



## Private equity benchmarks and portfolio optimization



Douglas Cumming<sup>a,\*</sup>, Lars Helge Haß<sup>b,1</sup>, Denis Schweizer<sup>c,2</sup>

<sup>a</sup>York University – Schulich School of Business, 4700 Keele Street, Toronto, Ontario, Canada M3J 1P3

<sup>b</sup>Lancaster University Management School, Lancaster University, Lancaster, LA1 4YX, United Kingdom

<sup>c</sup>WHU – Otto Beisheim School of Management, Burgplatz 2, 56179 Vallendar, Germany

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### ABSTRACT

Portfolio optimization using private equity is typically based on one of three indices: listed private equity, transaction-based private equity, or appraisal value-based private equity indices. However, we show that none of these indices is fully suitable for portfolio optimization. We introduce here a new benchmark index for venture capital and buyouts, which is updated monthly, adjusted for autocorrelation (de-smoothing), and available contemporaneously. We illustrate how our benchmark enables superior quantitative portfolio optimization.

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## 1. Introduction

Private equity (PE) has played an increasingly important role in the portfolios of institutional investors such as endowments, pension funds, insurance companies, and high net worth individuals (see, e.g., Keuschnigg and Nielsen, 2003; Kanniainen and Keuschnigg, 2004; Nahata, 2008; Groh et al., 2010a; Groh et al., 2010b; and Groh and Liechtenstein, 2011a, 2011b, 2011c). In fact, according to the Boston Consulting Group (2009), as of year-end 2009, approximately US \$1 trillion was invested in PE. However, institutional investments in PE are both long-term and illiquid, and it is thus somewhat difficult to establish optimal portfolio weights, particularly relative to more liquid asset classes.

The importance of benchmarking PE investments in theory and in practice cannot be overstated. Recent work by Groh et al. (2012) focuses on benchmarking PE at a country level. This issue is followed closely by institutional investors worldwide,<sup>3</sup> who require representative benchmarks or PE indices in order to determine the optimal proportion of PE to be allocated to their portfolios (see, e.g., Woodward and Hall, 2003; Woodward, 2004; Tierney

and Bailey, 1997; and Nesbitt and Reynolds, 1997). Moreover, suitable benchmarks are also needed to calculate risk exposures, such as value-at-risk (VaR) or conditional VaR, and risk capital requirements, such as those mandated under Basel III. Without appropriate benchmarks, institutional investors are at risk of misallocating their capital to the PE asset class as a whole, as well as among various PE funds.

Benchmarking, therefore, is fundamental to the entire PE market and all firms and stakeholders connected with it. It can be considered one of the most important aspects of PE research.

This paper addresses several interrelated issues. First, are institutional investors using the most appropriate PE benchmarks in portfolio optimization? Second, if not, what are the most appropriate benchmarks? And third, how large are the differences in portfolio construction for the appropriate versus inappropriate PE benchmarks?

Institutional investors generally use one of three concepts when constructing PE indices: (1) listed PE indices, (2) transaction-based PE indices, and (3) appraisal value-based PE indices. Each index has advantages and disadvantages for capturing PE risk/return profiles. In this paper, we show, however, that none of the indices fully captures appropriately input quantities for portfolio optimization or for risk models.

For example, listed PE indices contain up-to-date data, but are insufficient for portfolio optimization because they overestimate the volatility of the underlying investments, and hence underestimate the optimal percentage of PE to be allocated to a given

\* Corresponding author. Tel.: +1 416 736 2100x77942; fax: +1 416 736 5687.

E-mail addresses: [dcumming@schulich.yorku.ca](mailto:dcumming@schulich.yorku.ca) (D. Cumming), [l.h.hass@lancaster.ac.uk](mailto:l.h.hass@lancaster.ac.uk) (L. Helge Haß), [Denis.Schweizer@whu.edu](mailto:Denis.Schweizer@whu.edu) (D. Schweizer).

URL: <http://ssrn.com/author=75390> (D. Cumming).

<sup>1</sup> Tel.: +44 1524 593981; fax: +44 1524 847321.

<sup>2</sup> Tel.: +49 261 6509 724; fax: +49 261 6509 729.

<sup>3</sup> See, for example, <http://blog.iese.edu/vcpeindex>.

portfolio. Transaction-based indices, on the other hand, use realized cash flows of prior PE transactions, but the time lag of their data availability is suboptimal, and they may thus misallocate portfolio weights, particularly during financial crises. Meanwhile, appraisal value-based PE indices use quarterly evaluations of the book value of PE portfolio companies, along with changes in actual cash flow. But they likewise feature a time lag in data availability, and may also have appraisal-smoothing problems. A mismatch in data timing and smoothed returns can create spurious portfolio optimization results.

Thus, none of the three index concepts currently in popular use is fully capable of capturing the risk/return profile of PE, and none provides the necessary input quantities for portfolio optimization or risk models. We elaborate on this point further in the first part of this paper. We also improve upon these methods by introducing a new representative benchmark for PE that specifically considers segments of the PE asset class for venture capital and buyouts as a means to more appropriately capture the risk/return profile.

Our new benchmark index works as follows. We first calculate appraisal value-based PE benchmarks using those indices. We rescale quarterly returns to monthly returns by using *Getmansky et al.'s (2004)* method, which corrects for positive autocorrelation in returns (see also *Koijen et al., 2009*). Second, we use capital market information in a forecast model that includes listed PE and macroeconomic variables in order to close the quarterly time gap and obtain up-to-date performance. In our final step, we calculate a superior benchmark that features monthly frequency and contemporary performance reporting.

We demonstrate that our new benchmark is suitable for use in portfolio optimization and risk models. Because portfolio optimization varies in an economically significant way in relation to index choice, we find that our new index provides a quantitative improvement. Furthermore, by using a Monte Carlo simulation and historical US returns from the January 1999–December 2008 period, we show that the portfolio exhibits statistically significantly higher levels of risk when listed PE is used as a proxy than when our modified appraisal value-based benchmark is used. We also find lower stated Sharpe ratios when using listed PE than when we use our modified appraisal value-based benchmark. This choice could cause disproportionately low levels of new capital inflows compared to peers that use the appropriate PE benchmarks for performance assessment. Overall, our results confirm that our new index improves risk management for PE limited partners, thus facilitating the flow of funds into the PE industry.

The remainder of this paper is structured as follows. Section 2 describes the different index concepts. Section 3 introduces our methodology for constructing the appraisal value-based PE benchmarks, and presents the results of the forecast model. Section 4 demonstrates the impact of index selection on the resulting asset allocation. Section 5 concludes, and provides a summary of our most important findings.

## 2. Alternative private equity performance indices

This section describes the various indices used for portfolio optimization in more detail, as well as the data sources used for constructing these indices.

### 2.1. Listed private equity indices

Listed PE indices track listed indirect private investment companies (“funds of funds”), listed direct private capital investment companies, and listed PE fund managers (*Bergmann et al., 2009*). Data on listed PE are available from stock exchanges and from

the Listed Private Equity Association (see [Appendix A](#) for more details).

Listed PE indices have the liquidity advantage of reflecting current values because they feature daily trading. However, this advantage can also be a point of criticism for index construction. With daily trading comes daily price changes. Because the expectations of market participants affect pricing, especially during times of crisis, listed PE indices can be more volatile than actual PE valuations. Moreover, listed PE vehicles are not necessarily representative of the entire PE universe, because the decision to list is not random, and the type of investors attracted to listed PE differs from those attracted to, e.g., limited partnership PE vehicles (*Cumming et al., 2011*).

In contrast, actual PE valuations are typically carried out on an annual basis, although a small percentage of funds conducts semi-annual valuations (*Cumming and Walz, 2010*). Actual PE valuations are based not only on realized investments that have been exited (e.g., by IPO, acquisition, secondary sale, buyback, or liquidation), but also on unrealized valuations on unexited investments. It typically takes 3–5 years from the date of first investment to exit an investment (see *Nahata (2008)* for venture capital, *Hege et al. (2009)*, *Giot and Schwienbacher (2007)*, *Metrick and Yasuda (2010b)*, and *Schwienbacher (2008)* for venture capital and buyouts, and *Cao (2011)* and *Cao and Lerner (2009)* for buyouts).

Moreover, private entrepreneurial firms are valued on the basis of long-run performance expectations, and thus their valuations do not fluctuate with daily market swings (*Metrick and Yasuda, 2010a*). Therefore, PE fund managers do not carry out daily valuations. However, non-daily pricing in actual PE investments is not the same thing as stale pricing. Rather, actual PE valuations change only when substantial new information exists that would influence long-run expectations. In the case of entrepreneurial firms, this substantial change is typically attributable to something that would not be related to a daily swing in public stock markets but is a material event that is important to shareholders, such as the hiring of a key employee or the attainment of a patent or a strategic alliance (see *Anand and Khanna, 2000*).<sup>4</sup> The correct frequency for PE pricing therefore reflects actual practice: Typically once or twice per year, as well as during the period of the announcement of material information, fund managers report to their institutional investors any information that could result in a change in expected valuation. Therefore, listed PE indices are insufficient for portfolio optimization because the volatility from the underlying stock market fluctuations is greater than the more episodic realization of cash flows from illiquid alternative investments. Listed PE indices would tend to underestimate the optimal portfolio share for PE due to the overestimation of risk.

### 2.2. Transaction-based private equity indices

Cumulative cash flows of portfolio companies from non-listed PE funds or limited partnerships (as described in *Metrick and Yasuda, 2010a*) are used to determine transaction-based index performance. Transaction-based PE indices are available commercially from organizations such as Ceres ([Appendix A](#)). And, because they are based on realized cash flows, they avoid the problem of risk overestimation.

The calculation of monthly Ceres indices is straightforward. As soon as a portfolio company in any fund is sold (exited), or a distribution is made, the resulting performance is distributed over the

<sup>4</sup> Because all private equity funds are listed on stock exchanges, they are subject to certain disclosure requirements. For example, in the US, managers of listed private equity funds must disclose any material event that would be important to shareholders or the United States Securities and Exchange Commission in an 8-K filing, which is available to investors in the EDGAR database.

holding period of the investment.<sup>5</sup> The Ceprex method thus enables the development of the portfolio company to be mapped over time. Because portfolio company performance is primarily fixed as of the day of sale, and is distributed over the entire holding period, ex post changes will arise from the active portfolio companies in the data record.

However, one concern with transaction-based indices is the potential difficulty of tracking “living” dead investments, or investments that should or will be written off, but are still being held by fund managers who do not wish to report them as failures yet. Empirical research on the Ceprex index has considered this issue, assessing the index’s sensitivity to non-random decisions to exit investments (Cumming and Walz, 2010).

One of the advantages of the Ceprex index is that it is able to consider sensitivity to this issue.<sup>6</sup> Other indices, such as the Cambridge Associates (CA) Index, have an unknown sensitivity to assessing non-random exit decisions. Moreover, the CA index is based on US-only samples, while the Ceprex Index uses an international sample of PE transactions, and is much more internationally known than any other transaction-based PE index.

The Ceprex index also does not account for valuation-dependent changes in the net asset value of the portfolio companies. Hence, distortions resulting from valuations or managed pricing do not occur. In other words, smoothed returns are not a problem with this index. However, one limitation is the considerable time lag of two to three quarters in the calculations. This time lag prevents up-to-date portfolio optimizations, as well as contemporaneous determinations of risk exposure, which is particularly problematic during times of crisis when risk capital budgets are tight.

### 2.3. Appraisal value-based private equity indices

Appraisal value-based PE indices are calculated quarterly, based on evaluation changes in the net asset value (book value) of portfolio companies, along with cumulative cash flows. Appraisal value-based (AVB) indices reflect changes in appraisals and in selling prices. Thomson Reuters’ VentureXpert database is the leading data provider for this type of index (see Appendix A), followed by Cambridge Associates, State Street, and more recently LPX, with their NAV index family.

The calculation method of AVB PE indices, compared with transaction-based indices, features relatively up-to-date (one-quarter delay) performance determination, achieved by the quarterly rather than monthly granularity of the observation moments. However, despite these many advantages, the problem of smoothed returns may occur as a result of deformation, possibly through appraisal smoothing (the estimated value method for determining the NAVs of portfolio companies), quarterly data availability, and/or stale pricing (e.g., price distortions due to illiquid and less than daily evaluated positions). Statistically, this may lead to positive autocorrelation.

Such relationships are common for illiquid investments such as PE, individual hedge fund strategies, and real estate. They typically arise due to (1) irregular price determination, (2) overly long time

<sup>5</sup> Data vendors such as the frequently used Thomson Reuters’ VentureXpert database provide information that, e.g., an exit took place, but can only note that the value is “undisclosed”, or that it is “estimated” (a rough indicator). In the case of Cepres, the information provided is much richer, because Cepres is a spin-off from a fund of private equity funds. As a result, Cepres has access to due diligence reports, including audited filings of investment firms and precisely dated information about the cash flows for each financing tranche (see Krohmer et al., 2009). Such information is needed to calculate a transaction-based index.

<sup>6</sup> This sensitivity check requires access to the underlying data, such as that used by, e.g., Cumming and Walz (2010), which are available from the premium content at Cepres.

periods between price determination, and (3) the use of book value instead of market prices (Geltner, 1991; Gompers and Lerner, 1997). The resulting autocorrelation can cause a significant underestimation of risk, e.g., volatility.

In summary, as we noted earlier, none of the three commonly used index concepts can fully capture the risk/return profile of PE funds, and they are all likely to provide inappropriate portfolio optimization weights. In the next section, we describe our new method for calculating a benchmark index for portfolio optimization with PE.

### 3. Data

We use the Thomson Reuters VentureXpert database for the construction of our AVB PE benchmark. In Section 4, we will calculate AVB indices (US venture capital and US buyouts); we will then use these indices to compute the AVB benchmarks for venture capital and buyouts. To bridge the one-quarter gap, we use the following explanatory variables in the regression: the LPX Venture Capital Index and LPX Buyout Index to proxy for listed PE; US industrial production as an indicator of economic activity; gross domestic product as a gauge of a country’s economy health; US Treasury 1- and 10-Year Yields to proxy for the yield curve; the US consumer price index to identify periods of inflation or deflation; the Nasdaq Composite Price Index and the NYSE Composite Price Index to proxy for the condition of the exit channel for technology and (more) mature companies; and liquidity, measured by the average bid-ask spread for all Nasdaq and NYSE companies (see also Table 5 for the regression results, as well as Appendix A).

In Section 5, where we contrast the portfolio weights resulting from the optimizations with the AVB benchmark, index, or listed PE, we use the following indices as the investors’ choice sets: stock markets (NIKKEI 500, S&P 500, DJ STOXX 600), bond markets (JPM Japan Government Bond Index, JPM United States Government Bond Index, JPM Europe Government Bond Index, JPM UK Government Bond Index), alternative investments (FTSE EPRA/NAREIT for real estate, S&P GSCI Commodity Total Return for commodities, and HFRI Funds of Funds for hedge funds), and money market (London Interbank Offered Rate, see Table 7 for descriptive statistics). All time series in this study are on a monthly basis, except for the AVB indices, which are quarterly before transformation, and US gross domestic product, which is also quarterly.<sup>7</sup> The inception date is January 1995, because all indices report data from this date onward, and the end date is December 2008. Although the previously described indices are the most common for their asset classes, other indices exist for different asset classes.

### 4. Construction of adequate appraisal value-based private equity benchmarks

In light of the advantages of the AVB PE indices noted in Section 2, we believe they are the most suitable starting point from which to begin our determination of PE benchmarks. We can calculate the AVB PE subindices very straightforwardly by using the Thomson Reuters VentureXpert database for US buyouts and venture capital (henceforth referred to as VC). We focus on both segments because they constitute the largest PE market segments and have the highest capitalizations (see, e.g., Credit Suisse, 2012):

$$AVBR_t = \frac{NAV_t + Cash\ Outflow_t - Cash\ Inflow_t}{NAV_{t-1} + Cash\ Outflow_{t-1} - Cash\ Inflow_{t-1}} - 1, \text{ with} \\ = 1, \dots, T, \quad (1)$$

<sup>7</sup> To transform quarterly US gross domestic product growth into monthly observations, we distribute growth rates equally over the three months of the quarter.

where  $AVBR_t$  (appraisal value-based return) stands for the return in quarter  $t$ , and  $NAV$  represents the book value of portfolio companies, as well as the payments and dividends. In order to calculate the index properly, we must connect the quarter returns ( $AVBR_t$ ), as follows:

$$AVBI_T = 100 \cdot \prod_{t=1}^T (1 + AVBR_t). \quad (2)$$

Due to the evaluation impacts (appraisal smoothing, stale pricing, managed pricing), all returns are smoothed, and show up as positive autocorrelation. If we test the AVB PE indices for significant positive autocorrelation, we find that the first two lags are significant for both PE substrategies, as shown in Table 1, and based on data in Section 3 and Appendix A. This implies that, because of the smoothed returns, the risks may be underestimated without separate consideration of this autocorrelation structure. A correction must thus be made.

For this correction, we use Getmansky et al.'s (2004) method, and "calculate" the autocorrelation out of the returns. This is the equivalent of "desmoothing" the time series. More specifically, we apply the following three-step procedure to construct our time series. First, we evenly distribute each quarterly return over three monthly returns. Second, we determine the underlying autocorrelation structure, which follows a moving average process, via maximum likelihood estimation. Third, we use the estimated parameters to obtain the "desmoothed" time series.<sup>8</sup>

Table 2 gives the descriptive statistics for each AVB PE index, as well as the desmoothed and rescaled AVB PE subindex and the LPX indices (for listed PE). The risk profile measured by all risk indicators increases remarkably throughout the desmoothing and rescaling. Furthermore, we can now compare the differences between the risk/return profiles of listed PE and PE, because the individual subindex returns have the same frequency and their smoothing effects are corrected.

Note that the listed PE indices exhibit considerably higher risk for all values, while the comparison between the maximum drawdowns of the AVB subindices and the LPX indices exhibit dramatically different results. The LPX buyout has reached a maximum drawdown of about 80% in the past, but that of the AVB buyout index has only increased to 25%. Thus, the risk level of PE is considerably overestimated, and the optimal percentage with listed PE as a benchmark is generally underestimated.

The higher moments of the listed PE indices exhibit negative skewness, as well as positive excess kurtosis. Thus, the "extreme" return realisations in the listed PE indices are found on the loss side, and the risk measures exhibit much higher risk levels, especially maximum drawdown. We note that, among other things, this illustrates how listed PE is an insufficient proxy for the PE asset class, because the risk profiles of stock market investments do not sufficiently proxy for the risk profiles of PE investments.

After desmoothing and rescaling the AVB PE indices, we must bridge the 3-month time delay (the one-quarter gap). To accomplish this, we use a forecasting model for listed PE and macroeconomic variables that contains, e.g., market expectations about the future of PE. We include the following factors in the forecast model (see Appendix A and Section 3 for the data sources):

<sup>8</sup> Getmansky, Lo, and Makarov (2004) argue that, in general, two conditions of their desmoothing technique must be satisfied: (1) the maximum likelihood estimation must converge and (2) the smoothing parameters must have a positive sign. We performed tests that suggest both conditions hold (the tables are available from the authors upon request). Furthermore, Getmansky, Lo, and Makarov (2004) show theoretically that their desmoothing technique does not change the mean of the time series. We therefore adjust the mean of the desmoothed time series so that it matches the mean of the original series (for additional details, see Busack et al., 2011).

**Table 1**  
Autocorrelation structure of the appraisal value-based private equity indices.

	Lag 1	Lag 2	Lag 3	Lag 4
US buyout	<b>0.3355</b>	<b>0.3005</b>	0.2314	0.1508
US venture capital	<b>0.6153</b>	<b>0.4998</b>	<b>0.3905</b>	0.0450

This table shows the autocorrelation coefficient of the quarterly distribution of returns for our appraisal value-based PE indices (US buyout and venture capital) from January 1995–December 2008 for lags 1–4. Values in boldface are significant to at least a 95% level.

- Listed private equity (LPX):** Listed PE is evaluated on a daily basis, so expectations about its future prospects for the PE industry (as measured by the AVB indices) are included in the price determination. The LPX buyout and the LPX VC are considered proxies for listed PE.
- Economic activity (EA, GDP):** When industry growth is strong, we expect to see a higher number of attractive investment opportunities for entrepreneurs. We also expect to see an increase in business activity and the number of start-ups, which are associated with an increase in financing needs and PE demand. We expect economic activity to have a positive influence on PE performance (Gompers and Lerner, 1998; Jeng and Wells, 2000). The proxies for economic activity include the US industrial production index, and US GDP.
- Interest (Interest<sub>short</sub>, Interest<sub>long</sub>):** Interest rates are positively correlated with PE financing. Thus, an increase in the interest rate is associated with an increase in the demand-side effect of the attractiveness of VC versus bank financing for entrepreneurs (Gompers and Lerner, 1998; Bonini, 2012). (For highly leveraged buyouts, however, we may observe a negative relationship due to the higher cost of debt.) As proxies, we include the US Treasury 1-Year and 10-Year yields.
- Price level (CPI):** Given sticky prices (e.g., when nominal costs grow faster than output prices; see Fama, 1981, and Schwert, 1981), we expect the price level to have a negative impact on PE investments. As a proxy, we use the US consumer price index.
- Exit channel (Nasdaq, NYSE):** We expect that the performance of PE investments and the existence of a well-functioning exit channel will be highly interrelated. PE investment success implies higher rates of return, which in turn results in more IPO activity. Thus, the variables should show similar patterns. Higher stock market activity also implies higher PE returns (Black and Gilson, 1998; Rajan and Zingales, 2003). We use the Nasdaq as our proxy index for VC exit conditions, and the NYSE as the proxy index for buyout exit conditions.
- Information asymmetry (Liquidity):** We expect that during high levels of asymmetric information, PE experts may be better able to use their knowledge to generate higher returns. We use the average bid-ask spread for all Nasdaq and all NYSE shares, respectively, to proxy for the level of asymmetric information most relevant for VC and buyout investments.

Our forecast model is based on the following regression equation:

$$AVBR_\tau = \alpha + \beta_1 \cdot LPX \text{ Venture Capital}_\tau^{(VC)} + \beta_2 \cdot LPX \text{ Buyout}_\tau^{(BO)} + \beta_3 \cdot EA_\tau + \beta_4 \cdot GDP_\tau + \beta_5 \cdot Interest_{short,\tau} + \beta_6 \cdot Interest_{long,\tau} + \beta_7 \cdot CPI_\tau + \beta_8 \cdot Nasdaq_\tau^{(VC)} + \beta_9 \cdot NYSE_\tau^{(BO)} + \beta_{10} \cdot Liquidity(Nasdaq)_\tau^{(VC)} + \beta_{11} \cdot Liquidity(NYSE)_\tau^{(BO)} + \varepsilon_\tau, \text{ with } \tau = 1, \dots, T, \quad (3)$$

where  $AVBR_\tau$  represents the desmoothed and rescaled return of the AVB PE subindex in month  $\tau$ ,  $\beta_i$  represents the slope coefficient, and

**Table 2**  
Descriptive statistics of the monthly return distributions for all private equity indices.

	US buyout	US buyout (desmoothed)	LPX Buyout	US Venture Capital	US venture capital (desmoothed)	LPX Venture Capital
Mean	0.72%	0.72%	0.58%	1.14%	1.14%	0.16%
Standard deviation	1.42%	2.73%	6.09%	2.99%	5.72%	8.48%
Skewness	-0.65	-1.14	-3.70	1.64	1.51	-0.32
Kurtosis	3.31	7.68	23.05	8.86	19.88	4.28
LPM	0.30%	0.62%	1.63%	0.53%	1.12%	3.17%
CVaR ( $\alpha = 95\%$ )	-2.73%	-7.32%	-18.00%	-3.83%	-11.10%	-19.32%
Maximum drawdown	23.34%	25.69%	79.96%	52.65%	57.17%	89.07%
Jarque-Bera	1	1	0	1	1	0

A Jarque-Bera value of 1 means that the assumption of normal distribution at a 5% level can be rejected. This table shows the mean, monthly standard deviation, skewness, kurtosis, square root of lower partial moment 2 with threshold 0 (LPM), conditional value at risk (CVaR) at a 95% confidence coefficient, and maximum drawdown of the monthly return distributions for the appraisal value-based PE indices (US buyout and venture capital), as well as the de-smoothed and rescaled appraisal value-based PE indices and listed PE (LPX Buyout and LPX Venture Capital) for the January 1995–December 2008 period. All discrete returns are converted into logarithmic returns. Because the appraisal value-based indices are calculated on a quarterly basis, they are converted into monthly data via the [Getmansky et al.'s \(2004\)](#) method. The significance of the autocorrelation up to lag 4 was also desmoothed using this method. Finally, we test for the assumption of a normally distributed return distribution via the [Jarque and Bera \(1980\)](#) tests. 1 indicates a rejection at the 1% level of the null hypothesis that the return distribution follows a normal distribution.

$\varepsilon_t$  is the error term. All returns are calculated in US dollars.  $T$  shows the last month of the observation period that is 3 months prior to the current date (the one-quarter gap). The correlations of the explanatory variables are in [Table 3](#). We find statistically significant correlations between some explanatory variables, which may lead to multicollinearity in the regression. We account for this by calculating variance inflation factors.

In order to obtain meaningful results, all variables must be stationary. Therefore, we perform augmented Dickey-Fuller tests up to lag 6 to check for unit roots. [Table 4](#) gives the resulting test statistics.

For all variables except interest rates and liquidity, we can reject the hypothesis of a unit root. For interest rates and liquidity, we find that they are integrated of order 1, i.e., they become stationary after first-differencing.

[Table 5](#) gives an overview of the regression results where interest rates and liquidity have been first-differenced. The slope coefficients have the expected signs, except for inflation and NYSE liquidity. Economic activity, the short-term interest rate, and the exit channel are statistically and economically significant for US buyouts. The LPX, economic activity, and the short-term interest rate are statistically and economically significant for US venture capital.

As discussed previously, we compute variance inflation factors (VIFs) to check for multicollinearity ([Table 6](#)). All VIFs are smaller than 5, so we conclude there is no multicollinearity (see [Belsley](#)

[et al., 1980; Kutner et al., 2004](#)). As a robustness check, we calculate all of the following results with a more parsimonious model using exogenous variables that show statistical significance at least at the 5% level. The results remain qualitatively the same. Tables and figures are available from the authors upon request.

In a last step, we calculate the three missing returns (point estimator) for months  $T + 1$ ,  $T + 2$ , and  $T + 3$  via the forecasting model, as follows:

$$\begin{aligned} \widehat{AVBR}_\tau = & \hat{\alpha} + \hat{\beta}_1 \cdot LPX\ Venture\ Capital_\tau^{(VC)} + \hat{\beta}_2 \cdot LPX\ Buyout_\tau^{(BO)} \\ & + \hat{\beta}_3 \cdot EA_\tau + \hat{\beta}_4 \cdot GDP_\tau + \hat{\beta}_5 \cdot Interest_{short,\tau} \\ & + \hat{\beta}_6 \cdot Interest_{long,\tau} + \hat{\beta}_7 \cdot CPI_\tau + \hat{\beta}_8 \cdot Nasdaq_\tau^{(VC)} \\ & + \hat{\beta}_9 \cdot NYSE_\tau^{(BO)} + \hat{\beta}_{10} \cdot Liquidity(Nasdaq)_\tau^{(VC)} \\ & + \hat{\beta}_{11} \cdot Liquidity(NYSE)_\tau^{(BO)}, \end{aligned} \tag{4}$$

with  $\tau = T + 1, T + 2, T + 3$ .

It is now possible to determine the appraisal value-based benchmark (AVBB) for both PE segments:

$$AVBB_{T+3} = 100 \cdot \prod_{\tau=1}^{T+3} (1 + AVBR_\tau). \tag{5}$$

The AVBBs are applicable to the risk/return profiles of the various PE segments, because the returns are monthly, desmoothed, and contemporaneous. They can thus be used as input quantities for portfolio optimization and risk allocation models.

**Table 3**  
Correlation matrix for the explanatory variables.

	LPX Buyout	LPX Venture Capital	EA	GDP	Interest (short)	Interest (long)	Inflation	Nasdaq	NYSE	Nasdaq Liquidity	NYSE Liquidity
LPX Buyout	<b>1.00</b>										
LPX Venture Capital	<b>0.60</b>	<b>1.00</b>									
EA	0.16	0.09	<b>1.00</b>								
GDP	<b>0.46</b>	<b>0.35</b>	<b>0.48</b>	<b>1.00</b>							
Interest (short)	<b>0.24</b>	<b>0.22</b>	<b>0.24</b>	<b>0.42</b>	<b>1.00</b>						
Interest (long)	0.08	0.15	<b>0.21</b>	<b>0.30</b>	<b>0.67</b>	<b>1.00</b>					
Inflation	<b>0.32</b>	0.08	0.10	<b>0.33</b>	0.14	0.14	<b>1.00</b>				
Nasdaq	<b>0.52</b>	<b>0.72</b>	0.04	<b>0.33</b>	0.11	0.07	0.05	<b>1.00</b>			
NYSE	<b>0.70</b>	<b>0.59</b>	0.04	<b>0.36</b>	0.09	-0.05	0.15	0.70	<b>1.00</b>		
Nasdaq Liquidity	<b>-0.48</b>	<b>-0.47</b>	-0.14	<b>-0.22</b>	<b>-0.26</b>	<b>-0.25</b>	-0.11	<b>-0.44</b>	<b>-0.44</b>	<b>1.00</b>	
NYSE Liquidity	<b>-0.20</b>	-0.18	0.03	-0.07	-0.03	-0.09	-0.12	-0.18	-0.13	<b>0.46</b>	<b>1.00</b>

This table shows the correlations between the explanatory variables of regression model (3) for the January 1995–December 2008 period. Variable definitions and data sources are in [Appendix A, Table A-I](#). Values in boldface are significantly different from zero at the 1% level.

**Table 4**  
Augmented dickey–fuller test statistics for the explanatory variables.

	ADF(1)	ADF(2)	ADF(3)	ADF(4)	ADF(5)	ADF(6)
LPX Buyout	<b>-6.95</b>	<b>-4.90</b>	-2.93	-1.97	-1.86	-1.34
LPX Venture Capital	<b>-7.30</b>	<b>-5.85</b>	<b>-5.47</b>	<b>-4.40</b>	<b>-4.14</b>	<b>-3.83</b>
EA	<b>-5.70</b>	<b>-3.24</b>	-2.06	-1.17	-0.61	-0.88
GDP	<b>-3.03</b>	<b>-3.41</b>	-0.82	-0.86	-0.76	-0.46
Interest (short)	-0.78	-0.82	-0.92	-0.69	-0.75	-1.07
Interest (long)	-1.36	-1.20	-1.00	-0.76	-0.71	-0.80
Inflation	<b>-7.81</b>	<b>-5.42</b>	<b>-5.29</b>	<b>-5.73</b>	<b>-4.35</b>	<b>-4.43</b>
Nasdaq	<b>-8.35</b>	<b>-7.03</b>	<b>-5.93</b>	<b>-5.37</b>	<b>-4.14</b>	<b>-3.98</b>
NYSE	<b>-8.74</b>	<b>-7.04</b>	<b>-5.17</b>	<b>-4.11</b>	<b>-3.79</b>	<b>-3.10</b>
Liquidity (Nasdaq)	-2.15	-2.08	-2.02	-1.98	-1.96	-1.90
Liquidity (NYSE)	-1.57	-1.45	-1.26	-1.20	-1.23	-1.20

This table shows the augmented Dickey–Fuller test statistics for the explanatory variables of regression model (3) for the January 1995–December 2008 period. Variable definitions and data sources are in Appendix A, Table A-1 and Section 3. For values in boldface, the hypothesis of a unit root can be rejected at the 5% level.

**Table 5**  
Regression results for the forecast model.

	US buyout beta	US venture capital beta
Constant	0.0045**	0.006*
LPX Buyout	0.0204	
LPX Venture Capital		0.113**
EA	0.4623**	0.733*
GDP	0.6511	2.070
Interest (short)	1.2993*	2.929*
Interest (long)	0.0017	-1.026
Inflation	0.2797	0.180
Nasdaq		0.055
NYSE	0.1076**	
NYSE liquidity	-0.3196	
Nasdaq liquidity		1.204
Nobs	167	167
F-test	15.00**	11.68**
Adj. R <sup>2</sup>	40.28%	33.99%

This table shows the slope coefficients, the number of observations (nobs), the F-statistic, and the adjusted R<sup>2</sup> for the forecast model for the appraisal value-based PE subindex for US buyout and venture capital. The forecast model is based on the following regression equation:

$$AVBR_{\tau} = \alpha + \beta_1 \cdot LPX \text{ Venture Capital}_{\tau}^{(VC)} + \beta_2 \cdot LPX \text{ Buyout}_{\tau}^{(BO)} + \beta_3 \cdot EA_{\tau} + \beta_4 \cdot GDP_{\tau} + \beta_5 \cdot Interest_{short,\tau} + \beta_6 \cdot Interest_{long,\tau} + \beta_7 \cdot CPI_{\tau} + \beta_8 \cdot Nasdaq_{\tau}^{(VC)} + \beta_9 \cdot NYSE_{\tau}^{(BO)} + \beta_{10} \cdot Liquidity(Nasdaq)_{\tau}^{(VC)} + \beta_{11} \cdot Liquidity(NYSE)_{\tau}^{(BO)} + \varepsilon_{\tau}, \text{ with } \tau = 1, \dots, T$$

where  $AVBR_{\tau}$  represents the desmoothed and rescaled return of the appraisal value-based PE subindex in month  $\tau$ ,  $\beta_i$  represents the slope coefficient, and  $\varepsilon_{\tau}$  is the error term. All returns are calculated in US dollars.  $T$  shows the last month of the observation period that is three months prior to the current date for the January 1995–December 2008 estimation period. Variable definitions and data sources are in Section 3 and Appendix A, Table A-1.

\* Statistical significance at the 5% level.

\*\* Statistical significance at the 1% level.

However, the forecasts of the missing 3 months are not predictable with absolute certainty. We show results only within the 2.5% (down) and 97.5% (up) confidence bands, meaning that the returns will lie within these bands for 95% of the expected future

**Table 6**  
Variance inflation factors for the explanatory variables.

	LPX Buyout	LPX Venture Capital	EA	GDP	Interest (short)	Interest (long)	Inflation	Nasdaq	NYSE	Nasdaq liquidity	NYSE liquidity
US buyout	2.35		1.31	1.12	1.99	1.91	1.12		2.11		1.07
US venture capital		2.28	1.31	1.13	1.91	1.87	1.10	2.24		1.40	

This table shows the variance inflation factors for the explanatory variables of regression model (3) for the January 1995–December 2008 period. Variable definitions and data sources are in Appendix A, Table A-1.

realisations. These confidence bands can also be used as input quantities in the portfolio optimization or the risk models in order to simulate the worst and best case scenarios (Fig. 1).

Fig. 1 illustrates the ex post realized development of the AVB PE indices in order to provide a first impression of forecasting quality. Note that the realized values are above the up-confidence band, which we would not a priori expect. For that reason, we test the prediction precision of the forecasting model using a 4-year estimation period for the slope coefficients, and we compare the forecasted and ex post realized returns. We distinguish between whether the regression parameter is estimated on a monthly (1-month forecast) or quarterly (3-month forecast) basis. Fig. 2 shows that we can forecast the realized returns for the AVBBs accurately, especially during times of lower volatility.

To summarize, the precision (the ratio of accurate forecasts within the 95% confidence interval, divided by all forecasts) is 93% for US buyouts and 92% for US VC. To evaluate the performance of our forecast model, we next calculate the mean squared error of our forecast. We obtain a value of 0.0001 for US buyouts and 0.0009 for US VC. When we use simple historic values to forecast, we obtain mean squared errors of 0.0002 for US buyouts and 0.0012 for US VC. The differences between the mean squared errors for US buyouts and US VC are statistically significant at the 1% level, according to the Diebold and Mariano (1995) test. Therefore, our forecast model can predict future values with significantly lower mean squared errors than we would obtain by naively using historical values. This implies that our forecast model is indeed superior to the naïve use of historical values.

## 5. Portfolio optimization with appraisal value-based benchmarks

In this section, we use the AVBBs calculated above for US VC and buyouts for multi-asset portfolio optimization. We then compare our results to those obtained from using the PE asset class that we proxy for with listed PE (LPX indices) or AVB indices. Because of the non-normal return distributions (Table 2), we must consider higher moments in the portfolio optimization. Failure to do so would increase the probability of maintaining biased and suboptimal weight estimations, as well as underestimating tail losses.

We use three different risk measures (RM): (1) variance (Markowitz, 1952), (2) lower partial moments (Harlow, 1991), and (3) conditional value-at-risk (Rockafellar and Uryasev, 2000, 2002). The extra two measures are used to capture potential tail risks. Higher moments are implicitly considered in the optimization, which is important when return distributions are characterized by negative skewness and positive excess kurtosis.

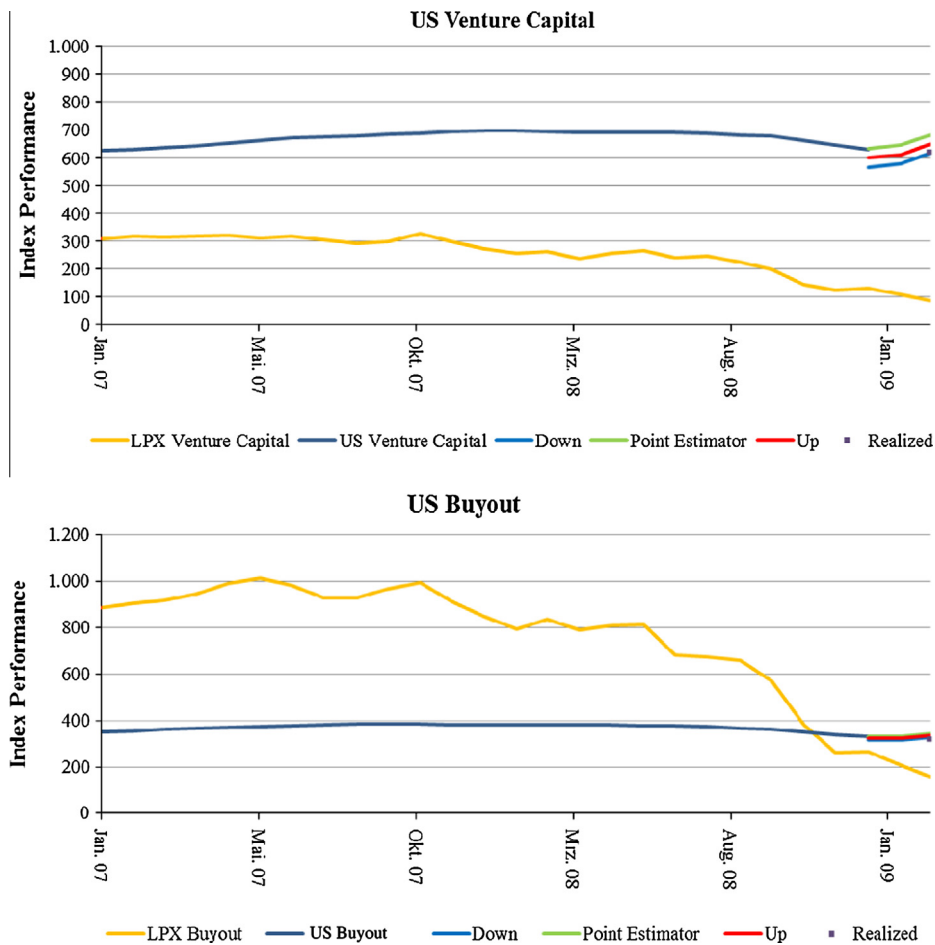
In addition to PE, other investment opportunities in the portfolio optimization include equity and bond markets, alternative investments, and money market (see Section 3 for a complete description of considered asset classes). Table 7 provides a detailed summary of the descriptive statistics for the proxy indices.

We show the results for the efficient multi-asset portfolios for our alternative risk measures (RM), and considering (1) AVBBs (US VC/buyouts), (2) AVB index (US VC/buyouts), and (3) listed

**Table 7**  
Descriptive statistics for the monthly return distributions of all asset classes.

	NIKKEI 500	S&P 500	DJ EURO STOXX	JPM EUROPE	JPM US	JPM UK	JPM Japan	NAREIT	S&P GSCI	HFRI	LIBOR
Mean	0.01%	-0.12%	0.06%	0.51%	0.52%	0.32%	0.37%	0.55%	0.59%	0.47%	0.41%
Standard deviation	6.19%	4.65%	5.54%	3.05%	1.46%	2.79%	2.88%	7.44%	7.55%	1.80%	0.07%
Skewness	0.12	-0.90	-1.09	0.21	-0.05	-0.42	-0.07	-1.85	-1.05	-0.51	-0.06
Kurtosis	2.87	5.04	5.56	3.65	5.04	4.82	2.98	18.55	6.06	6.59	1.90
LPM	2.44%	1.80%	2.02%	0.92%	0.34%	0.90%	0.95%	1.98%	2.64%	0.45%	0.00%
CVaR ( $\alpha = 95\%$ )	-11.39%	-11.43%	-14.58%	-5.43%	-2.67%	-6.01%	-5.59%	-18.55%	-17.35%	-3.86%	0.29%
Maximum drawdown	70.02%	48.29%	57.24%	23.62%	4.95%	22.14%	21.50%	73.83%	67.03%	21.03%	0.00%
Jarque-Bera	0	1	1	1	1	1	0	1	1	1	0

A Jarque-Bera value of 1 means that the assumption of a normal distribution at a 1% level can be rejected. This table shows the mean, monthly standard deviation, skewness, kurtosis, square root of lower partial moment 2 with threshold 0 (LPM), conditional value at risk (CVaR) at a 95% confidence coefficient, and the maximum drawdown of the monthly return distributions for the equity markets (NIKKEI 500, S&P 500, DJ STOXX 500), bond markets (JPM Japan Government Bond Index, JPM United States Government Bond Index, JPM Europe Government Bond Index, JPM UK Government Bond Index), alternative investments (FTSE EPRA/NAREIT, S&P GSCI Commodity Index, HFRI Fund of Funds Composite Index), and the money market (London Interbank Offered Rate - LIBOR) for the January 1999–December 2008 period. All indices are total return indices or retained earnings. All non-US dollar-based indices have been converted into US dollars, and all discrete returns are converted into logarithmic returns. Finally, the assumption of a normally distributed return distribution is proven via *Jarque and Bera (1980)* tests.

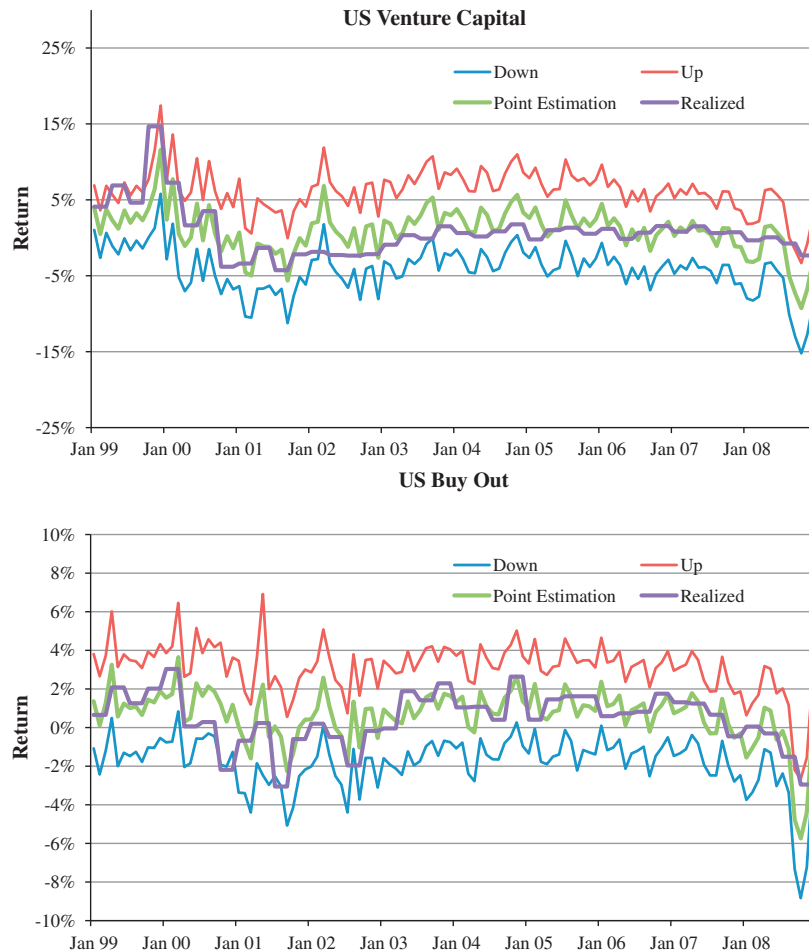


**Fig. 1.** US buyout and venture capital appraisal value-based private equity benchmarks. This figure shows two appraisal value-based PE benchmarks for the January 2007–December 2008 period. To calculate the benchmarks, we (1) scaled the quarterly data of the appraisal value-based PE Indices into monthly data and (2) corrected for stale pricing and appraisal smoothing, both by using *Getmansky et al.’s (2004)* method, in order to counteract potential distortions. As a last step, we (3) used the January 2009–March 2009 (one-quarter gap) period for the forecasting model. Here, the point estimator describes the 3-month forecast from the regression analysis, e.g., 2.5% down the confidence band, and 97.5% up the confidence band. The “realized” dot denotes the ex post realized index return. For all indices, January 1995 = 100.

PE index (LPX VC/buyouts). For the AVB index (US VC), we use the original Thomson Reuter VentureXpert index; for the AVBB (US VC/buyouts), we use our benchmark, including desmoothing and forecasts. In the portfolio optimization, we calculate the multi-asset portfolio with the minimal possible risk (for all risk measures separately) for a given expected portfolio return  $\mathbb{E}[r_p]$ . We can thus write the optimization problem as follows:

$$\begin{aligned}
 & \min_x RM(\bar{r}), \\
 & \text{subject to } \mathbb{E}[r_p] = r, x_1 + \dots + x_n = 1 \text{ and } x_i \leq 20\% \\
 & \quad \forall i = 1, \dots, n.
 \end{aligned} \tag{6}$$

where  $x_i$  represents the fraction of asset class  $i$  in the portfolio. In this optimization, we impose some standard assumptions, such as



**Fig. 2.** Precision of the forecasting model for US buyout and venture capital appraisal value-based private equity benchmarks. This figure shows the monthly return realizations for both appraisal value-based PE benchmarks (US buyout and US venture capital) for the January 1999–December 2008 period. The estimation period for the forecasting model is 5 years. Here, the point estimator describes the 1-month forecast from the regression analysis, e.g., 2.5% down the confidence band, 97.5% up the confidence band, and where “realized” equals the ex post return.

a budget constraint and short-selling restrictions. Mathematically, this means that the portfolio weights must sum to 1 and cannot be negative. Furthermore, we impose a 20% minimum diversification constraint for all proxy indices. The aim of this restriction is to avoid having the portfolio dominated by a single asset class. Moreover, when imposing the minimum diversification restriction, the results are not as prone to corner solutions as they would otherwise be, because optimal portfolio weights do not rely in a comparable way on the past performance of respective assets. When we relax this restriction, however, the results for our PE proxies do not qualitatively change.

The findings resulting from this optimization are threefold. First, the efficient frontier using the AVBB is above that obtained using LPX, but below that obtained using the AVB Index. This finding holds for both US buyouts and VC (see Fig. 3, panels A and B).

Second, listed PE (VC and buyouts) is not included in the optimal portfolios for the chosen time period, while the AVBB and the AVB index are included up to the maximum weights of 20%. All results hold for US buyouts and VC (see Fig. 3).

Third, for a given expected return, the portfolio weight of the AVB index is always equal to or above its respective AVBB counterpart (one exception is the CVaR optimization for VC, where the portfolio weight for AVBB for expected returns ranging from 5.6% to 5.9% p.a. is slightly higher than that for the AVB Index).

Against this backdrop, we can consider the role of listed PE in optimal portfolios as secondary. In contrast, we find that the

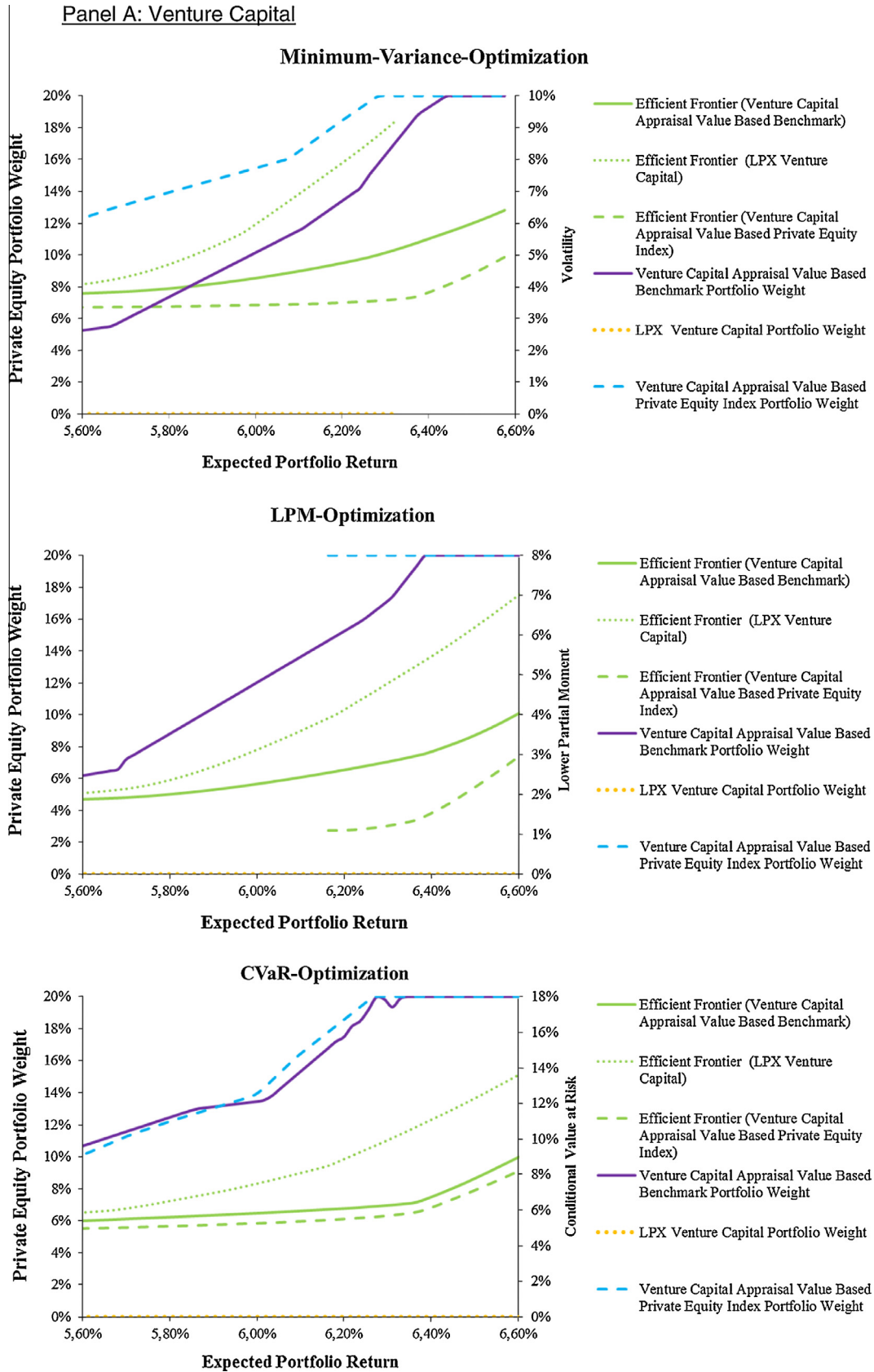
optimal percentage of US VC, using the AVBB, increases steadily, from about 5% to 10% (for low expected portfolio returns), to the maximum 20% allocation (high expected portfolio returns) for all risk measures used in the optimizations.

We observe similar behavior for the AVB index, but beginning instead from about 11%, and reaching the maximum weight constraint even for lower expected portfolio returns (except for LPM optimization, where the AVB index is also always allocated with the maximum portfolio weight). Furthermore, the AVBB increases slightly in importance when we focus on downside risk, especially for the lower partial moment. In this case, the percentage of US VC in the optimal portfolios with low levels of expected portfolio returns is higher than for the minimum-variance optimization (see Fig. 3, panel A).

A similar picture is also shown in panel B of Fig. 3 when analyzing US buyouts. The major differences are (1) the maximum 20% portfolio weight is allocated to the AVB index for low levels of expected portfolio returns and all considered risk measures, (2) the optimal percentage of US buyouts, using the AVBB, increases steadily for low expected portfolio returns, from a higher level of 15% to 18% compared to VC to a maximum allocation of 20% (high expected portfolio returns) for all risk measures used in the optimizations, and (3) the portfolio weight of the AVB index is always above the corresponding AVBB weight (see Fig. 3, panel B).

These results can be explained by examining the higher moments in Table 2. The AVBB not only exhibit higher past returns,





**Fig. 3.** Efficient Multi-Asset Portfolios and Optimal Private Equity Portfolio Weights. This figure shows the resulting efficient frontiers and the implied optimal PE portfolio weights alongside the efficient frontier for three optimization approaches (1) volatility, (2) lower partial moment (the square root of lower partial moment 2 with threshold 0), and (3) conditional value at risk (90% confidence level) in absolute terms. The efficient frontier begins with the minimum-risk portfolio and ends with the maximum expected return portfolio. The optimization covers the January 1999–December 2008 period. Panel A gives the results for venture capital; panel B gives the results for buyouts.

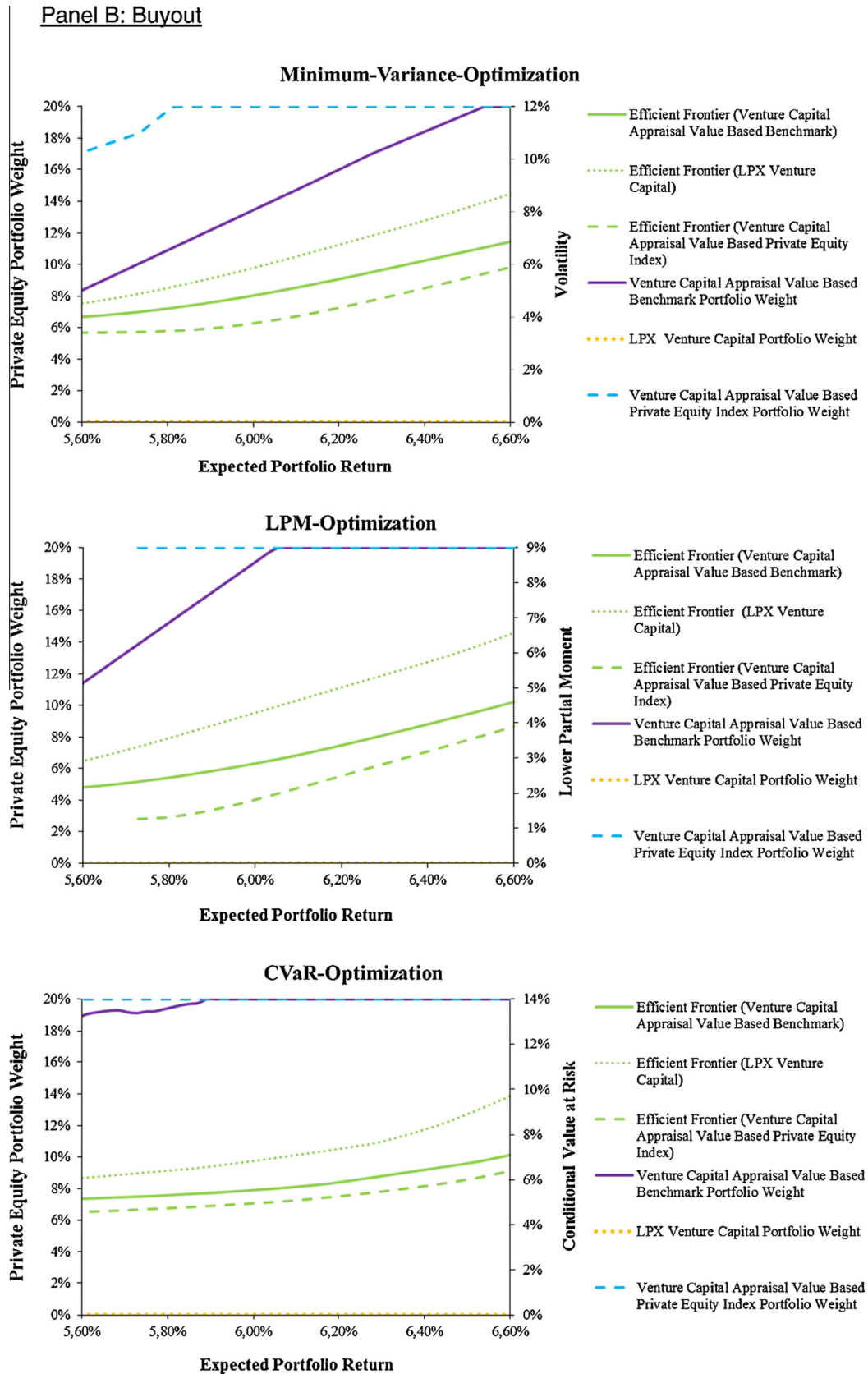


Fig. 3. (continued)

but also significantly more beneficial (higher) moment characteristics than listed PE indices. Relative to the other index concepts, the AVB index is included in all efficient portfolios with the highest

portfolio weights and, for the majority of portfolios, with the maximum possible allocation of 20%, regardless of the risk measure. This implies that, if we ignore the risks of the AVB index and use

**Table 8**  
Differences in the risk/return profile for different choices of the private equity proxies.

Portfolio weights for private equity proxies	Mean (%)	Standard deviation (%)	LPM (%)	CVaR (%)	VaR (%)	MaxDD (%)	Sharpe ratio
<i>Panel A: Venture Capital</i>							
20% LPX Venture Capital	1.99	10.74	0.04	-4.58	-3.16	9.17	-0.11
20% AVBB Venture Capital	4.82**	7.90**	0.02**	-3.07**	-1.93**	5.73**	0.21**
15% LPX Venture Capital	2.41	9.71	0.03	-4.13	-2.78	8.13	-0.07
15% AVBB Venture Capital	4.54**	7.53**	0.02**	-2.99**	-1.89**	5.58**	0.19**
10% LPX Venture Capital	2.84	8.74	0.03	-3.70	-2.43	7.16	-0.03
10% AVBB Venture Capital	4.26**	7.26**	0.02**	-2.93**	-1.85**	5.47**	0.15**
<i>Panel B: Buyout</i>							
20% LPX Buyout	3.54	9.11	0.03	-3.98	-2.17	7.38	0.05
20% AVBB Buyout	4.14**	6.51**	0.02**	-2.67**	-1.64**	4.90**	0.16**
15% LPX Buyout	3.58	8.56	0.03	-3.69	-2.05	6.85	0.05
15% AVBB Buyout	4.03**	6.62**	0.02**	-2.71**	-1.66**	5.00**	0.14**
10% LPX Buyout	3.62	8.04	0.03	-3.42	-1.94	6.36	0.06
10% AVBB Buyout	3.92**	6.76**	0.02**	-2.76**	-1.69**	5.13**	0.12**

The mean, standard deviation, square root of lower partial moment 2 with threshold 0 (LPM), CVaR, VaR, and MaxDD for the benchmark portfolio with AVBB Venture Capital as a PE proxy differ compared to the benchmark portfolio with LPX Venture Capital as a PE proxy and a higher Sharpe ratio.

The mean, standard deviation, square root of lower partial moment 2 with threshold 0 (LPM), CVaR, VaR, and MaxDD for the benchmark portfolio with AVBB Buyout as a PE proxy differ compared to the benchmark portfolio with LPX Buyout as a PE proxy and a higher Sharpe ratio.

This table gives the mean, standard deviation, square root of lower partial moment 2 with threshold 0 (LPM), conditional value-at-risk (CVaR) with a 90% confidence level, value at risk (VaR) with a 90% confidence level, maximum drawdown (MaxDD), and the Sharpe ratio (the risk-free rate equals 3.13% (EURIBOR)) for a naively diversified benchmark portfolio (e.g., the portfolio composition is  $x\%$  PE and  $(1-x)/11\%$  of all other asset classes in Table 7), when we include PE proxies (LPX Venture Capital, LPX Buyout, AVBB venture capital, or AVBB buyout) with portfolio weights of 20%, 15%, and 10%, and reduce the weights of the former benchmark portfolio accordingly. Calculations are based on Efron and Tibshirani's (1994) standard block-bootstrap Monte Carlo simulation for the January 1999–December 2008 period, with five lags and 5,000 runs. For the tests in differences for the mean and risk measures, we use t-values; for the Sharpe ratio, we use the tests proposed by Jobson and Korkie (1981) and Ledoit and Wolf (2008).

\*\* Statistical significance at the 1% level.

it naively in an asset allocation, it is likely to significantly overestimate the optimal portfolio weight.

In an untabulated robustness check, we consider three different time periods, (1) January 1999–September 2007, (2) January 1999–December 2008, and (3) January 1999–December 2010, in order to investigate whether the relationships continue to hold. We find that the relationships among the three indices are comparable for all three periods. The major difference occurs during period 3 for buyouts, where the AVB index and AVBB always show the maximum possible allocation of 20% regardless of the risk measure, and the corresponding listed PE index has portfolio allocations larger than zero. Furthermore, we find allocations ranging from zero (low expected portfolio return) to the 20% maximum (high expected portfolio return) for the LPX buyout index during the pre-crisis period (period 1).

To summarize, the approximation of the PE asset class, via listed PE, results in a biased risk/return profile, where other influencing factors are also reflected. The risk of PE in general is overestimated up to a risk level for which the PE asset class cannot be justified, as shown by quantitative calculations in a multi-asset portfolio. In contrast, with the AVB index, the optimal PE portfolio weight equals the upper 20% restriction in the majority of portfolios, because the true PE risk/return profile is incorrectly reflected. For these reasons, we can consider the AVBB as a compromise between listed PE and AVB indices, with the main advantage that it can properly measure the underlying risk/return profile of the PE asset class.

Given our analysis of the characteristics of the different index concepts in the portfolio optimization, we can now explore how using listed PE indices or AVBBs as proxies for PE can impact portfolio risk. We add PE (based on listed PE) and AVBB to a naively diversified portfolio, and evaluate portfolio risk using a Monte Carlo simulation. We use optimized (minimum-risk portfolios and randomly selected efficient portfolios) instead of naively diversified portfolios, and find that our results remain qualitatively stable (tables and figures are available from the authors upon request). We use historical returns from January 1999 through December 2008 to generate 5,000 time series of returns following the

block-bootstrap Monte Carlo approach of Efron and Tibshirani (1994).

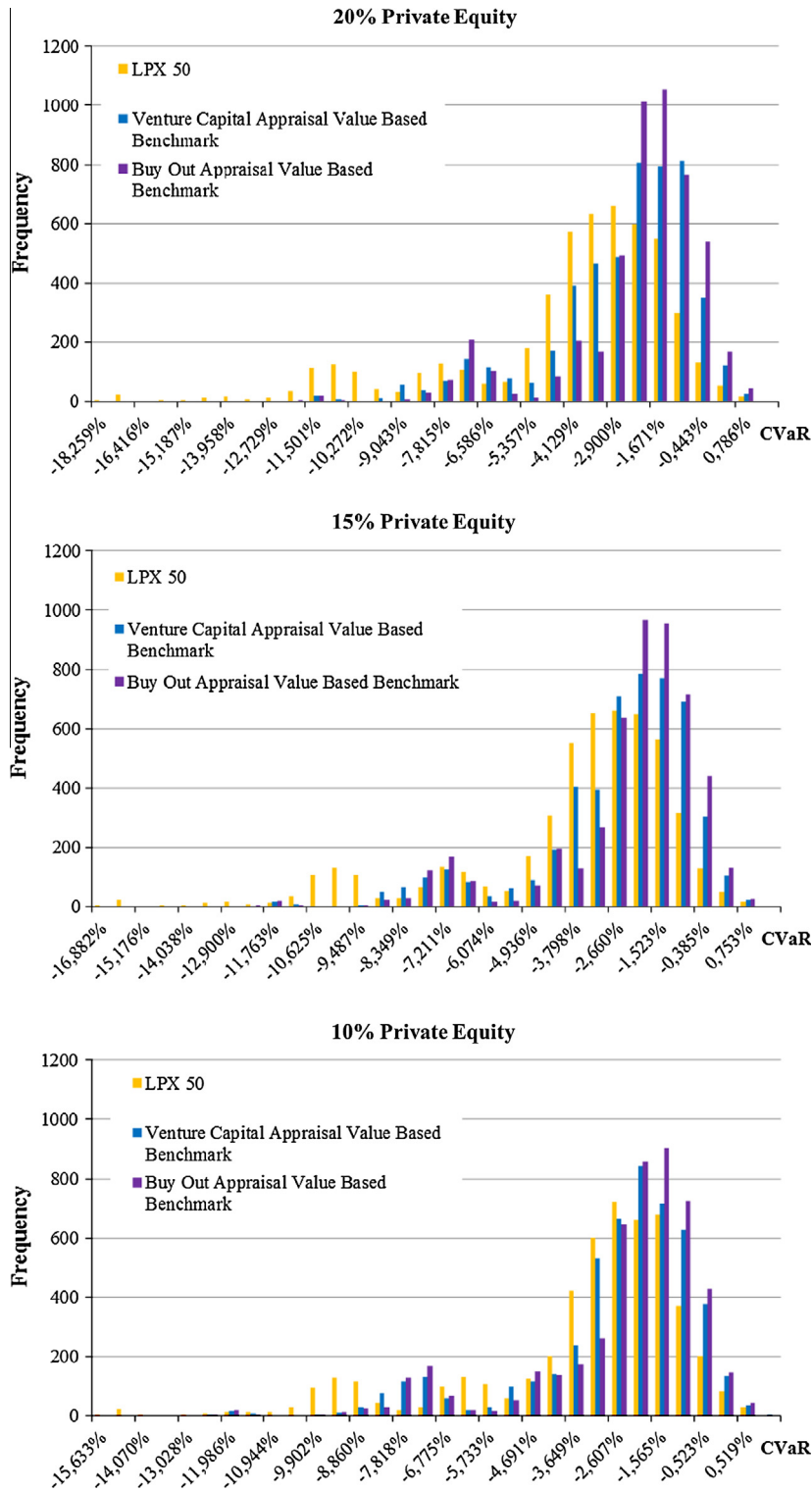
Table 8 clearly shows that all risk measures indicate a statistically significantly higher risk level in the portfolio when using listed PE as a proxy compared to using the AVBB. Furthermore, the resulting Sharpe ratio for the portfolio is significantly lower when using listed PE as a proxy for PE. This means that, for example, VaR levels may be unnecessarily high, which could induce reductions in risk positions, such as those triggered by the requirements of Basel II and III for some institutional investors. This effect would not be observed when using an appropriate PE benchmark, however. Fig. 4 gives a graphic depiction of the distribution of the risk measure for the example of CVaR.

It is important to note that the choice of index can cause disproportionately low levels of new capital inflows compared to using appropriate PE benchmarks for performance assessment. In other markets, such as that for mutual funds, research (e.g., Choi and Kahan, 2007) has shown a similar result, that the presence of misinformed investors is likely to result in inappropriate capital allocations. In the case of PE, however, the problem may be much more pronounced if the misallocation of capital is attributable to the lack of a benchmark index, and therefore affects the entire industry.

## 6. Extensions

This paper introduces a new PE benchmark concept for portfolio optimization. We used popular indices to represent other asset classes in order to demonstrate how our new index improves portfolio optimization for PE allocation. A natural extension of this work would be to consider not only indices but the different sub-index types for various fund styles and specific other alternative investments.

The use of indices has several advantages over individual assets (single PE or hedge funds, or hedge fund styles/different commodities). First, it is not necessary to account for differences in liquidity. Furthermore, trading costs at the index level are comparable. Portfolio allocation models at an individual asset level are not comparable across different types of alternative investments, and



**Fig. 4.** Downside risk distribution for various private equity proxies. This figure shows the distribution of the risk measure conditional value-at-risk (CVaR), based on annual portfolio returns, with a 90% confidence level for a naively diversified benchmark portfolio (portfolio composition is  $x\%$  PE, and  $(1 - x)/11\%$  of all other asset classes in Table 5), when we include PE proxies (LPX 50, AVBB venture capital, or AVBB buyout) with portfolio weights of 20%, 15%, and 10%. The distribution is based on Efron and Tibshirani's (1994) standard block-bootstrap Monte Carlo simulation for the January 1999–December 2008 period, with five lags and 5,000 runs.

would thus need to account for liquidity and trading costs. Second, the indices are calculated net of fees and taxes. Portfolio allocation based on specific underlying assets, by contrast, requires accounting for differential fees, transaction costs, and tax structures specific to the particular asset (Best and Hlouskova, 2005; DeMiguel et al., 2009).

Despite the additional complexity when using individual assets in our approach, there are some promising advantages. For example, there are many styles of hedge funds (and, to a lesser extent, types of PE funds), with differing strategies and risk/return profiles. There are also many different commodities featuring different risk/return profiles. Using aggregated indices for these asset classes

**Table A-I**  
Variable description.

Private equity indices			
Type	Data provider	Data source	Time period
Listed Private Equity	LPX Group	<a href="http://www.lpx-group.com">http://www.lpx-group.com</a>	January 1995–December 2008
Transaction-Based Private Equity	Ceprex Indices	<a href="http://www.cepres.com">http://www.cepres.com</a>	January 1995–December 2008
Appraisal Value-Based Private Equity	Thomson Reuters VentureXpert	<a href="http://www.thomsonreuters.com">http://www.thomsonreuters.com</a>	January 1995–December 2008
Explanatory variables			
Variable name	Description	Data provider	Time period
LPX Venture Capital	LPX Venture Capital Index	LPX Group	January 1995– December 2008
LPX Buyout	LPX Buyout Index	LPX Group	January 1995– December 2008
EA	US Industrial Production	Thomson Reuters Datastream	January 1995– December 2008
GDP	Gross Domestic Product	Thomson Reuters Datastream	January 1995– December 2008
Interest (short)	US Treasury Bill Rate	Thomson Reuters Datastream	January 1995– December 2008
Interest (long)	Government Bond Yield 10Y	Thomson Reuters Datastream	January 1995– December 2008
CPI	US Consumer Price Index	Thomson Reuters Datastream	January 1999– December 2008
Nasdaq	Nasdaq Composite Price Index	Thomson Reuters Datastream	January 1995– December 2008
NYSE	NYSE Composite Price Index	Thomson Reuters Datastream	January 1995– December 2008
Liquidity (NYSE)	Average bid-ask spread for all NYSE companies	CRSP (Center for Research in Security Prices)	January 1995– December 2008
Liquidity (Nasdaq)	Average bid-ask spread for all Nasdaq companies	CRSP (Center for Research in Security Prices)	January 1995– December 2008

This table provides the type, data provider, data source, and time period for the PE indices, as well as the variable name, description, data provider, and time period for the explanatory variables in the regression in Eq. (3).

means losing the ability to combine individual assets to achieve the best investor-specific risk/return profile. This is especially problematic when we consider that individual assets in alternative investments exhibit higher moments, which can be used in our suggested approach to achieve superior portfolio diversification. This is also true for derivative securities, which may be worthwhile considering in future extensions of our approach.

Another promising extension would be to differentiate between listed and non-listed private equity deals in order to analyze any differences in results. We know from Klein and Zur (2009) and Mietzner et al. (2011) that listed and non-listed private equity funds acquire minority and majority stakes in listed and private companies, and that sometimes PE companies themselves may change status. Consider, for example, the case of Blackstone (a private company), which acquired a 4.5% stake in Deutsche Telecom (a public company) at the end of April 2006. On March 22, 2007, Blackstone filed with the SEC to raise \$4 billion in an initial public offering and became public. Considering this example, it would be interesting and useful to explore how and whether a change in status of a portfolio company impacts our results or the risk/return profile.<sup>9</sup>

Another promising avenue for extension is the introduction of dynamics in our approach. This seems like a potentially useful way to incorporate higher moments into dynamic asset allocation models, in addition to using a dynamic objective function.

## 7. Conclusion

Portfolio optimization with PE in practice has been based on one of three indices: listed PE, transaction-based PE, or appraisal value-based PE. This paper explains why these indices are insufficient for portfolio optimization. We also illustrate how we can cal-

culate adequate benchmarks for different PE segments by using appraisal value-based PE Indices. Our benchmarks, in comparison to the three indices commonly used now, have the advantages of being (1) available on a monthly basis, (2) desmoothed for autocorrelation, and (3) up-to-date. To close the one-quarter gap, we used a forecasting model (e.g., a point estimator), flanked by an up and down confidence band, in order to estimate the best and worst case developments meaningfully and conservatively. Our benchmarks meet all the demands necessary to serve as adequate input quantities in portfolio optimization or for risk models.

We further show that the choice of PE proxy has a major impact on portfolio performance and risk/return profile. The index we develop here would yield more accurate financial reporting and portfolio optimization than those currently in use. This accuracy would in turn facilitate the development of PE markets and appropriate institutional risk management for PE limited partners. The empirical methods we develop can be applied in future work to PE, as well as to other illiquid alternative investment markets, such as art, real estate, and timber, among others.

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## Appendix A.

See Table A-I.

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## References

- Anand, Bharat.N., Khanna, Tarun., 2000. Do firms learn to create value? The case of alliances. *Strategic Management Journal* 21, 295–315.
- Belsley, David.A., Kuh, Edwin., Welsch, Roy.E., 1980. *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. John Wiley & Sons, New York.
- Bergmann, Bastian, Hans, Christophers, Matthias, Huss, Zimmermann, Heinz, 2009. Listed private equity. In: Cumming, Douglas (Ed.), *Private Equity: Fund Types, Risk, Return and Regulation*. Wiley Press, New York, pp. 53–70 (Chapter 4).
- Best, Michael J., Hlouskova, Jaroslava, 2005. An algorithm for portfolio optimization with transaction costs. *Management Science* 51, 1676–1688.
- Black, Bernard S., Gilson, Ronald J., 1998. Venture capital and the structure of capital markets: banks versus stock markets. *Journal of Financial Economics* 47, 243–277.
- Bonini, Stefano, 2012. The development of venture capital: macroeconomic, political and legal determinants. In: Douglas, Cumming (Ed.), *Oxford Handbook of Venture Capital*. Oxford University Press, New York, pp. 824–863 (Chapter 28).
- Boston Consulting Group, 2009. *Boston Consulting Group Global Asset Management Report*. BCG, Boston, Massachusetts.
- Busack, Michael, Lars H. Hass, Denis Schweizer, 2011. *Konstruktion zeitnaher Private Equity Benchmarks zur Portfolio-Optimierung und Risikomodellierung*. Unpublished Working Paper.
- Cao, Jerry Xiaping, 2011. IPO timing, buyout sponsor's exit and the performance of RLBO companies. *Journal of Financial and Quantitative Analysis* 46, 1001–1024.
- Cao, Jerry Xiaping, Lerner, Josh, 2009. The performance of reverse leveraged buyouts. *Journal of Financial Economics* 91, 139–157.
- Choi, Stephen J., Kahan, Marcel, 2007. The market penalty for mutual fund scandals. *Boston University Law Review* 87, 1021–1057.
- Credit Suisse, 2012. *Credit Suisse Alternative Investments Research Monthly Report*, December 2012.
- Cumming, Douglas, Fleming, Grant, Johan, Sofia, 2011. Institutional investment in listed private equity. *European Financial Management* 17, 594–618.
- Cumming, Douglas, Walz, Uwe, 2010. Private equity returns and disclosure around the world. *Journal of International Business Studies* 41, 727–754.
- DeMiguel, Victor, Garlappi, Lorenzo, Nogales, Francisco J., Uppal, Raman, 2009. A generalized approach to portfolio optimization: improving performance by constraining portfolio norms. *Management Science* 55, 798–812.
- Diebold, Francis X., Mariano, Roberto S., 1995. Comparing Predictive accuracy. *Journal of Business and Economic Statistics* 13, 253–263.
- Efron, Bradley, Tibshirani, Robert J., 1994. *An Introduction to the Bootstrap*. Chapman & Hall, New York.
- Fama, Eugene F., 1981. Stock returns, real activity, inflation, and money. *American Economic Review* 71, 545–565.
- Geltner, David M., 1991. Smoothing in appraisal-based returns. *Journal of Real Estate Finance & Economics* 4, 327–345.
- Getmansky, Mila, Andrew, W. Lo, Makarov, Igor, 2004. An econometric model of serial correlation and illiquidity in hedge fund returns. *Journal of Financial Economics* 74, 529–609.
- Giot, Pierre, Schwienbacher, Armin, 2007. IPOs, trade sales and liquidations: modelling venture capital exits using survival analysis. *Journal of Banking and Finance* 31, 679–702.
- Gompers, Paul.A., Lerner, Josh., 1997. Risk and reward in private equity investments: the challenge of performance assessment. *Journal of Private Equity* 1, 5–12.
- Gompers, Paul A., Josh Lerner, 1998. What Drives Venture Capital Fundraising? *Brookings Papers on Economic Activity-Microeconomics* (July), pp. 149–192.
- Groh, Alexander, Liechtenstein, Heinrich, 2011a. International allocation determinants of institutional investments in venture capital and private equity limited partnerships. *Journal of International Banking, Accounting and Finance* 3, 176–200.
- Groh, Alexander, Liechtenstein, Heinrich, 2011b. Determinants for allocations to CEE venture capital and private equity limited partnerships. *Venture Capital: An International Journal of Entrepreneurial Finance* 13, 175–194.
- Groh, Alexander, Liechtenstein, Heinrich, 2011c. The first step of the capital flow from institutions to entrepreneurs: the criteria for sorting venture capital funds. *European Financial Management* 17, 532–559.
- Groh, Alexander, Liechtenstein, Heinrich, Lieser, Karsten, 2010a. The European venture capital and private equity country attractiveness indices. *Journal of Corporate Finance* 16, 205–224.
- Groh, Alexander, Liechtenstein, Heinrich, Canela, Miguel, 2010b. Limited partners' perceptions of the central eastern european venture capital and private equity market. *Journal of Alternative Investments* 12, 96–112.
- Groh, Alexander, Heinrich Liechtenstein, Karsten Lieser, 2012. *The Global Venture Capital and Private Equity Country Attractiveness Index, 2012 Annual*, Ernst & Young.
- Harlow, W.V., 1991. Asset allocation in a downside-risk framework. *Financial Analysts Journal* 47, 28–40.
- Hege, Ulrich, Palomino, Frederic, Schwienbacher, Armin, 2009. Venture capital performance: the disparity between Europe and the United States. *Revue Finance* 30, 7–50.
- Jarque, Carlos M., Bera, Anil K., 1980. Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters* 6, 255–259.
- Jeng, Leslie A., Wells, Philippe C., 2000. The determinants of venture capital funding: evidence across countries. *Journal of Corporate Finance* 6, 241–289.
- Jobson, J. Dave, Korkie, Bob M., 1981. Performance hypothesis testing with the Sharpe and Treynor measures. *Journal of Finance* 36, 889–908.
- Kannianen, Vesa, Keuschnigg, Christian, 2004. Start-up investment with scarce venture capital support. *Journal of Banking and Finance* 28, 1935–1959.
- Keuschnigg, Christian, Nielsen, Soren Bo, 2003. Taxes and venture capital support. *European Finance Review* 7, 515–539.
- Klein, April, Zur, Emanuel, 2009. Entrepreneurial shareholder activism: hedge funds and other private investors. *Journal of Finance* 64, 187–229.
- Koijen, Ralph S.J., Rodriguez, Juan Carlos, Sbuels, Alessandro, 2009. Momentum and mean reversion in strategic asset allocation. *Management Science* 55, 1199–1213.
- Krohmer, Philipp, Lauterbach, Rainer, Calanog, Victor, 2009. The bright and dark side of staging: investment performance and the varying motivations of private equity firms. *Journal of Banking and Finance* 33, 1597–1609.
- Kutner, Michael, Neter, John, Nachtsheim, William, 2004. *Applied Linear Statistical Models*. McGraw-Hill Irwin, New York.
- Ledoit, Oliver, Wolf, Michael, 2008. Robust performance hypothesis tests with the sharpe ratio. *Journal of Empirical Finance* 15, 850–859.
- Markowitz, Harry M., 1952. Portfolio selection. *Journal of Finance* 7, 77–91.
- Metrick, Andrew, Yasuda, Ayako, 2010a. The economics of private equity funds. *Review of Financial Studies* 23, 2303–2341.
- Metrick, Andrew, Yasuda, Ayako, 2010b. *Venture Capital and the Finance of Innovation*. John Wiley & Sons, Hoboken, NJ.
- Mietzner, Mark, Schweizer, Denis, Tyrell, Marcel, 2011. Intra-industry effects of shareholder activism in germany – is there a difference between hedge fund and private equity investments? *Schmalenbach Business Review* 63, 155–185.
- Nahata, Raj, 2008. Venture capital reputation and investment performance. *Journal of Financial Economics* 90, 127–151.
- Nesbitt, Steve, Reynolds, Hal, 1997. Benchmarks for private market investments: public indexes to gauge private investments. *Journal of Portfolio Management* 23, 85–90.
- Rajan, Raghuram G., Zingales, Luigi, 2003. The great reversals: the politics of financial development in the twentieth century. *Journal of Financial Economics* 69, 5–50.
- Rockafellar, Tyrrell R., Uryasev, Stanislav, 2000. Optimization of conditional value-at-risk. *Journal of Risk* 2, 21–41.
- Rockafellar, Tyrrell R., Uryasev, Stanislav, 2002. Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance* 26, 1443–1471.
- Schwert, G. William, 1981. The adjustment of stock prices to information about inflation. *Journal of Finance* 36, 15–29.
- Schwiebacher, Armin, 2008. Venture capital investment practices in Europe and in the United States. *Financial Markets and Portfolio Management* 22, 195–217.
- Tierney, David, Bailey, Jeffrey, 1997. Opportunistic investing: performance measurement, benchmarking and evaluation. *Journal of Portfolio Management* 23, 69–78.
- Woodward, Susan, 2004. *Measuring Risk and Performance for Private Equity*, Sand Hill Econometrics. <www.sandhillecon.com>.
- Woodward, Susan, Robert Hall, 2003. *Benchmarking the Returns to Venture*, NBER Working Paper No. 10202.