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Downside risk management and VaR-based optimal portfolios for precious metals, oil and stocks

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ABSTRACT

Value-at-Risk (VaR) is used to analyze the market downside risk associated with investments in six key individual assets including four precious metals, oil and the S&P 500 index, and three diversified portfolios. Using combinations of these assets, three optimal portfolios and their efficient frontiers within a VaR framework are constructed and the returns and downside risks for these portfolios are also analyzed. One-day-ahead VaR forecasts are computed with nine risk models including calibrated RiskMetrics, asymmetric GARCH type models, the filtered Historical Simulation approach, methodologies from statistics of extremes and a risk management strategy involving combinations of models. These risk models are evaluated and compared based on the unconditional coverage, independence and conditional coverage criteria. The economic importance of the results is also highlighted by assessing the daily capital charges under the Basel Accord rule. The best approaches for estimating the VaR for the individual assets under study and for the three VaR-based optimal portfolios and efficient frontiers are discussed. The VaR-based performance measure ranks the most diversified optimal portfolio (Portfolio #2) as the most efficient and the pure precious metals (Portfolio #1) as the least efficient.

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1. Introduction

In this high risk and volatile environment, the time is right to examine the downside risk/return profiles for major commodities and stocks. In particular, the downside risk pertains well to the four major precious metals – gold, silver, platinum and palladium – which have risen significantly in terms of global trading and portfolio investments in the recent years, as well as to oil and stocks. The financial and commodity markets had undergone a severe financial crisis in 2007/2008, which turned into a Great Recession, fostering risk aversion and preferences toward safe havens. Despite the ensuing recovery, the mounting risk and uncertainty have confounded investors, portfolio managers and policy-makers. In such an environment, it will be valuable and useful to examine asset behaviors that are not only volatile but also characterized by extreme events like the 2007/2008 financial crisis that affected essentially all asset markets.

Standing as hedges and safe havens against risk and during uncertainty, commodities like the precious metals and oil have experienced extraordinary surges in prices and returns in the last few years, which have elevated the potential downside risk and subjected them to black swan-types of events. These assets have therefore become important elements of diversified portfolios. Additionally, stocks have also become very volatile on both sides of the return aisle and had underdone severe extreme events; with high and opposing wild swings being part of their daily trading. Under such circumstances, significant and extreme drops in prices and returns of these assets have become more probable, with potentially damaging consequences on portfolios of individuals and institutions. These circumstances have also made risk management strategies for these high flying commodities and highly volatile stocks more challenging, particularly as the percentages of violations of confidence targets have compounded.

The quantification of the potential size of losses and assessing risk levels for individual precious metals, oil, stocks and portfolios composed of them is fundamental in designing prudent risk management and portfolio strategies. Value-at-Risk (VaR) models have become an important instrument within the financial markets for quantifying and assessing market downside risks associated with financial and commodity asset price fluctuations. They determine the maximum expected loss an asset or a portfolio can generate over a certain holding period, with a pre-determined probability value. Thus, a VaR model can be used to evaluate the performance of portfolio managers by providing downside risk quantification, together with asset and portfolio returns. It can also help investors and portfolio managers to determine the most effective risk management strategy for a given situation. Moreover, quantification of the extreme losses in asset markets is important in the current market environment. Extreme value theory (EVT) provides a comprehensive theoretical forum through which statistical models describing extreme scenarios can be developed.

There is a cost of inaccurate estimation of the VaR in financial markets which affects efficiency and accuracy of risk assessments. Surprisingly, despite the increasing importance of precious metals and the diversified portfolios that include them as well as other assets and their highly volatile nature, to our knowledge there is only one study that analyzes the VaR for precious metals (Hammoudeh, Malik, & McAleer, 2011), while there are several studies that have worked on oil and stocks' VaRs. Hammoudeh et al. (2011) concentrate on the four major precious metals only, use relatively older VaR techniques and do not deal with VaR-based optimal portfolio constructions and efficient VaR frontiers. These authors do not distinguish between the risk associated with positive and negative returns which usually display asymmetric behavior. Their study also does not deal directly with volatility clustering. Moreover, it does not include EVT methods which provide quantification of the stochastic behavior of a process at unusually large or small levels. On the contrary, our current study expands the spectrum of asset diversification and deals with events that are more extreme than any others that have been previously observed. Most importantly, it constructs VaR-based optimal portfolios and efficient VaR frontiers of different degrees of diversification and examines their characteristics and performances. It also ranks those optimal portfolios using a VaR-based risk performance measure.

The broad objective of this paper is to fill this void in the financial risk management and modern portfolio analysis literature by using more up-to-date techniques and designing optimal diversified portfolios that take into account volatility asymmetry and clustering, with relatively strong emphasis on precious metals which have not been researched adequately despite their potential to provide diversification within broad investment portfolios and hedging capability (Draper, Faff, & Hillier, 2006). To achieve these objectives, the paper computes VaRs for gold, silver, platinum, palladium, oil and the S&P 500 index, using nine estimation methods including RiskMetrics, Duration-based Peak Over Threshold (DPOT), conditional EVT (CEVT), APARCH models (using normal and skewed *t*-distributions), GARCH-based filtered historical simulation and median strategy. Using different and multiple VaR techniques are of particular importance during high volatility periods like the one the markets experienced during the 2007/2009 Great Recession and its ensuing weak and choppy recovery. The VaR estimates for the different models diverge considerably during these periods, and thus should have pertinent implications for capital charges and profitability. The paper also uses several risk performance evaluations of these techniques including an unconditional coverage test, an independence test and a conditional coverage test. The risk models are also compared under the Basel Accord rules. The optimal VaR-based portfolios and their efficient VaR frontiers are constructed.

The portfolio weights suggest that optimal portfolios have more gold than any of the six assets under study. The average portfolio daily returns of the three optimal portfolios differ only slightly. As an annual approximation, we obtain the average returns 9%, 8.625% and 8.5%, for optimal portfolios #1, #3 and #2, respectively. In terms of standard deviation, the most diversified optimal portfolio (#2) has the lowest standard deviation as expected. In terms of statistical properties, the best performers are the conditional EVT and the Median Strategy. Under the Basel II Accord, the performance diverges between the individual assets and optimal portfolios. With individual assets, the RiskMetrics performs poorly and the best performer is the CEVT-sstd model. However, with optimal portfolios the RiskMetrics model is the best performer under the Basel rules, followed by the Median Strategy and the conditional EVT models. In the case of the well-known RiskMetrics model applied to optimal portfolios, there is a discrepancy between the performance using the statistical properties and the performance under the Basel rules.

As indicated above, such a study is valuable and useful in light of increases in the weights of commodities, particularly precious metals, in portfolios, especially hedge funds and exchange-traded funds (ETFs). More stringent changes in the Basel accords can have adverse effects on banks, their stocks and the value of their trading portfolios which likely include precious metals and oil, as well as stocks.

This paper is organized as follow. After this introduction, Section 2 provides a review of the literature. Section 3 presents the VaR models under comparison. In Section 4 we construct optimal portfolios and their efficient frontiers within a VaR framework. In Section 5, we compare the VaR models using the returns from individual models and form the optimal portfolios constructed in the previous section. Section 6 concludes.

2. Review of the literature

The commodity literature is expanding and gaining importance as a result of the increasingly significant role that these assets play in international financial markets and global economies. More exchange-traded commodities (ETCs) and exchange-traded funds (ETFs) are being created for specific commodities, being heavy on certain commodities or as hybrids of commodities and equities such as the CRB Global Commodity Equity Index Fund.[3] Barclays created an ETF based on the broad-based Goldman Sachs Commodity Index (GSCI), tracking 24 commodities across the energy, metal, and agriculture and livestock sectors. The most recent promising ETFs have been created for platinum and palladium.[4] In this section, we present a review of existing studies and highlight the economic significance regarding the particularly sparse literature related to precious metals, as well as the literature on energy commodities and stocks.

[3] Deutsche Bank introduced the first commodities ETF listed on a U.S. exchange in February 2006. This ETF tracks six highly-liquid futures contracts on crude oil, heating oil, aluminum, gold, corn, and wheat, and is rebalanced annually to weights of 35%, 20%, 12.5%, 10%, 11.25%, and 11.25%, respectively.

[4] The first gold ETF is the SPDR® Gold Shares (GLD) which was originally listed on the New York Stock Exchange in November of 2004. It is the largest physically backed gold exchange traded fund (ETF) in the world and has a value of more than $60 billion.

Jensen, Johnson, and Mercer (2002) find that commodity futures substantially enhance portfolio performance for investors, and show that the benefits of adding commodity futures accrue almost exclusively when the Federal Reserve is following a restrictive monetary policy. Overall, their findings indicate that investors should gauge monetary conditions to determine the optimal allocation of commodity futures within a portfolio. Draper et al. (2006) examine the investment role of precious metals in financial markets using daily data for gold, silver and platinum. They show that all three precious metals have low correlations with stock index returns, which suggests that these metals provide diversification within broad investment portfolios. They also show that all three precious metals have hedging capability for playing the role of safe havens, particularly during periods of abnormal stock market volatility.

Hammoudeh and Yuan (2008) apply univariate GARCH models to investigate the volatility properties of two precious metals, gold and silver, and one base metal, copper. Using the standard univariate GARCH model, they find that gold and silver had almost the same volatility persistence, while the persistence was higher for the pro-cyclical copper. Canover, Jensen, Johnsos, and Mercer (2009) present new evidence on the benefits of adding precious metals (gold, silver and platinum) to U.S. equity portfolios. They find that adding a 25% metals allocation to the equities of precious metals firms improves portfolio performance substantially, and that gold relative to platinum and silver has a better standalone performance and appears to provide a better hedge against the negative effects of inflationary pressures. They also show that while the benefits of adding precious metals to an investment portfolio varied somewhat over time, they prevailed throughout much of the 34-year period.

Prices of precious metals, oil and stocks have been highly volatile in the past, and even more so recently. The volatile precious metal price environment requires market risk quantification. VaRs have become an essential tool within financial markets for quantifying and assessing portfolio market risk, that is, the risk associated with price movements (see Christoffersen, 2009; Jorian, 2007 for a detailed overview of VaR). A VaR model determines the maximum expected loss a portfolio can generate over a certain holding period, with a pre-determined probability value. Therefore, VaR can be used, for instance, to evaluate the performance of portfolio managers by providing risk quantification, together with portfolio returns. Moreover, VaRs can help portfolio managers to determine the most suitable risk management strategy for a given situation.

VaRs have thus become a standard measure of downside market risk and are widely used by financial intermediaries and banks (see Basel Committee on Banking Supervision, 1988, 1995, 1996; Pérignon and Smith, 2010), equity markets (Bali, Moc, & Tanga, 2008; McAleer & da Veiga, 2008a, 2008b; McAleer, Jimenez-Martin, & Perez-Amaral, 2009; McAleer, Jimenez-Martin, & Perez-Amaral, 2010), energy markets (Cabedo & Moya, 2003; Marimoutou, Raggd, & Trabelsi, 2009), among others. As mentioned above, despite the importance of precious metals and their volatile nature, to the best of our knowledge there is only one study that estimates VaRs for precious metals. Hammoudeh et al. (2011) use VaR models to analyze the downside market risk associated with unilateral investments in gold, silver, platinum and palladium. The estimation models include RiskMetrics, Gaussian GARCH(1,1), GARCH-based FHS, GARCH with *t*-distribution and GARCH-FHS. Their results suggest that portfolio managers engaged in precious metals who wish to follow a conservative strategy should calculate the VaR using GARCH-*t* as this will yield fewer violations, though with lower profitability. As indicated before, Hammoudeh et al. (2011) does not use recent advances in estimation techniques and does not construct optimal VaR-based portfolios and efficient VaR frontiers.

VaR methods have also been used to measure and evaluate down side market risk for the energy markets. Hung, Lee, and Liu (2008) use three GARCH models (GARCH-*N*, GARCH-*t* and GARCH-HT h) to estimate and compare the accuracy and efficiency of the VaR models for daily spot prices of five energy commodities – WTI crude oil, Brent crude oil, heating oil No. 2, propane and New York Harbor Conventional Gasoline Regular. The results suggest that the VaR estimates generated by the GARCH-HT models have good accuracy at both low and high confidence levels. Additionally, they also imply that VaR models are suitable for energy commodities. Marimoutou et al. (2009) apply unconditional and conditional EVT models to forecast the VaR in the oil market. The results of these models are compared to those of conventional models such as GARCH, HS and FHS. The conditional EVT and FHS offer a major improvement over the other methods under study. However, GARCH(1,1)-*t* model may provide equally good results which are comparable to those of the conditional and FHS methods. These authors underscore the importance of filtering in forecasting VaRs. Aloui and Mabrouk (2010) compute the VaRs for three ARCH/GARCH-type models including FIGARCH, FIAPARCH and HYGARCH. They show that with consideration for long-range memory, fat-tails and asymmetric models perform better in predicting a one-day-ahead VaR for both short and long trading positions. Additionally, the FIAPARCH model outperforms the other models in the prediction of VaRs. Cabedo and Moya (2003) examine three VaR estimation methods: the historical simulation standard approach, the historical simulation with ARMA forecasts (HSAF) approach, and the variance-covariance method based on ARCH model forecasts to quantify the oil price risk. The results show that HSAF methodology provides a flexible VaR quantification, which fits the continuous oil price movements well and provides efficient risk quantification.

Recurring crashes in stock markets and returning stumbles in commodity markets have also brought to prominence the pertinence of analysis of extreme events and black swans. Extreme risk analysis using the General Pareto Distribution (GPD) model gained momentum in the past two decades as a result of high swings and violent crashes in stock and commodity prices. McNeil (1997, 1998) investigates extreme risks in financial time series, using extreme value theory. Embrechts (1999, 2000) shows robustness of EVT in risk estimates. McNeil and Frey (2000) extend the analysis of extreme risk by combining a GARCH filter with the extreme value theory. Muller, Dacorogna, and Pictet (1998) and Pictet, Dacorogna, and Mullar (1998) investigate extreme risk in foreign exchange markets using GARCH models. Gencay and Selcuk (2004) investigate the relative performance of VaR models using EVT, in a number of emerging markets after the 1997 Asian financial crisis. Giot and Laurent (2003) model VaR using a number of parametric univariate and multivariate models of the ARCH class with skewed student-*t* density.

Under the Modern Portfolio Theory, the weights of assets in a portfolio are obtained by maximizing the expected risk premium per unit of risk, where the standard deviation is the measure for risk. With the presence of asymmetric and heavy tailed distributions for returns, the standard deviation as a measure for risk can lead to inefficient strategies to optimize portfolios. In the recent literature, a newer approach emerged to maximize expected return subject to a downside risk constraint rather than the standard deviation. The construction of portfolios by maximizing expected return subject to a shortfall constraint has its origins in the work of Roy (1952). Leibowitz and Kogelman (1991) and Lucas and Klaassen (1998) define the shortfall constraint as a minimum return that should be gained over a given time horizon for a given confidence level. Campbell, Huisman, and Koedijk (2001) extend the literature on asset allocation subject to shortfall constraints, suggesting a portfolio construction model based on the VaR.

3. VaR estimation methods

In this section, we explicitly define the VaR followed by a brief review of the nine different methods that we use to estimate the VaR.[5] As usually, we consider the asset return process denoted by

$$R_t = \mu_t + \varepsilon_t \tag{3.1}$$

where $\varepsilon_t|\Omega_{t-1} \sim (0, h_t)$, Ω_{t-1} is the information set at time $t-1$ and h_t is the conditional variance at time t. The VaR measure with coverage probability, p, is defined as the conditional quantile, $VaR_{t|t-1|}(p)$, where

$$P(R_t \leq VaR_{t|t-1}(p)|\Omega_{t-1}) = p \tag{3.2}$$

The VaR is a quantile p of the return distribution and measures the worst expected loss over a given horizon at a given level of confidence $1-p$. It is usual to multiply this quantile by the amount invested and express the VaR in terms of this amount. Throughout the paper, we choose the coverage probability $p = 0.01$, which is consistently used in the literature and is the level established in the Basel Accord rules for computing capital requirements (see Basel Committee on Banking Supervision, 1988, 1995, 1996).

For the out-of sample study we choose the well-known Morgan (1996) RiskMetrics approach which assumes $\mu_t = 0$, a normal distribution for ε_t and $h_t = (1-\lambda)\varepsilon_{t-1}^2 + \lambda h_{t-1}$ where λ is set to 0.94 for daily

[5] The working paper with detailed information on those VaR methods is available upon request.

data. We choose two asymmetric GARCH type models based on the Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) model where μ_t is expressed as a first autoregressive process based on returns and h_t is expressed as in the APARCH(1,1) model proposed by Ding, Engle, and Granger (1993). We denote by APARCH-n the model with normal innovations z_t, and by APARCH-sst the one with skewed-*t* innovations. In a comparative study for the Asian markets, Tu, Wong, and Chang (2008) find that the APARCH model with the skewed-*t* distribution performs better than the one with the normal or with the student-*t* distribution. The GARCH-type models with skewed-*t* innovations have frequently been found to provide excellent forecast results (see, for example, Giot & Laurent, 2003; Mittnik & Paolella, 2000).

The Filtered Historical Simulation approach (FHS) was originally proposed by Barone-Adesi, Giannopoulos, and Vosper (1999). Under the FHS approach, we pre-filter the data using a location-scale model based on an AR(s) process and the GARCH(1,1) model. We choose one model with the filter AR(1)-GARCH(1,1), another with normal innovations (denoted by FHS-n) and a third with the filter AR(1)-GARCH(1,1) with skewed-*t* innovations (denoted by FHS-sstd).

In the group of the EVT models, we choose the Conditional EVT (CEVT) and the Duration based Peaks-Over-Threshold (DPOT). The first is well known and is widely used in the literature, while the second was recently proposed by Araújo Santos and Fraga Alves (2011). This CEVT is a two-stage hybrid approach which combines a time-varying volatility model with the Peaks-Over Threshold method derived from the EVT method (for details about the POT method, see Embrechts, Klüppelberg, & Mikosch, 1997). Diebold, Schuermann, and Stroughair (1998) propose in a first step the standardization of the returns through the conditional means and variances estimated with a time-varying volatility model and in a second step the estimation of a p-quantile using EVT and the standardized returns. McNeil and Frey (2000) combine an AR(1)-GARCH(1,1) process, assuming normal innovations, with the POT method. The filter with normal innovations, while capable of removing the majority of clustering, will frequently be a misspecified model for returns. In order to address this misspecification, Kuester, Mittik, and Paolella (2006) suggest a filter with the skewed *t* distribution. We will denote this model as CEVT-n and CEVT-sst, with normal and with skewed *t* innovations, respectively. Several studies have concluded that the conditional EVT is the method with the better out-of-sample performance to forecast the one-day-ahead VaR (see, for example, Bekiros & Georgoutsos, 2005; Bystrom, 2004; Ghorbel & Trabelsi, 2008; Kuester et al., 2006; McNeil & Frey, 2000; Ozun, Cifter, & Yilmazer, 2010).

The POT method is based on the excesses over a threshold u and on the Pickands–Balkema–de Haan Theorem (see Balkema & de Haan, 1974; Pickands, 1975). For distributions in the maximum domain of attraction of an extreme value distribution, this theorem states that when u converges to the right-end point of the distribution, the excess distribution $P[X-u \le y | X>u]$ converges to the Generalized Pareto Distribution (GPD):

$$G_{\gamma,\sigma}(y) = \begin{cases} 1-(1+\gamma y/\sigma)^{-1/\gamma}, & \gamma \neq 0 \\ 1-\exp(-y/\sigma), & \gamma = 0 \end{cases} \tag{3.3}$$

where $\sigma > 0$, and the support is $y \ge 0$ when is $\gamma \ge 0$ and $0 \le y \le -\sigma/\gamma$ when is $\gamma < 0$. With financial time series, a relation between the excesses and the durations between excesses are usually observed. Araújo Santos and Fraga Alves (2011) propose using this dependence to improve the risk forecasts with duration-based POT models (DPOT). For estimation, these models use the durations, at time of excess i, as the preceding v excesses ($d_{i,v}$). At time t, $d_{i,v}$ denotes the duration until t as the preceding v excesses. The DPOT model assumes the GPD for the excess Y_t above u, such that

$$Y_t|\Omega_t \sim \text{GPD}\left(\gamma, \sigma_t = \frac{\alpha}{(d_{t,v})^c}\right) \tag{3.4}$$

where γ and α are parameters to be estimated. We choose $v=3$ and $c=3/4$, as values of c close or equal to 3/4 have been shown to exhibit the best results (see Araújo Santos & Fraga Alves, 2011).

Finally we choose the median strategy. In McAleer et al. (2010) a risk management strategy proposed under the Basel II Accord is described as being robust to a global financial crisis. These authors define a robust risk management strategy as a strategy that provides stable results in terms of the daily capital requirements and the number of violations, regardless of the economic turbulence (tranquil or turbulent periods). The empirical results suggest that the strategy based on the median of the point VaR forecasts of a set of risk models was robust in this sense.

4. Optimal portfolios

Applying the portfolio construction model proposed by Campbell et al. (2001), we derive three optimal portfolios with the provision that the maximum expected loss would not exceed the VaR for a chosen investment horizon at a given confidence level. This is a general model for an optimal portfolio selection developed under the framework of Arzac and Bawa (1977), and this model under certain assumptions collapses to the CAPM, as developed by Sharpe (1964), Lintner (1965) and Mossin (1966). In what follows, the notation is similar to those presented in Campbell et al. (2001). The amount invested is denoted by $W(0)$ and the time horizon by T. The amount B represents borrowing ($B > 0$) and lending ($B < 0$), while r_f is the interest rate at which the investor can borrow and lend for the period T. With the n assets, $\gamma(i)$ denotes the fraction invested in the asset I, while $q(p,P)$ stands for the quantile p of the return distribution for portfolio P and $VaR(p,P)$ is the Value-at-Risk for portfolio P. The following performance measure for risk

$$\varphi(p, P) = W(0)r_f - VaR(p, P) \tag{4.1}$$

was proposed. The mathematical problem is to find the optimal portfolio P' by choosing the fractions $\gamma(i)$ that maximize the return-risk ratio $S(P)$. This ratio can be written as

$$P' : \max_p S(P) = \frac{r(P) - r_f}{\varphi(p, P)} \tag{4.2}$$

Investors will first choose the fractions $\gamma(i)$ and then the amount of borrowing or lending will depend on how much the VaR of the portfolio P' differs from the VaR limit defined by the investor (VaR^*). Taking into account VaR^*, the amount to be borrowed can be obtained with the following equation:

$$B = \frac{W(0)(VaR^* - VaR(p, P'))}{\varphi(p, P')} \tag{4.3}$$

In this study, six assets have been used to consider three types of portfolios and in order to construct three optimal portfolios under the framework presented above. Optimal portfolio #1 is the pure precious metals portfolio and includes the four precious metals – gold, silver, platinum and palladium. Optimal portfolio #2 is the most diversified and comprised of six assets that include the four precious metals, Brent oil and the S&P 500 index. Portfolio #3 targets the three asset classes and encompasses gold, oil and the S&P 500 index. We use the daily time horizon (T), the coverage probability (p) that is equal to 0.01 and the risk-free return (r_f) equal to the 10-year Treasury rate available on the last day of the sample period (which is equal to 3.16%).[6] The used daily returns are based on the closing spot prices for the four precious metals (gold, silver, platinum, and palladium), oil and the S&P 500 index for the period January 2, 1995 to July 5, 2011. In order to construct the efficient frontiers without the risk-free asset, we apply genetic algorithms, and then we apply Eq. (4.2) to obtain the optimal portfolios. To construct the efficient frontier with the risk-free asset, we apply Eq. (4.3). Considering the period under study, the efficient frontiers for the three types of portfolios are presented in Figs. 1–3. In these Figures, we also represent the return and risk as defined in Eq. (4.1) for each individual asset, using gray points. Tables 1, 2 and 3 present the fractions of each individual asset in the optimal portfolios #1, #2 and #3, respectively.

In Eq. (4.2), $S(P)$ is a performance measure like the Sharpe ratio, which can be used to evaluate and rank the efficiency of portfolios. We apply this measure to the three optimal portfolios achieving the ratios 9.63605E−06, 8.69753E−06 and 7.43642E−06, respectively for portfolios #2, #3 and #1. As

[6] We also computed optimal portfolios using others values for the risk-free return and the results are not very sensitive to values close to the 3.16% rate.

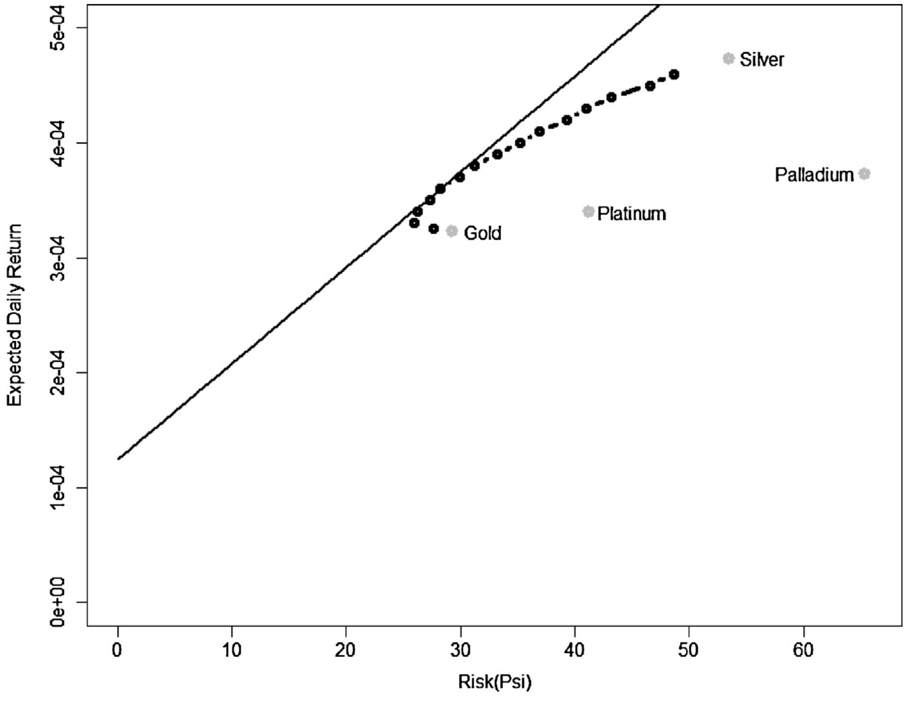

Fig. 1. Efficient VaR frontier for optimal Portfolio #1. Notes: Portfolio #1 encompasses gold, silver, platinum and palladium. The efficient VaR frontier is for the empirical distribution, using daily data and a daily VaR at the 99% confidence level.

expected, the better ratio is achieved with the most diversified portfolio (Portfolio #2) that includes the six assets, followed by portfolios #2 and #3 in this sequence. In the next section, we compare the nine risk models using the returns of the six individual assets and to the three constructed optimal portfolios.

Table 1
Estimated VaR for optimal Portfolio #1.

Gold (%)	Silver (%)	Platinum (%)	Palladium (%)	Portfolio VaR ($)
58.2%	21.8%	18.2%	1.8%	−28.09

Notes: Portfolio #1 encompasses gold, silver, platinum and palladium. Data on the precious metals returns are used to find the optimal portfolio at the point where the risk-return trade-off in Eq. (4.2) is maximized. The risk-free return is the last 10-year Treasury rate available in the sample period (equivalent to 3.16%). The VaR for $1000 held in the portfolio is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR.

Table 2
Estimated VaR for optimal Portfolio #2.

Gold (%)	Silver (%)	Platinum (%)	Palladium (%)	Brent (%)	SP 500 (%)	Portfolio VaR ($)
44.6%	3.7%	25.7%	2.0%	11.6%	12.5%	−22.24

Notes: Portfolio #2 is comprised of gold, silver, platinum, palladium, Brent and the S&P 500 index. Daily returns are used to find the optimal portfolio at the point where the risk-return trade-off in Eq. (4.2) is maximized. The level of the risk-free return is the last data available on the 10-year Treasury rate of the sample period (which is equal to 3.16%). The VaR for $1000 held in the portfolio is given for a daily time horizon and the 99% confidence level. The historical distribution is used to estimate the VaR.

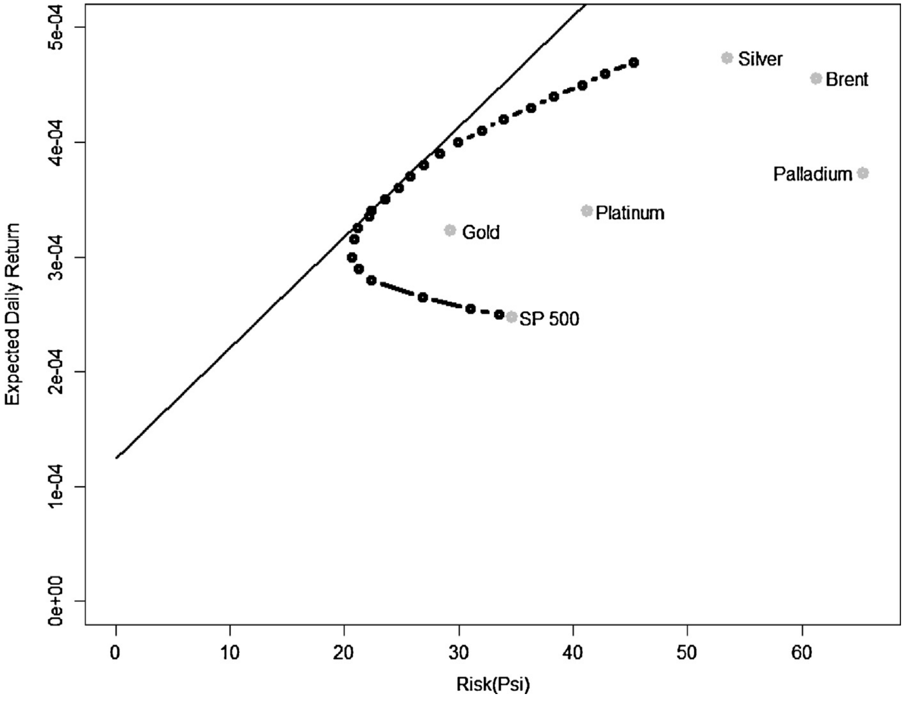

Fig. 2. Efficient VaR frontier for optimal Portfolio #2. Notes: Portfolio # 2 includes gold, silver, platinum, palladium, Brent and the S&P 500 index. The efficient VaR frontier is for the empirical distribution, daily data and a daily VaR at the 99% confidence level.

Table 3
Estimated VaR for optimal Portfolio #3.

Gold (%)	Brent (%)	SP 500 (%)	Portfolio VaR ($)
66.4%	22.6%	10.9%	−25.01

Notes: Portfolio #3 includes gold, Brent and the S&P 500 index. Daily returns are used to find the optimal portfolio at the point where the risk-return trade-off Eq. (4.2) is maximized. The risk-free return is the 10-year Treasury rate available on the last day of the sample period which is equal to 3.16%. The VaR for $1000 held in the portfolio is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR.

5. Empirical results

In this section, we present the descriptive statistics for the individual assets and for the optimal portfolios constructed in the previous section, the results of the tests from the out-of-sample investigation, and the performance of the models under the Basel Accord.

5.1. Descriptive statistics

We use daily returns based on closing spot prices for the four precious metals (gold, silver, platinum, and palladium), the oil price and the S&P 500 index[7] for the period January 2, 1995 to July 5, 2011. Our sample period is particularly interesting to study because it is sourced for a diversified portfolio of

[7] We estimated univariate VaRs for exchange rates. While the VaR results were reasonable, the weights of the exchange rates in the optimal portfolio were very small because of the fact that for the period under study the average returns are very close to zero. The very small weights led us to exclude these assets in this paper.

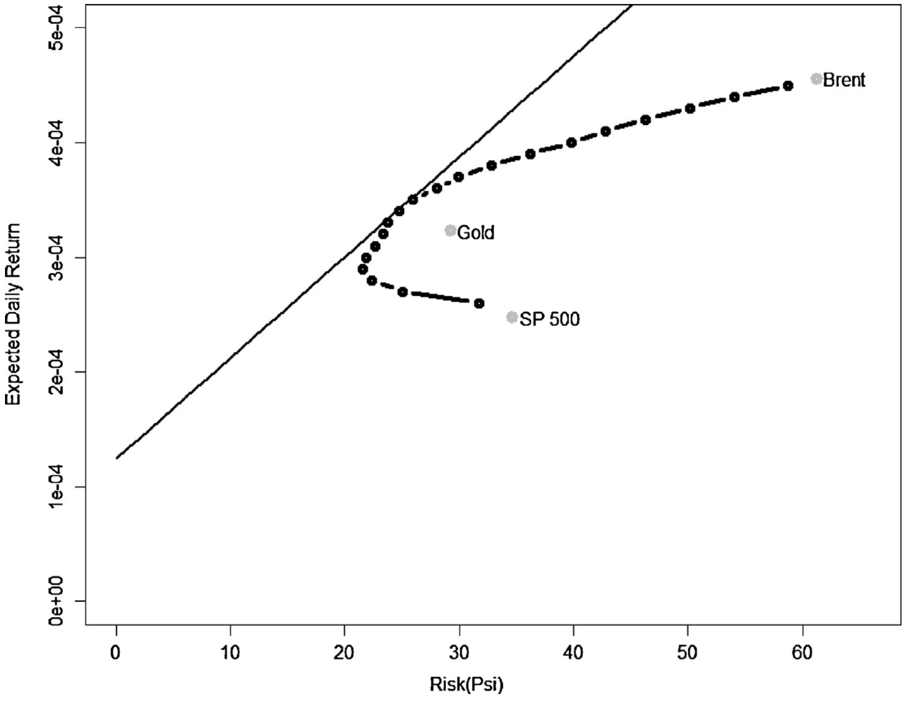

Fig. 3. Efficient VaR frontier for optimal Portfolio #3. Notes: Portfolio #3 includes gold, Brent and the S&P 500. The VaR frontier is for the empirical distribution, the daily data and a VaR at the 99% confidence level.

asset classes, with a strong precious metals flavor because of the strong performance of those metals during the period under study and the dearth of studies on market risk quantification for these metals and their combinations with other key assets. It also includes the financial crisis of 2008–2009.

All precious metals are traded at COMEX in New York, and their prices are measured in US dollars per troy ounce. The oil price is represented by the North Sea Brent which is traded at the Intercontinental Exchange (ICE) and measured in dollars per barrel. The price has proven recently to be the better reflector of oil market fundamentals and a better processor of geopolitical risk than West Texas Intermediate (WTI) which is highly sensitive to level of storage at Cushing, Oklahoma. Despite this recent important difference, their contemporaneous correlation over the sample period is about 0.96 because Brent and WTI belong to one common pool (Bhar, Hammoudeh, & Thompson, 2008).

The descriptive statistics for the six assets are given in Table 4. Over the sample period, silver has the highest historical average return, while the S&P 500 index exhibits the lowest among the six prices.

Table 4
Descriptive statistics for individual assets.

	Gold	Silver	Platinum	Palladium	Brent	S&P 500
Mean	0.000323	0.000474	0.000340	0.000373	0.000456	0.000248
Median	0.000000	0.000978	0.000245	0.000000	0.000103	0.000346
Maximum	0.070060	0.131632	0.100419	0.191608	0.181297	0.109572
Minimum	−0.079719	−0.203851	−0.096731	−0.169984	−0.198906	−0.094695
Std. Dev.	0.010178	0.018531	0.014103	0.022275	0.023453	0.012401
Skewness	0.052157	−1.099834	−0.373034	−0.046521	−0.111741	−0.207868
Kurtosis	6.381428	11.128350	5.729614	6.585190	8.039865	11.413160
Jarque–Bera	7314.86	23,103.20	5995.47	7788.82	4564.07	12,724.45
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 5
Descriptive statistics for optimal portfolios.

	Portfolio #1	Portfolio #2	Portfolio #3
Mean	0.000360	0.000340	0.000345
Median	0.000377	0.000433	0.000430
Maximum	0.064713	0.062196	0.068879
Minimum	−0.068637	−0.062630	−0.055381
Std. Dev.	0.010064	0.008623	0.009368
Skewness	−0.354589	−0.250287	0.009460
Kurtosis	8.502819	7.651170	7.215008
Jarque–Bera	5520.588	3924.522	3186.152
Probability	0.000000	0.000000	0.000000

Notes: Portfolio #1 encompasses gold, silver, platinum and palladium. Portfolio #2 includes gold, silver, platinum, palladium, Brent and the S&P 500 index. Portfolio #3 includes gold, Brent and the S&P 500 index.

In terms of historical volatility, Brent has the highest standard deviation on average while gold has the lowest among the asset classes. The low volatility of gold price is consistent with the fact that gold has an important monetary component, and is not used frequently in exchange market interventions like currencies. Silver is, however, more commodity-driven than gold as its monetary element has been gradually phased out and it has become more of an industrial metal. Brent production has been on a declining streak since 1995, adding to its scarcity, sensitivity and volatility. In fact, the North Sea production has fallen 45 percent since April 2003 (Hammoudeh, 2011). Moreover, the oil market is global and all oil strands belong to one common pool. It's traded on both NYMEX and ICE. The 2005 hurricane Katrina and the BP oil spell all impacted Brent as they affected the prices of other oil types.

The Jarque–Bera statistic indicates that all series are not normally distributed. All series have negative skewness, with the exception of gold which is positively skewed. The right tail for the positively skewed gold is longer; the mass of the distribution is concentrated on the left of the distribution figure. The return has a few relatively high values, which means most of series is bunched up on the low end of the spread scale. Silver has also the highest negative skewness. Moreover, all the series have high kurtosis, suggesting that their distributions are leptokurtic (peaked).

The descriptive statistics for the returns of the optimal portfolios constructed in Section 4 are given in Table 5. The average daily returns of the three VaR-based optimal portfolios differ only slightly. By multiplying the average daily returns by 250 as an annual approximation, we obtain around 9%, 8.625% and 8.5% annual returns for optimal portfolios #1, #3 and #2, respectively. In terms of standard deviation, the most diversified optimal portfolio (#2) has the lowest standard deviation, as expected. Also, for the optimal portfolios returns, the Jarque–Bera statistic shows strong evidence against the normal distribution.

5.2. Out-of-sample study

In order to compare the downside risk models under study for each individual asset and the three optimal portfolios, we use the 4304 daily returns from the period January 2, 1995 to July 5, 2011. With a moving window of size 1000 days, we obtain 3304 one-day-ahead VaR forecasts for each model presented in Section 4. As in previous studies, for the EVT methods, we choose the number of top order statistics $k = 100$ (see McNeil & Frey, 2000 for a simulation study that supports this choice). The programs that are used in estimating the risk models and in applying the accuracy tests are written in the R language (R Development Core Team, 2008) and with the fGarch (Wuertz, Chalabi, & Miklovic, 2008) and POT (Ribatet, 2009) packages. The primary tool for assessing the accuracy of the interval forecasts is to monitor the binary sequence generated by observing whether the return r_t on day t is in the tail region specified by the VaR at time $t-1$. This sequence is referred to as the hit sequence:

$$I_t(p) = \begin{cases} 1, & r_t < VaR_{t|t-1}(p) \\ 0, & r_t \geq VaR_{t|t-1}(p) \end{cases} \tag{5.1}$$

Table 6
Backtesting VaR for precious metals. Brent and SP500 index.

	RiskMetrics	APARCH-n	APARCH-sstd	FHS-n	FHS-sstd	DPOT	CEVT-n	CEVT-sstd	Median
Gold									
% of viol.	1.8160%	1.9673%	1.0291%	1.1804%	1.2409%	1.0291%	1.0291%	0.9988%	1.0593%
Kupiec uc	17.898***	24.360***	0.028	1.027	1.799	0.028	0.028	0.000	0.115
MM ind	2.002	1.698	1.428	0.180	1.241	1.377	−0.223	−0.193	1.399
Christ. cc	22.791***	24.742***	0.854	1.962	2.185	13.043*	0.736	0.666	0.865
Silver									
% of viol.	2.2094%	2.2094%	1.2712%	1.0896%	1.1804%	1.1501%	1.1199%	1.2712%	1.1804%
Kupiec uc	36.310***	36.310***	2.260	0.260	1.027	0.717	0.462	2.260	1.027
MM ind	−1.653	−1.248	0.292	0.385	0.366	1.406	0.163	0.097	0.507
Christ. cc	36.615***	43.879***	12.042***	0.939	1.517	1.267	1.302	4.721*	1.517
Platinum									
% of viol.	2.1489%	2.1792%	1.2712%	1.4225%	1.2712%	1.0593%	1.2712%	1.2107%	1.3317%
Kupiec uc	33.147***	34.714***	2.260	5.268**	2.260	0.115	2.260	1.388	3.326*
MM ind	−0.043	0.738	−0.131	1.216	0.790	0.787	0.355	0.403	0.559
Christ. cc	44.442***	49.161***	4.721*	13.417***	4.721*	3.755	4.721*	4.151	5.508*
Palladium									
% of viol.	2.0278%	1.5436%	1.1804%	1.2409%	1.2409%	1.0593%	1.1199%	1.1199%	1.1804%
Kupiec uc	27.168***	8.458***	1.027	1.799	1.799	0.115	0.462	0.462	1.027
MM ind	0.474	0.335	0.603	−0.091	-0.091	1.708	0.012	0.012	−0.041
Christ. cc	33.349***	9.831***	3.951	2.185	2.185	12.676*	1.073	1.073	1.517
Brent									
% of viol.	1.5436%	1.3317%	1.0291%	0.9988%	1.0593%	1.1199%	0.9685%	0.9080%	1.0291%
Kupiec uc	8.458***	3.326*	0.028	0.000	0.115	0.462	0.033	0.292	0.028
MM ind	-0.719	2.603	2.435	1.565	2.406	2.260	1.081	1.686	1.020
Christ. cc	9.831***	3.582	0.854	0.666	0.866	1.302	0.659	0.840	0.736
SP500									
% of viol.	2.0581%	2.1186%	1.0593%	1.2712%	1.1804%	0.9988%	1.1199%	0.9080%	1.1804%
Kupiec uc	28.618***	31.608***	0.115	2.260	1.027	0.000	0.462	0.292	1.027
MM ind	1.788	10.245***	1.276	0.723	1.598	3.675*	1.990	1.422	2.324
Christ. cc	28.877***	31.816***	0.866	3.348	1.962	0.666	1.302	0.840	1.962

* represent significance at 10%.
** represent significance at 5%.
*** represent significance at 1%.

Table 7
Backtesting VaR for optimal portfolios.

	RiskMetrics	APARCH-n	APARCH-sstd	FHS-n	FHS-sstd	DPOT	CEVT-n	CEVT-sstd	Median
Portfolio #1									
% of viol.	1.6344%	1.8160%	1.3015%	1.1501%	1.1804%	1.2107%	1.1199%	1.1501%	1.1199%
Kupiec uc	11.271***	17.898***	2.770*	0.717	1.027	1.388	0.462	0.717	0.462
MM ind	0.236	0.540	0.937	−0.050	0.299	−0.364	−0.183	−0.209	−0.183
Christ. cc	12.369***	20.296***	8.256**	1.267	1.962	4.151	1.073	1.605	1.073
Portfolio #2									
% of viol.	1.7554%	1.5436%	1.2409%	1.1804%	1.2712%	1.2107%	1.2107%	1.1804%	1.2409%
Kupiec uc	15.547***	8.458***	1.799	1.027	2.260	1.388	1.388	1.027	1.799
MM ind	−0.768	−0.688	0.597	−0.094	1.102	1.260	0.048	0.335	0.482
Christ. cc	16.333***	8.523**	2.835	1.962	2.599	1.824	1.824	1.517	2.185
Portfolio #3									
% of viol.	1.6646%	1.6646%	1.1199%	1.3015%	1.3923%	1.1501%	1.1199%	1.1199%	1.2107%
Kupiec uc	12.285***	12.285***	0.462	2.769*	4.576**	0.717	0.462	0.462	1.388
MM ind	0.219	−0.920	0.180	1.513	1.559*	3.230*	0.507	0.507	−0.068
Christ. cc	13.299***	15.414***	1.073	3.065	4.762*	3.810	1.073	1.073	1.824

* represent significance at 10%.
** represent significance at 5%.
*** represent significance at 1%.

Christoffersen (1998) shows that evaluating interval forecasts can be reduced to examining whether the hit sequence satisfies the unconditional coverage (UC) and independence (IND) properties. When both properties are validated, we say that the hit sequence satisfies the conditional coverage (CC) property. In order to test the UC hypothesis, we apply the Kupiec test (Kupiec, 1995), while to test the CC hypothesis we apply the conditional coverage test developed by Christoffersen (1998). To test the IND hypothesis alone, we apply the independence test that was recently introduced in the literature by Araújo Santos and Fraga Alves (2010). This test is based on durations between consecutive violations and until the first violation. We refer to this test as the MM ratio test.

The results are presented in Tables 6 and 7. In terms of the percentage of violations and UC property, the RiskMetrics and APARCH-n models perform very poorly both with the individual assets and with the optimal portfolios. The percentage of violations is much higher than 1%, and in all cases with the exception of Brent, the UC hypothesis is rejected at the 1% significance level. With the FHS-n model and the Median Strategy, the UC hypothesis is rejected when we use the platinum returns, with the significance levels equal to 5% and 10%, respectively. With the APARCH-sst model, the UC hypothesis is rejected when we use the optimal portfolio #1's returns and for the higher significance level equal to 10%. All other models perform well in terms of the UC property, without a rejection of the UC hypothesis. It is interesting to note the very good performance of the DPOT model, with the percentage of violations being always very close to 1%. In terms of the CC property, the RiskMetrics and APARCH-n models perform very poorly both with individual assets and with optimal portfolios. With the APARCH-sstd model, the CC hypothesis is rejected for silver, platinum and optimal portfolio #1. The best performers are the CEVT models and the Median Strategy, with the rejection of the CC hypothesis occurring only in one case and at the higher significance level of 10%.

The results for the MM test are presented in Tables 6 and 7. All the models under study perform well or reasonably well in terms of not producing clusters of violations. The APARCH-sstd model in the case of the S&P 500 index and with the lower significance level of 1% fails the MM IND test. In the case of optimal portfolio #3, the FHS-sstd and the DPOT models fail the MM IND test, with the higher significance level equal to 10%. The DPOT model also fails the MM IND test with the returns from the S&P 500 index with the significance level equal to 10%.

5.3. Daily capital charges based on VaR forecasts

Under the Basel II Accord, the VaR forecasts of the banks must be reported to the regulatory authority. These forecasts are used to compute the amount of capital requirements used as a cushion against adverse market conditions. The Basel Accord stipulates that the daily capital charge must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor k (see Table 8). The Basel Accord imposes penalties in the form of a higher multiplicative factor k on banks which use models that lead to a greater number of violations than would be expected given the specified coverage probability $p = 0.01$. Considering the individual assets, only the DPOT model and CEVT-sstd never enter the red zone of the Basel rules. However DPOT produce higher averages daily capital charges with both individual assets and optimal portfolios. The best performer with the individual assets is the CEVT-sstd model. The results still are very different when optimal portfolios

Table 8
Basel accord penalty zone.

Zone	Number of violations	k
Green	0–4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days.

Table 9
Daily capital charges.

Model	Number of days in the red zone	Daily capital charges		
		Mean	Maximum	Minimum
Panel A: S&P 500				
RiskMetrics	426	0.092	0.400	0.035
AR APARCH-n	616	0.089	0.395	0.034
AR APARCH-sstd	3	0.094	0.374	0.038
Filtered HS-n	209	0.093	0.428	0.038
Filtered HS-sstd	81	0.094	0.416	0.038
DPOT	0	0.099	0.267	0.035
CEVT-n	170	0.098	0.440	0.037
CEVT-sstd	0	0.096	0.382	0.036
Median strategy	139	0.094	0.406	0.037
Panel B: Portfolio #1 (gold, silver, platinum and palladium)				
RiskMetrics	0	0.0777	0.2388	0.0299
AR APARCH-n	166	0.0784	0.2543	0.0419
AR APARCH-sstd	10	0.0837	0.2693	0.0420
Filtered HS-n	0	0.0820	0.2419	0.0409
Filtered HS-sstd	0	0.0818	0.2379	0.0389
DPOT	0	0.0946	0.2401	0.0362
CEVT-n	0	0.0828	0.2306	0.0442
CEVT-sstd	0	0.0823	0.2266	0.0416
Median strategy	0	0.0814	0.2262	0.0410
Panel C: Portfolio #2 (gold, silver, platinum, palladium, Brent and S&P 500 index)				
RiskMetrics	0	0.0673	0.2031	0.0357
AR APARCH-n	0	0.0655	0.1981	0.0397
AR APARCH-sstd	0	0.0706	0.2223	0.0457
Filtered HS-n	0	0.0691	0.2008	0.0414
Filtered HS-sstd	0	0.0701	0.1997	0.0418
DPOT	0	0.0781	0.1856	0.0380
CEVT-n	0	0.0694	0.1904	0.0439
CEVT-sstd	0	0.0695	0.1895	0.0448
Median strategy	0	0.0693	0.1876	0.0418

are considered. Considering the optimal portfolios, the best performer is RiskMetrics followed by the Median Strategy, the Conditional EVT and FHS models. In the case of RiskMetrics model when applied to optimal portfolios, there is clearly a discrepancy between the performance based on the statistical properties and the performance under the Basel rules. Without any loss of generality, in Table 9 we report the results for the S&P 500 index, Portfolio #1 and Portfolio #2.

6. Conclusions

In this paper, Value-at-Risk (VaR) is used to analyze the downside market risk associated with four precious metals, oil and the S&P 500 index. We also construct and rank three VaR-based optimal portfolios and efficient frontiers using these assets. We compute the VaR for the individual precious metals, oil, S&P 500 index and the portfolios, using the calibrated RiskMetrics, the APARCH model, the Filtered Historical Simulation approach, the duration-based POT method, the conditional EVT approach and the Median Strategy. The economic importance of our results is highlighted by calculating the daily capital requirements using the different models. In terms of statistical properties, the best performers are the conditional EVT and the Median Strategy. Under the Basel II Accord, the performance of the different methods in terms of the regulatory capital requirements and days in the red zone diverges between individual assets and optimal portfolios. For individual assets and based on the statistical properties, the RiskMetrics performs poorly while the best performer is the CEVT-sstd model. Based on the average capital requirements and days in the red zone, the performance of RiskMetrics for the individual assets is mixed, giving the lowest average for gold, silver and Brent and the second lowest for the rest of the assets. However, the best performance is still marred with several days in the red zone for silver. Surprisingly, with the three optimal portfolios the RiskMetrics model is the best performer under the Basel rules in terms of both the number of days in the red zone and the average capital requirements. This result has important implications for profitability of the portfolio.

The optimal portfolio weights suggest that the three optimal portfolios should have more gold than any of the other assets under study over the sample period. This result contradicts the conventional wisdom which suggests that about 10% of a diversified portfolio should be in gold. The VaR-based performance measure ranks the most diversified optimal portfolio (Portfolio #3 which includes gold, oil and the S&P 500) as the most efficient, and the pure precious metals portfolio (Portfolio #1) as the least efficient. This result underscores the importance of diversifying across different asset classes over diversifying within an asset class even if this class includes a star asset like gold or oil. It has also implications for ETFs which are based on one physical commodity or one asset class. Last but not least, the optimal portfolios give the best performance under the Basel rules for the RiskMetrics model which performs poorly in terms of the statistical properties of individual assets, and thus does not have good reputation.

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