 1, \cdots, m \quad (7)$$

[8] The QP method is based on a measure of central tendency (mean) and it penalizes deviations from the center with an escalating rate due to the square feature of the quadratic objective function.

where $\mu_p$ is the return of the portfolio. Eqs. (5)–(7) are common to all portfolio specification problems in this section and respectively represent the targeted return of the portfolio, the constraint on the sum of the portfolio weights $w_j$ to be equal to one, and the constraint on every weight to be positive semi-definite (i.e., short sales are not considered). These three equations are omitted in subsequent portfolio problem specifications to avoid repetition.

In contrast to the variance risk measure applied in the optimization problem defined by Eqs. (4)–(7), the *MAD* risk measure, which is applied in the linear portfolio optimization problem (8) below, penalizes deviations from the center with a linear rate. This feature, while allowing for faster solutions to large optimization problems, does not adequately represent most investors' preferences and demands. However, since the risk measure does not scale or penalize leptokurtic observations as heavily as the QP does, it could be considered as more robust. The optimization problem to be solved is

$$\min_{w,d} \frac{1}{n} \sum_{i=1}^{n} d_i \quad (8)$$

subject to

$\sum_{j=1}^{m}\left(r_{i,j} - \mu_j\right) w_j \leq y_i, \; \forall_i \in \{1, \cdots, n\}$

$\sum_{j=1}^{m}\left(r_{i,j} - \mu_j\right) w_j \geq - y_i, \; \forall_i \in \{1, \cdots, n\}$ where the parameter $d_i$ accounts for absolute deviations of returns from the forecast mean. The constraints with the summation terms delineate the lower boundary $y_i$ and the upper boundary $y_i$ for the optimization problem. Constraints (6)–(8) are also part of formulation (9).

The *Minimax* risk measure for portfolio optimization in the linear programming problem (9) below is perhaps the most conservative because according to the first constraint, the difference between the maximum loss of the portfolio $M_p$ and the forecast return of the portfolio is targeted to be less than or equal to zero.

$$\min_{M_p,w} M_p \quad (9)$$

subject to

$$M_p - \sum_{j=1}^{m} w_j r_{ij} \leq 0, \; \forall_i \in \{1, \cdots, n\}$$

The *CVaR* measure applied in the portfolio optimization problem (10) is desirable not only because it is a coherent measure of risk (e.g., when the probability distribution function is continuous) but also because it is more in tune with the loss function of the tail distribution. The optimization problem to be solved is

$$\min_{w,d,v} \frac{1}{na} \sum_{i=1}^{n} d_i + v \quad (10)$$

subject to

$$\sum_{j=1}^{m} w_j r_{i,j} + v \geq - d_i, \; \forall_i \in \{1, \cdots, n\}$$

$$d_i \geq 0, \; \forall_j \in \{1, \cdots, n\}$$

where $\mu_p$ as explained above represents the target return of the portfolio, $v$ is the *VaR* at the $a$-coverage rate, and $d_i$ accounts for the deviation values below the *VaR*.

The *CDaR* measure for portfolio optimization in the linear programming problem (11) expresses the path dependent nature of the measure. The draw down events in the historical return distribution play an important role in the *CDaR* weight allocation. The optimization problem to be solved is

**Table 1**
Stock names and codes of the gold and iron ore-nickel portfolios.

| Gold stock's codes | Gold stock's names | Iron ore–nickel stocks' codes | Iron ore–nickel stocks' names |
|---|---|---|---|
| C1:D10:SBMX | ST Barbara | C1:D12:BHPX | BHP Billiton |
| C2:D9:NWRX | Northwest resources | C2:D19:GBGX | Gindalbie metals |
| C3:D5:NSTX | Northern star | C3:D14:MCRX | Mincor resources |
| C4:D12:SHKX | Stone resources of Australia | C4:D8:WSAX | Western areas |
| C5:D8:DEGX | Degrey mining | C5:D6:AGOX | Atlas iron |
| C6:D13:RSGX | Resolut mining | C6:D11:FMSX | Flinders mines |
| C7:D4:AXMX | Apex minerals | C7:D20:GRRX | Grange resources |
| C8:D16:ORNX | Orion gold | C8:D7:ARHX | Australasian resources |
| C9:D11:RCFX | Redcliffe resources | C9:D5:ARI | Arrium |
| C10:D6:EXMX | Excalibur mining | C10:D2:FCNX | Falcon minerals |
| C11:D1:TAMX | Tanami gold | C11:D13:POSX | Poseidon nickel |
| C12:D14:GLNX | Gleneagle gold | C12:D9:HRRX | Heron resources |
| C13:D3:MOYX | Millenium minerals | C13:D1:MGXX | Mount Gibson iron |
| C14:D20:EVNX | Evolution mining | C14:D15:ADYX | Admiralty resources |
| C15:D7:AUZX | Australian mines | C15:D4:FMGX | Fortescue metals |
| C16:D2:HEGX | Hill end gold | C16:D17:ILUX | Iluka resources |
| C17:D15:KMCX | Kalgoorlie Mining | C17:D3:IGOX | Independence group |
| C18:D18:IRCX | Intermin resources | C18:D16:SHDX | Sherwin iron |
| C19:D19:HAOX | Haoma mining | C19:D10:MLMX | Metallica minerals |
| C20:D17:CTOX | Citigold | C20:D18:MOLX | Moly mines |

$$\min_{w,u,v,z} v+\frac{1}{na}\sum_{i=1}^{n} z_i \tag{11}$$

subject to

$$\sum_{j=1}^{m} w_j r_{i,j}+u_i-u_{i-1}\geq 0,$$

$$z_i-u_i+v\geq 0,\ \forall_i \in\{1,\cdots,n\}$$

$$z_i\geq 0;\ u_i\geq 0,\ \forall_i \in\{1,\cdots,n\}$$

$$u_0=0,\ \forall_i \in\{1,\cdots,n\}$$

where parameters $z$ and $u$ are auxiliary vectors representing the lower and upper bounds. The parameter $v$ accounts for the *CDaR* at the $a$ quantile level (Ghalanos, 2013).

## 3. Data

The portfolios are comprised of 20 gold and 20 iron ore-nickel stock return series accessed from the Australian stock market. It is worth noting that iron ore is the second most traded commodity worldwide after oil. The series are daily and the sample period spans from January 2005 to July 2012. Thus, the data set accounts for three financial period scenarios revolving around the 2008/2009 global financial crisis. We specifically divide the entire return series period (i.e. the full sample) into the pre-GFC (January 2005–July 2007), the coincident GFC (July 2007–December 2009) and the post-GFC (January 2010–July 2012) financial period scenarios. The logarithmic returns are first computed and then filtered with an ARMA (1,1)-GARCH (1,1) process with Student-$t$ innovations. This filtering model is selected because it offers the possibility of capturing not only the distributional characteristics of mining portfolio returns, but also the potential of analyzing the tail behavior that is prevalent during turbulent times. As shown later, our gold stocks tend to display a strong positive tail behavior during crisis periods, while the iron ore-nickel stocks have a negative tail behavior in similar market conditions. A probability integral transform is then applied to the standardized residuals in order to obtain the “copula data” to be used in the pair vine copula modeling.

The stock names and codes are listed in Table 1. For the d-vine modeling, the group of stocks in the first tree influences the rest of the stocks in the portfolios through high correlation values. For the c-vine modeling, the rootstock of the first tree has a key role in the shaping of the entire vine structure due to the high correlation values it has with the rest of the stocks in the portfolio. According to the c-vine column order of the data sets, the St. BARBARA (SBMX) and BHP BILLITON (BHPX) stocks occupy the first columns in the gold and iron ore-nickel portfolios, respectively.[9]

## 4. Empirical applications

### 4.1. Dependence matrix estimation

Since they are essential to portfolio optimization, multiple scenario dependence matrices of the gold and the iron ore-nickel portfolios are estimated. This task is carried out through the application of the c-vine, d-vine and r-vine copula models which make use of a wide range of bivariate copulas families listed together with their conventional numbers in Table 2. The numbering of the bivariate copula families will become useful when estimating and analyzing the dependence structure contained in the diagonal matrices presented in this section. Since all the stocks in both portfolios are found to correlate positively under all financial period scenarios, we only present Kendall's tau correlation matrix corresponding to the full sample period scenario.

It should be noted that all copulas in Table 2 are bivariate copulas and should not be confused with pair vine copulas. The bivariate Gaussian and Frank copulas are suitable to model greater dependence in the center of joint return distribution. The main difference between them is that the Frank bivariate copula can also account for the nonlinearities in the center of the joint distribution, while the Gaussian bivariate copula can only focus on linear dependence relationships. The Student-t copula specializes in modeling symmetrically greater dependence in both, the positive and negative tails. The bivariate Clayton, the 180° rotated Gumbel and Joe copulas are adequate to model greater

[9] SBMX started as an oil endeavor in 1969 and then refocused its operations on gold in the 2000s. BHPX calls itself the world leading diversified resources company. It develops and converts natural resources to fuel development and growth all over the world. It is among the world’s largest producers of iron ore.

**Table 2**
Repertoire of bivariate copula families employed by the vine models.

| One Par | | Archimedean 2Par | | 90 Rotated | | 180 Rotated | | 270 Rotated | |
|---|---|---|---|---|---|---|---|---|---|
| Gaussian | (1) | Clayton-Gumbel(BB1) | (7) | Clayton | (23) | Clayton | (13) | Clayton | (33) |
| Student-*t* | (2) | Joe-Gumbel(BB6) | (8) | Gumbel | (24) | Gumbel | (14) | Gumbel | (34) |
| Clayton | (3) | Joe-Clayton(BB7) | (9) | Joe | (26) | Joe | (16) | Joe | (36) |
| Gumbel | (4) | Joe-Frank(BB8) | (10) | Clayton-Gumbel (BB1) | (7) | Clayton-Gumbel (BB1) | (17) | Clayton-Gumbel(BB1) (37) | (37) |
| Frank | (5) | | | Joe-Gumbel (BB6) | (28) | Joe-Gumbel(BB6) | (18) | Joe-Gumbel(BB6) | (38) |
| Joe | (6) | | | Joe-Clayton(BB7) | (29) | Joe-Clayton(BB7) | (19) | Joe-Clayton(BB7) | (39) |
| | | | | Joe-Frank(BB8) | (30) | Joe-Frank(BB8) | (20) | Joe-Frank(BB8) | (40) |

*Notes:* These copulas allow one to capture different forms of dependence including symmetric and asymmetric in the center and in the tails.The numbering of the bivariate copula families facilitates the analysis, evaluation and interpretation of the dependence structure contained in the diagonal matrices presented below.

**Table 3**
Gold portfolio summary of vine models' bivariate copula selection.

| Bivariate copula | Full sample | | Pre-GFC | | GFC | | Post-GFC | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Vine model | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine |
| Clayton | 6 | 8 | 11 | 12 | 18 | 19 | 9 | 11 | 12 | 15 | 12 | 18 |
| Gumbel180 | 16 | 16 | 15 | 22 | 14 | 14 | 14 | 15 | 12 | 9 | 12 | 11 |
| Studen-t | 20 | 23 | 21 | 14 | 14 | 17 | 16 | 19 | 21 | 19 | 17 | 19 |
| Joe 180 | 1 | 8 | 8 | 15 | 15 | 10 | 3 | 7 | 6 | 0 | 0 | 8 |
| Joe-Frank 180 | 26 | 28 | 19 | 0 | 0 | 8 | 8 | 8 | 11 | 0 | 0 | 6 |
| Clayton 270 | 0 | 0 | 0 | 5 | 8 | 0 | 0 | 0 | 0 | 5 | 7 | 0 |
| Frank | 54 | 46 | 54 | 48 | 49 | 51 | 85 | 69 | 72 | 58 | 59 | 53 |
| Gaussian | 17 | 17 | 15 | 27 | 25 | 22 | 17 | 21 | 18 | 30 | 26 | 28 |
| Gumbel | 15 | 14 | 11 | 13 | 4 | 10 | 0 | 0 | 3 | 9 | 11 | 9 |
| Clayton 180 | 0 | 0 | 6 | 11 | 18 | 14 | 8 | 6 | 13 | 10 | 11 | 9 |
| Clayton 90 | 4 | 5 | 0 | 4 | 4 | 0 | 0 | 0 | 0 | 7 | 8 | 0 |
| Studen-t | 20 | 23 | 21 | 14 | 14 | 17 | 16 | 19 | 21 | 19 | 17 | 19 |
| Joe | 0 | 0 | 3 | 0 | 0 | 3 | 0 | 0 | 5 | 0 | 0 | 6 |
| Joe-Frank | 15 | 16 | 20 | 7 | 3 | 2 | 7 | 8 | 4 | 0 | 0 | 4 |

*Notes:* the top row of the table states the four financial period scenarios modeled with the pair vine copulas. The three types of vine copula models implemented are the c-vine, d-vine and r-vines. The first column of the table lists the bivariate copulas used most frequently by the vine copula models to measure the dependence between pair of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The full sample period spans from January 2005 to July 2012, the pre-GFC stretches from Jan 2005 to July 2007; the GFC period covers from July 2007 to Dec 2009, and the post-GFC period accounts for the volatility between Jan 2010 and July 2012.

dependence in the negative tail. On the contrary, the bivariate Gumbel, Joe and the180° rotated Clayton bivariate copulas are suitable to model greater dependence in the positive tail.

*4.1.1. The gold portfolio*

According to the cross financial period scenario vine models' frequency of the bivariate copula selection displayed in Table 3 and partially in Fig. 1, the Frank bivariate copula is the most predominant in all financial period scenarios of the gold portfolio, suggesting that most of the dependence in the gold portfolio is concentrated in the center of the joint return distribution. This finding implies that the return on gold portfolio is likely to fluctuate in normal market conditions and is not exposed to extreme values. It can thus be deduced that the gold stocks are less volatile and less risky in financial crisis periods characterized by low or no confidence in the financial markets. Arriving at these findings through the application of pair vine copulas is significant because gold indeed has gained the reputation of being a safe haven in times of financial uncertainty, commonly featured by low levels of confidence in the financial markets. This hedging feature of gold was particularly observed during the global financial crisis of 2008–2009 where the price of this precious metal experienced extreme appreciation movements as the confidence in the financial stock markets eroded.

A comparison between the pre-GFC and the post-GFC financial period scenarios of the gold portfolio indicates that the number of stocks having a nonlinear or non-normal relationship is larger in the pre-GFC. This fact is also particularly observed through a larger presence of the Gaussian copula in the post-GFC period. The pre-GFC experienced an unprecedented boom in commodity prices. The Frank copula has its largest presence in the GFC period, implying that the returns of the gold portfolio during the period are driven by complex relationships of dependence. In general, the level of complexity in the dependence relationships of the gold stocks appears to decrease as the financial stock market confidence increases. The noticeable decrease of the copulas for the modeling of asymmetric dependence in the negative tail confirms the immunity of gold to financial crisis periods.

With respect to model selection, the r-vine model selects the Frank copula most frequently in the full sample and pre-GFC period scenarios; however, the c-vine model selects it in more occasions in the GFC period, while the d-vine does this selection in the post-GFC. Hence, the r-vine copula model overall best accounts for the multivariate dependence structure in the center of the gold portfolio. In order to confirm empirically our judgment about the r-vine being the most adequate model to capture the multivariate dependence structure of the gold portfolio, we run on the fit of the c-vine, d-vine and r-vine to the portfolios the *ECP* and *ECP2* goodness-of-fit tests, which are based on the empirical copula processes. For further details on the goodness of fit tests see Schepsmeier (2013, 2014), and Genest et al. (2009). The *ECP* and *ECP2* goodness-of-fit tests implemented are non-parametric and are based on the Cramer-von Mises (*CvM*) and Kolmogorov-Smirnov (*KS*) test statistics.

The resulting $p$-values from the goodness-of-fit testing displayed in Table 4 confirm that the r-vine is the most suitable model to capture the dependence structure of the gold portfolio.

a

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | | | | | | | | | | | | | | | | | | |
| 20 | 2 | | | | | | | | | | | | | | | | | |
| 2 | 2 | 20 | | | | | | | | | | | | | | | | |
| 3 | 2 | 20 | 10 | | | | | | | | | | | | | | | |
| 2 | 5 | 14 | 1 | 5 | | | | | | | | | | | | | | |
| 1 | 2 | 5 | 5 | 20 | 1 | | | | | | | | | | | | | |
| 20 | 10 | 5 | 2 | 5 | 10 | 10 | | | | | | | | | | | | |
| 20 | 2 | 5 | 20 | 5 | 5 | 2 | 5 | | | | | | | | | | | |
| 10 | 10 | 6 | 20 | 5 | 1 | 4 | 20 | 10 | | | | | | | | | | |
| 1 | 20 | 13 | 1 | 5 | 20 | 13 | 5 | 14 | 3 | | | | | | | | | |
| 5 | 10 | 5 | 5 | 5 | 5 | 4 | 10 | 5 | 4 | 1 | | | | | | | | |
| 14 | 2 | 5 | 5 | 2 | 1 | 5 | 5 | 5 | 5 | 10 | 1 | | | | | | | |
| 20 | 2 | 20 | 1 | 10 | 2 | 10 | 4 | 13 | 23 | 20 | 5 | 10 | | | | | | |
| 20 | 10 | 5 | 5 | 5 | 4 | 20 | 20 | 5 | 4 | 20 | 4 | 5 | 4 | | | | | |
| 14 | 14 | 14 | 20 | 2 | 1 | 2 | 33 | 14 | 5 | 14 | 5 | 4 | 10 | 5 | | | | |
| 20 | 10 | 5 | 5 | 5 | 5 | 5 | 3 | 10 | 3 | 34 | 14 | 1 | 24 | 4 | 3 | | | |
| 14 | 2 | 5 | 5 | 5 | 23 | 20 | 5 | 5 | 10 | 23 | 1 | 2 | 5 | 5 | 1 | 5 | | |
| 5 | 5 | 5 | 3 | 10 | 4 | 5 | 14 | 14 | 4 | 1 | 24 | 20 | 5 | 16 | 23 | 4 | 5 | |
| 2 | 14 | 1 | 3 | 20 | 20 | 20 | 2 | 4 | 5 | 14 | 4 | 14 | 2 | 14 | 16 | 1 | 33 | 5 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | | | | | | | | | | | | | | | | | | |
| 14 | 20 | | | | | | | | | | | | | | | | | |
| 2 | 2 | 10 | | | | | | | | | | | | | | | | |
| 2 | 2 | 3 | 5 | | | | | | | | | | | | | | | |
| 30 | 1 | 5 | 5 | 5 | | | | | | | | | | | | | | |
| 4 | 10 | 20 | 5 | 3 | 20 | | | | | | | | | | | | | |
| 10 | 10 | 2 | 20 | 2 | 2 | 5 | | | | | | | | | | | | |
| 2 | 5 | 14 | 20 | 5 | 14 | 14 | 3 | | | | | | | | | | | |
| 20 | 3 | 20 | 10 | 20 | 1 | 20 | 14 | 5 | | | | | | | | | | |
| 20 | 2 | 5 | 5 | 10 | 5 | 2 | 20 | 14 | 14 | | | | | | | | | |
| 2 | 2 | 2 | 5 | 5 | 20 | 3 | 5 | 5 | 14 | 1 | | | | | | | | |
| 14 | 5 | 2 | 5 | 14 | 4 | 1 | 20 | 1 | 1 | 1 | 20 | | | | | | | |
| 5 | 5 | 5 | 1 | 5 | 4 | 4 | 4 | 5 | 4 | 1 | 5 | 1 | | | | | | |
| 14 | 20 | 5 | 10 | 23 | 20 | 5 | 4 | 36 | 6 | 4 | 3 | 5 | 23 | | | | | |
| 20 | 10 | 5 | 2 | 5 | 20 | 10 | 5 | 20 | 5 | 5 | 10 | 5 | 23 | 5 | | | | |
| 20 | 20 | 4 | 20 | 3 | 4 | 10 | 13 | 16 | 14 | 5 | 4 | 20 | 16 | 16 | 14 | | | |
| 14 | 5 | 10 | 1 | 6 | 2 | 20 | 16 | 2 | 20 | 20 | 3 | 5 | 2 | 2 | 1 | 23 | | |
| 5 | 5 | 20 | 4 | 16 | 13 | 1 | 5 | 13 | 5 | 6 | 16 | 4 | 5 | 1 | 5 | 23 | 1 | |
| 5 | 2 | 2 | 5 | 5 | 1 | 20 | 2 | 14 | 2 | 16 | 10 | 4 | 1 | 3 | 10 | 10 | 10 | 16 |

b

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.100 | | | | | | | | | | | | | | | | | | |
| 0.133 | 0.518 | | | | | | | | | | | | | | | | | |
| 0.140 | 0.164 | 0.078 | | | | | | | | | | | | | | | | |
| 0.073 | 0.480 | 0.232 | 0.051 | | | | | | | | | | | | | | | |
| 0.247 | 0.119 | 0.046 | 0.060 | 0.033 | | | | | | | | | | | | | | |
| 0.139 | 0.349 | 0.143 | 0.054 | 0.082 | 0.046 | | | | | | | | | | | | | |
| 0.112 | 0.299 | 0.158 | 0.026 | 0.155 | 0.047 | 0.055 | | | | | | | | | | | | |
| 0.083 | 0.313 | 0.144 | 0.089 | 0.129 | 0.031 | 0.057 | 0.037 | | | | | | | | | | | |
| 0.044 | 0.087 | 0.001 | 0.068 | 0.048 | 0.010 | 0.001 | 0.033 | 0.058 | | | | | | | | | | |
| 0.166 | 0.142 | 0.036 | 0.049 | 0.040 | 0.115 | 0.009 | 0.049 | 0.035 | 0.003 | | | | | | | | | |
| 0.051 | 0.164 | 0.060 | 0.040 | 0.051 | 0.067 | 0.003 | 0.111 | 0.060 | 0.003 | -0.002 | | | | | | | | |
| 0.128 | 0.218 | 0.077 | 0.092 | 0.041 | 0.049 | 0.051 | 0.063 | 0.034 | 0.025 | 0.039 | -0.011 | | | | | | | |
| 0.207 | 0.121 | 0.095 | 0.048 | 0.082 | 0.087 | 0.046 | 0.007 | 0.027 | -0.003 | 0.042 | -0.010 | 0.040 | | | | | | |
| 0.072 | 0.228 | 0.063 | 0.023 | 0.069 | 0.002 | 0.031 | 0.051 | 0.021 | 0.005 | 0.031 | 0.003 | 0.022 | 0.012 | | | | | |
| 0.108 | 0.070 | 0.008 | 0.079 | 0.028 | 0.043 | 0.017 | -0.006 | 0.016 | 0.054 | 0.050 | 0.030 | 0.007 | 0.041 | -0.011 | | | | |
| 0.020 | 0.284 | 0.073 | 0.042 | 0.082 | 0.031 | 0.061 | 0.017 | 0.065 | 0.007 | -0.006 | 0.003 | 0.017 | -0.003 | 0.003 | 0.002 | | | |
| 0.075 | 0.574 | 0.298 | 0.052 | 0.222 | -0.012 | 0.098 | 0.104 | 0.072 | 0.034 | -0.021 | -0.023 | 0.024 | -0.018 | 0.041 | -0.019 | 0.077 | | |
| 0.027 | 0.273 | 0.108 | 0.003 | 0.071 | 0.002 | 0.056 | 0.004 | 0.002 | 0.005 | 0.009 | -0.007 | 0.019 | 0.005 | 0.005 | -0.006 | 0.003 | 0.093 | |
| 0.140 | 0.086 | 0.049 | 0.060 | 0.034 | 0.084 | 0.055 | 0.028 | 0.013 | 0.034 | 0.032 | 0.004 | 0.017 | 0.022 | 0.004 | 0.026 | 0.008 | -0.009 | -0.026 |

**Fig. 1.** Dependence structure & Kendall's tau correlation matrix of the gold stock portfolio. Notes: Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) copula family specification matrices. Panel (b) shows the c-vine estimated Kendall's tau correlation matrix. The numbers contained in the diagonal matrices of Panel (a) are the bivariate copula families selected by the c-vine and d-vine models to measure the dependence and correlation between the pairs of gold stocks. The function of every bivariate copula has been described beneath Table 2.

#### *4.1.2. The iron ore-nickel portfolio*

A comparison between the dependence structure of the portfolios displayed in Tables 3–5 indicates that the presence of the Frank copula in all financial periods of the iron ore-nickel portfolio is significantly smaller. With specific focus on the dependence structure of the iron ore-nickel portfolio, the number of selected bivariate copulas which model greater dependence in the negative tail is larger than that of the selected bivariate copulas which model greater dependence in the center of the joint return distribution. Thus, most of the dependence structure in the iron ore-nickel portfolio is concentrated in the left tail of the joint return distribution. As compared to the gold portfolio, this result implies that the iron ore-nickel portfolio becomes riskier in times of financial turbulences and crises featured by low or absence of confidence in the financial markets. In this regard, it is now well known that the price of the iron ore commodity indeed suffered significant declines during the global financial crisis of 2008–2009.

When we look at the results over different subperiods, this portfolio shows dependence concentration shifts from the positive tail to the negative tail. The shifts occur specifically from the pre-GFC to the GFC, indicating the presence of the left-tail dependence risk between stocks of the iron ore-nickel portfolio during the GFC period potentially due to investors' loss of confidence, and thus an increase in the probability that the iron ore-nickel portfolio realizes negative returns. On the other hand, the shifts of dependence concentration from the negative tail to the positive tail which took place from the GFC to the post-GFC reflect an improvement in investors' confidence, economic recovery, and a decrease in the probability that the iron ore-nickel portfolio is to suffer large losses.

As to model selection, the results indicate that the c-vine, relative to the d-vine and r-vine, is more frequently selected under all financial period scenarios for the modeling of negative tail dependence. Thus, it can be inferred that that the c-vine copula model better accounts for the multivariate dependence structure of the iron ore-nickel portfolio. The goodness-of-fit testing results from Table 4 confirm that the c-vine is the model that best fits the multivariate dependence of the iron ore-nickel portfolio.

### *4.2. Portfolio optimization*

We optimize the gold and iron ore-nickel portfolios under consideration by inserting different risk measures (variance, *MAD*, *Minimax*, *CVaR* and *CDaR*) into the linear and nonlinear portfolio optimization models in order to estimate the minimum risk optimal portfolios. The integration of the pair r-vine and c-vine copulas (i.e., several types of bivariate copula families including the Frank copula are used in the vine structure) into the portfolio optimization is only possible when we estimate the variance risk measure which is inserted into the *QP* optimization models. From a methodological point of view, the combination of pair vine copulas and portfolio optimization models allows one to capture the dependence in the center, and the negative and the positive tails.

In practice, we rely on the relationship between the pair-copula dependence parameters in Section 4.1 and the corresponding Kendall's tau to compute the correlation coefficients among the

**Table 4**
Gold & iron ore–nickel portfolios' goodness-of-fit testing for the c-vine, d-vine & r-vine.

| Portfolios and copulas | Gold | | | Iron ore–nickel | | |
|---|---|---|---|---|---|---|
| Vine model | c-vine | d-vine | r-vine | c-vine | d-vine | r-vine |
| **Full sample** | | | | | | |
| ECP(*CvM*) | *ts*=0.016 | *ts*=0.003 | *ts*=0.004 | *ts*=0.023 | *ts*=0.039 | *ts*=0.028 |
| | *p*=0.44 | *p*=0.975 | *p*= 0.985 | *p*=0.71 | *p*=0.70 | *p*=0.515 |
| ECP2(*CvM*) | *ts*=0.000 | *ts*=0.000 | *ts*=0.000 | *ts*=0.000 | *ts*=0.000 | *ts*=0.000 |
| | *p*=1.00 | *p* =1.00 | *p*=1.00 | *p*=1.00 | *p* =1.00 | *p*=1.00 |
| ECP (*KS*) | *ts* =1.825 | *ts*=0.952 | *ts*=1.339 | *ts*=2.028 | *ts*=3.018 | *ts*=2.072 |
| | *p* =0.23 | *p*=0.425 | *p*=0.045 | *p*=0.73 | *p*=0.04 | *p*=0.645 |
| ECP2(*KS*) | *ts*=0.022 | *ts*=0.022 | *ts*=0.022 | *ts*=0.055 | *ts*=0.066 | *ts*=0.047 |
| | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 |
| **Pre-GFC** | | | | | | |
| ECP(*CvM*) | *ts*=0.003 | *ts*=0.003 | *ts*=0.003 | *ts*=0.008 | *ts*=0.008 | *ts*=0.009 |
| | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=0.96 | *p*=0.98 | *p*=0.95 |
| ECP2(*CvM*) | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 |
| | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 |
| ECP (*KS*) | *ts*=0.607 | *ts*=0.849 | *ts*=0.824 | *ts*=0.438 | *ts*=0.354 | *ts*=0.336 |
| | *p*=0.285 | *p*=0.27 | *p*=0.345 | *p*=0.53 | *p* =0.78 | *p*=0.80 |
| ECP2(*KS*) | *ts*=0.039 | *ts*=0.039 | *ts*=0.039 | *ts*=0.039 | *ts*=0.079 | *ts*=0.078 |
| | *p*=1.00 | *p* =1.00 | *p*=1.00 | *p*=1.00 | *p* =1.00 | *p*=1.00 |
| **GFC** | | | | | | |
| ECP(*CvM*) | *ts*=0.012 | *ts*=0.004 | *ts*=0.003 | *ts*=0.015 | *ts*=0.016 | *ts*=0.018 |
| | *p*=0.77 | *p*=1.00 | *p*=1.00 | *p*=0.88 | *p*=0.965 | *p*=0.78 |
| ECP2(*CvM*) | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 |
| | *p*=1.00 | *p* =1.00 | *p*=1.00 | *p*=1.00 | *p* =1.00 | *p*=1.00 |
| ECP (*KS*) | *ts*=1.010 | *ts*=0.770 | *ts*=0.367 | *ts*=1.308 | *ts*=1.580 | *ts*=1.553 |
| | *p*=0.395 | *p* =0.22 | *p*=0.78 | *p*=0.575 | *p* =0.200 | *p*=0.320 |
| ECP2(*KS*) | *ts*=0.077 | *ts*=0.039 | *ts*=0.076 | *ts*=0.062 | *ts*=0.039 | *ts*=0.039 |
| | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p* =1.00 | *p*=1.00 |
| **Post-GFC** | | | | | | |
| ECP(*CvM*) | *ts*=0.002 | *ts*=0.001 | *ts*=0.002 | *ts*=0.013 | *ts*=0.013 | *ts*=0.010 |
| | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 |
| ECP2(*CvM*) | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 | *ts*=0.001 |
| | *p*=1.00 | *p* =1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 | *p*=1.00 |
| ECP (*KS*) | *ts*=0.431 | *ts*=0.131 | *ts*=0.304 | *ts*=1.053 | *ts*=0.983 | *ts*=0.849 |
| | *p*=0.055 | *p*=1.00 | *p*=0.43 | *p*=0.425 | *p*=0.29 | *p*=0.625 |
| ECP2(*KS*) | *ts*=0.078 | *ts*=0.039 | *ts*=0.039 | *ts*=0.039 | *ts*=0.078 | *ts*=0.039 |
| | *p*=1.00 | *p* =1.00 | *p*=1.00 | *p*=1.00 | *p* =1.00 | *p*=1.00 |

*Notes:* The abbreviations *p* and *ts* stand for p-value and t-statistic. The *ECP* and *ECP2* refer to the empirical copula processes. The *CvM* and *KS* stand for the Cram von Mises and Kolmogorov-Smirnov test statistics.

assets of the same portfolio (see, Heinen and Valdesogo, 2009). Note also that the use of the *QP* nonlinear optimization method with pair vine copulas does not constrain the distribution of the gold and iron ore-nickel returns to be normal but can capture the stylized facts of the univariate return distributions as well as the asymmetric and nonlinear dependence between stocks. It is

**Table 5**
Iron ore–nickel portfolio summary of vine models' bivariate copula selection.

| Bivariate Copula | Full sample | | | Pre-GFC | | | GFC | | | Post-GFC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Vine model | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine | C-vine | D-vine | R-vine |
| Clayton | 18 | 17 | 23 | 24 | 22 | 20 | 30 | 22 | 30 | 19 | 14 | 14 |
| Gumbel 180 | 16 | 15 | 23 | 23 | 17 | 20 | 27 | 28 | 30 | 22 | 20 | 14 |
| Student-t | 36 | 35 | 31 | 10 | 11 | 12 | 8 | 11 | 10 | 20 | 16 | 24 |
| Joe 180 | 13 | 12 | 9 | 22 | 22 | 23 | 13 | 10 | 14 | 19 | 9 | 19 |
| Joe-Frank 180 | 32 | 34 | 31 | 0 | 0 | 6 | 12 | 8 | 3 | 7 | 13 | 9 |
| Clayton 270 | 0 | 0 | 3 | 0 | 0 | 6 | 0 | 0 | 5 | 0 | 0 | 4 |
| Frank | 22 | 33 | 17 | 32 | 38 | 37 | 34 | 36 | 30 | 37 | 47 | 37 |
| Gaussian | 6 | 13 | 11 | 15 | 16 | 16 | 16 | 25 | 23 | 14 | 25 | 16 |
| Gumbel | 0 | 0 | 9 | 0 | 0 | 6 | 0 | 0 | 6 | 0 | 0 | 5 |
| Clayton 180 | 0 | 0 | 13 | 17 | 13 | 16 | 0 | 0 | 11 | 0 | 0 | 12 |
| Clayton 90 | 0 | 0 | 2 | 0 | 0 | 5 | 0 | 0 | 4 | 0 | 0 | 7 |
| Student-t | 36 | 35 | 31 | 10 | 11 | 12 | 8 | 11 | 10 | 20 | 16 | 24 |
| Joe | 0 | 0 | 5 | 0 | 0 | 11 | 0 | 0 | 8 | 0 | 0 | 5 |
| Joe-Frank | 0 | 0 | 8 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 0 | 2 |

*Notes:* the top row of the table states the four financial period scenarios modeled with the pair vine copulas. The three types of vine copula models implemented are the c-vine, d-vine and r-vines. The first column of the table lists the bivariate copulas used most frequently by the vine copula models to measure the dependence between pair of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The full sample period spans from January 2005 to July 2012, the pre-GFC stretches from Jan 2005 to July 2007; the GFC period covers from July 2007 to Dec 2009, and the post-GFC period accounts for the volatility between Jan 2010 and July 2012.

therefore expected that the portfolio optimization which uses pair vine copulas performs better than alternative linear portfolio optimization models with their corresponding risk measures.

As stated earlier, another important feature of our portfolio optimization framework is related to the interpretation of the multiple optimal weight allocations produced by the various portfolio optimization models applied. The perspective we adopt looks at the array of investment possibilities in terms of "average model convergence". There are positive implications and advantages for portfolio optimization from the adoption of this perspective. Firstly, the stocks where some, most or all of the models converge are spotlighted. This fact gives a reason to consider a more thorough investigation of those stocks since they could be good candidates for investment. Secondly, the relevance of the problem (a problem with a subjective and relativistic solution) of trying to find the best risk measure to be used for the optimization of portfolios is reduced significantly by shifting the focus on the search for average model convergence. Finally, the average model convergence identified provides objective confidence for the mining investors with respect to where to invest and how much to invest. The multiple risk measure-based portfolio optimization that does not look at the weights allocations in terms of average model convergence would thus not provide the confidence required by mining investors.

We consider the full sample period and arbitrarily set the target portfolio return $\mu_p$ to be equal to 4.2% for both portfolios. By doing so we can compare the portfolios in terms of investment risk and maintain a realistic investment scenario.

The use of the average model convergence on the optimal weight allocations of the gold portfolio displayed in Table 6 indicates that most of the optimization methods and the risk measures converge on average in the ST. BARBARA (SBMX) stock, when the portfolio optimization with respect to the *CDaR* is ignored. If the model specifications with respect to the *CDaR* and *Minimax* are ignored, the remaining models' optimal weights converge on average in NORTHWEST RESOURCES (NWRX) and RESOLUTE MINING (RSGS). This type of model convergence could be discerned by gold portfolio investors as the model consensus and be used to select stocks. In addition, those stocks could be good investment choices. Equally important, the different weight allocations produced by the various portfolio optimization models would allow the investors in the mining sector to select a particular risk measure for the optimization of their portfolios, depending on their return appetite and risk tolerance.

Out of those stocks only RESOLUTE MINING (RSGS) has a negative mean return value, while ST. BARBARA (SBMX) offers the best risk-return trade-off in the entire portfolio. The average model convergence is able to identify two stocks that could be good investment choices in terms of the mean return and variance. The ST. BARBARA (SBMX) stock is allocated large weights by most of the optimization methods and risk measures. The model specifications with respect to the *CDaR* and *Minimax* produce the most extreme weight allocations. The most balanced weight allocations result from the optimizations based on the *MAD* and *variance* risk measures. These patterns are encountered in both portfolios.

An analysis of the risk and return of the gold portfolio shows that the modeling framework, which combines the pair r-vine copula with portfolio optimization, outperforms the mean variance quadratic portfolio optimization, indicating that the pair vine copulas are capable of better capturing complex dependence structures. Hence, while the average model convergence identifies ST. BARBARA (SBMX) and RESOLUTE MINING (RSGS) as potentially good investment choices, the descriptive statistics indicate that NORTHERN STAR (NSTX) could also be a good investment choice.

With respect to the minimum risk optimal weight allocations of the iron ore-nickel portfolio displayed in Table 7, the optimization methods and risk measures are observed to converge on average in the BHP BILLITON (BHPX) stock, when the optimal weight allocations with respect to the *CDaR* and *CVaR* are ignored. In addition, this stock is allocated large weights by most of the optimization methods and the risk measures most likely because it

**Table 6**
Optimal weights and performance of the gold portfolio under multiple risk measures.

| Codes | Portfolio optimization | | Weights' average | | Stocks' descriptive statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold stocks | CVaR (LP) | CDaR(LP) | Minimax (LP) | MAD (LP) | Var (QP) | QP R-Vine | MW | MW ex. CDaR | MW ex. Minimax & CDaR | $\mu$ | $\sigma^2$ | $K$ | SK |
| SBMX | 30.01 | 44.28 | 24.25 | 24.93 | 29.23 | 5.04 | 26.29 | 22.69 | 22.30 | 0.07 | 0.18 | 4.56 | −0.05 |
| NWRX | 3.53 | 0 | 0 | 4.18 | 4.53 | 4.81 | 2.84 | 3.41 | 4.26 | −0.02 | 0.44 | 26.64 | −1.10 |
| NSTX | 19.62 | 6.39 | 31.72 | 23.75 | 19.95 | 0 | 16.91 | 19.01 | 15.83 | 0.11 | 0.37 | 10.66 | 0.16 |
| SHKX | 0 | 0 | 0 | 0 | 0 | 1.96 | 0.33 | 0.39 | 0.49 | −0.17 | 0.30 | 4.29 | 0.47 |
| DEGX | 0 | 0 | 0 | 0 | 0 | 3.56 | 0.59 | 0.71 | 0.89 | −0.18 | 0.32 | 11.40 | 1.08 |
| RSGX | 13.54 | 0 | 0 | 14.15 | 13.28 | 0 | 6.83 | 8.19 | 10.24 | 0.01 | 0.15 | 5.75 | −0.23 |
| AXMX | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0.00 | −0.22 | 0.44 | 16.79 | −0.15 |
| ORNX | 0 | 0 | 0 | 0 | 0 | 6.25 | 1.04 | 1.25 | 1.56 | −0.16 | 0.36 | 6.61 | −0.03 |
| RCFX | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0.00 | −0.14 | 0.61 | 5.67 | 0.65 |
| EXMX | 0 | 0 | 0 | 0 | 0 | 11.76 | 1.96 | 2.35 | 2.94 | −0.17 | 1.78 | 13.85 | 0.02 |
| TAMX | 0 | 0 | 0 | 1 | 0 | 2.77 | 0.63 | 0.75 | 0.94 | −0.05 | 0.26 | 17.94 | 0.85 |
| GLNX | 0 | 0 | 0 | 0 | 0 | 5.8 | 0.97 | 1.16 | 1.45 | −0.41 | 1.14 | 563.41 | −17.93 |
| MOYX | 0 | 0 | 0 | 0 | 0 | 14.06 | 2.34 | 2.81 | 3.52 | −0.15 | 0.45 | 22.31 | 0.11 |
| EVNX | 6.91 | 14.28 | 0 | 4.21 | 5.98 | 2.26 | 5.61 | 3.87 | 4.84 | 0.00 | 0.32 | 10.79 | 0.74 |
| AUZX | 0 | 0 | 0 | 0 | 0 | 5.13 | 0.86 | 1.03 | 1.28 | −0.14 | 2.15 | 16.55 | −0.00 |
| HEGX | 0 | 0 | 0 | 0 | 0 | 7.35 | 1.23 | 1.47 | 1.84 | −0.09 | 0.29 | 3.09 | 0.45 |
| KMCX | 0 | 0 | 0 | 0 | 0 | 12.51 | 2.09 | 2.50 | 3.13 | −0.21 | 0.53 | 45.01 | −2.27 |
| IRCX | 13.66 | 35.05 | 0 | 13.9 | 15.63 | 6.17 | 14.07 | 9.87 | 12.34 | 0.01 | 0.28 | 10.24 | 0.70 |
| HAOX | 6.97 | 0 | 0 | 5.24 | 3.59 | 4.57 | 3.40 | 4.07 | 5.09 | −0.02 | 0.67 | 18.06 | 1.85 |
| CTOX | 5.77 | 0 | 44.03 | 8.66 | 7.8 | 6 | 12.04 | 14.45 | 7.06 | −0.02 | 0.19 | 27.91 | 2.05 |
| *P-Ret* | 0.042 | 0.042 | 0.042 | 0.042 | 0.042 | 0.042 | NA | NA | NA | NA | NA | NA | NA |
| *P-Risk* | 5.55 | 103.02 | 15.63 | 1.80 | 0.062 | 0.052 | NA | NA | NA | NA | NA | NA | NA |

*Notes:* This table reports the minimum risk optimal weights (%) of the gold sector portfolio. The abbreviations LP, QP, G, MW, C and VaR stand for the linear programing, the mean-variance quadratic programming, and the Gaussian, mean of weights, canonical and variance. The names and codes of the stocks are provided in Table 1. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. CDaR and MW ex. Minimax & CDaR refer to the mean of weights excluding the CDaR and, the Minimax and CDaR measures, respectively. The target portfolio return is 0.042%.

**Table 7**
Optimal weights and performance of the iron ore-nickel portfolio under multiple risk measures.

| Codes | Portfolio optimization | Weights' average | Stocks' descriptive statistics | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ore-nickel stocks | CVaR (LP) | CDaR (LP) | Minimax (LP) | MAD (LP) | Var (QP) | QP C-vine | MW | MW ex. CDaR | MW ex. Minimax & CDaR | $\mu$ | $\sigma^2$ | $K$ | SK |
| BHPX | 46.72 | 53.15 | 39.52 | 39.38 | 39.62 | 9.81 | 38.03 | 35.01 | 33.88 | 0.04 | 0.05 | 4.10 | −0.23 |
| GBGX | 0.00 | 0.00 | 0.00 | 2.27 | 0.62 | 0 | 0.48 | 0.58 | 0.72 | 0.09 | 0.20 | 5.25 | 0.32 |
| MCRX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.59 | 0.60 | 0.72 | 0.90 | 0.00 | 0.13 | 3.91 | 0.43 |
| WSAX | 1.70 | 0.00 | 0.00 | 5.50 | 2.74 | 7.77 | 2.95 | 3.54 | 4.43 | 0.05 | 0.10 | 4.62 | 0.08 |
| AGOX | 1.83 | 0.00 | 4.19 | 2.10 | 4.04 | 2.51 | 2.45 | 2.93 | 2.62 | 0.11 | 0.21 | 6.70 | 0.64 |
| FMSX | 1.48 | 0.86 | 0.00 | 3.15 | 2.21 | 0 | 1.28 | 1.37 | 1.71 | 0.08 | 0.64 | 283.20 | 10.78 |
| GRRX | 0.00 | 0.00 | 0.00 | 1.62 | 2.20 | 4.01 | 1.31 | 1.57 | 1.96 | −0.01 | 0.20 | 10.65 | 0.39 |
| ARHX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.82 | 0.30 | 0.36 | 0.46 | −0.13 | 0.33 | 8.81 | 1.01 |
| ARI | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 12.29 | 2.05 | 2.46 | 3.07 | −0.05 | 0.08 | 6.26 | −0.18 |
| FCNX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.3 | 0.22 | 0.26 | 0.33 | −0.18 | 0.37 | 12.14 | 0.72 |
| POSX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.76 | 0.29 | 0.35 | 0.44 | −0.05 | 0.38 | 22.34 | 1.94 |
| HRRX | 0.63 | 0.00 | 0.00 | 4.12 | 2.59 | 4.91 | 2.04 | 2.45 | 3.06 | −0.03 | 0.22 | 13.26 | 1.37 |
| MGXX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 5.47 | 0.91 | 1.09 | 1.37 | 0.05 | 0.16 | 6.59 | 0.12 |
| ADYX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.14 | 0.36 | 0.43 | 0.54 | −0.11 | 0.42 | 13.36 | 1.46 |
| FMGX | 6.81 | 4.32 | 0.55 | 5.35 | 5.41 | 5.11 | 4.59 | 4.65 | 5.67 | 0.15 | 0.19 | 10.82 | 0.44 |
| ILUX | 27.35 | 41.66 | 46.59 | 22.64 | 27.38 | 16.14 | 30.29 | 28.02 | 23.38 | 0.04 | 0.07 | 3.30 | 0.10 |
| IGOX | 1.44 | 0.00 | 9.14 | 5.88 | 3.84 | 8.46 | 4.79 | 5.75 | 4.91 | 0.06 | 0.12 | 3.31 | 0.22 |
| SHDX | 3.32 | 0.00 | 0.00 | 2.70 | 2.46 | 3.94 | 2.07 | 2.48 | 3.11 | −0.05 | 0.29 | 10.49 | 0.50 |
| MLMX | 8.71 | 0.00 | 0.00 | 5.27 | 6.87 | 5.38 | 4.37 | 5.25 | 6.56 | 0.02 | 0.22 | 2.91 | 0.36 |
| MOLX | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.58 | 0.60 | 0.72 | 0.90 | −0.12 | 0.29 | 5.61 | 0.67 |
| *P-Ret* | 0.042 | 0.042 | 0.042 | 0.042 | 0.042 | 0.042 | NA | NA | NA | NA | NA | NA | NA |
| *P-Risk* | 4.39 | 40.91 | 7.94 | 1.35 | 0.035 | 0.026 | NA | NA | NA | NA | NA | NA | NA |

*Notes*: This table reports the minimum risk optimal weights (%) of the iron ore-nickel sector portfolio. The abbreviations LP, QP, G, MW, C and VaR stand for the linear programing, the mean-variance quadratic programming, and the Gaussian, mean of weights, the canonical and variance. The names and codes of the stocks are provided in Table 1. R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean weights excluding *CDaR* and *Minimax* and *CDaR*, respectively. The target portfolio return is 0.042%.

is the rootstock of the iron ore-nickel portfolio according to the c-vine modeling. The descriptive statistics indicate that BHP BILLITON (BHPX) has the largest return relative to risk in the entire portfolio. The risk and return of the portfolios indicate that the modeling framework, which combines the pair c-vine copula with portfolio optimization, outperforms the quadratic portfolio optimization based on the empirical variance-covariance matrix, another indication that the pair vine copulas are suitable to capture complex dependence features from the marginal and joint distributions.

A synthesis of the portfolio optimization indicates that it is worth considering pair vine copulas in portfolio optimization given the edge they provide in the estimations. The interpretation of the multiple weight allocations in terms of "average model convergence" is shown to be an effective alternative way to deal with the investment confidence problem faced by the mining investors during the stock selection process. Our proposed modeling framework uncovers the dependence risk and resource allocation risk of the Australian gold and iron ore-nickel portfolios.

## 5. Conclusions

The research into the fields of asset dependence and portfolio optimization has attracted significant attention from both the finance academics and practitioners since the seminal work of Markowitz (1952). It is now common that while the Markowitz (1952)'s mean-variance optimization constitutes the foundation of the modern portfolio theory, it suffers from several drawbacks including, among others, the assumptions of normally distributed returns and a constant linear correlation structure of portfolio assets. Hence, an efficient approach to portfolio optimization must appropriately accommodate not only the stylized facts in the distributional characteristics of asset returns (Samuelson, 1970; Rubinstein, 1973; Clark, 1973) but also their conditional dependence structure (Longin and Solnik, 2001; Engle, 2002).

This article addresses the dependence characteristics and risks of Australian gold and iron ore-nickel stock portfolios under specific market conditions. The dependence structure and optimal weight allocation characteristics of the portfolios is also examined. The minimum risk optimal portfolios are estimated using multiple risk measures combined with linear and nonlinear optimizations methods. An integrated modeling framework of the pair vine copulas and portfolio optimization with respect to the variance risk measure is also implemented with the purpose of improving the accuracy of the estimations.

Our results confirm the suitability of the r-vine and c-vine copula models in gauging the complex and nonlinear dependence structure of the stocks across three financial period scenarios of the two portfolios: the pre-GFC (January 2005–July 2007), the coincident GFC (July 2007–December 2009) and the post-GFC (January 2010–July 2012). Based on the dependence risk of the stocks and the high and low dependence risk features of the portfolios in specific market conditions, the gold stocks are found to be less risky than the iron ore-nickel stocks during times of financial uncertainty and crisis periods characterized by low or no confidence in the stock markets. Gold therefore retains the reputation it has ever commanded of being a safe haven in crisis periods. On the other hand, the iron ore-nickel stocks are found to be a better investment choice when the markets behave normally since their dependence risk is rather high in times of financial uncertainty characterized by low or no confidence in the stock markets.

Regarding the multiple risk measure-based portfolio optimizations, the weight allocation is not similar across models where different risk measures are used. This finding suggests that investors in the Australian gold and iron ore-nickel sectors should be aware of these weight differences and look for average model convergence in order to mitigate the uncertainty they face when selecting stocks for investment. We show that the average model convergence occurs in some stocks which could be considered as good candidates for investment. The use of the average model convergence also has useful implications for portfolio management and optimization as it spotlights the stocks where some, most or all of the models converge. It also reduces the significance of the problem of trying to find the best risk measure to be used for the optimization of portfolios. From a practical point of view, the weight allocation differences should be considered in order to make an appropriate choice (based on the return appetite and risk tolerance) of the risk measure to be used for the optimization of portfolios. Finally, our results point out that the integration of the pair vine copulas and portfolio optimization does provide an edge in the estimations in terms of risk and return tradeoffs.

Overall, the financial methodology implemented in this paper is useful for policy purposes and could also provide suggestions for investing in the Australian mining sectors. The gold portfolio, for instance, could be used to hedge investment positions in equity sectors which have high dependence risk during crisis periods. Moreover, if multiple sector portfolios are modeled, their dependence risk characteristics and differences could be combined to design risk management frameworks and hedging strategies that can be used to manage the risk of loss during market downturns. For their part, policymakers can apprehend the level of systemic risk when the economy relies heavily on the performance of a specific market sector (e.g., the mineral and energy resources sectors in Australia and Canada), which has the potential to trigger strong contagion affects and lead to recessions. By taking this information into consideration, policymakers could device risk management frameworks to deal with extreme market downturn events. The fit of the multiple risk measure-based portfolio optimization also enables investors to identify, with greater confidence, the mining stocks that are good candidates for investment.

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