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A hub location inventory model for bicycle sharing system design: Formulation and solution

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ABSTRACT

This study addresses a strategic design problem for bicycle sharing systems incorporating bicycle stock considerations. The problem is formulated as a hub location inventory model. The key design decisions considered are: the number and locations of bicycle stations in the system, the creation of bicycle lanes between bicycle stations, the selection of paths of users between origins and destinations, and the inventory levels of sharing bicycles to be held at the bicycle stations. The design decisions are made with consideration for both total cost and service levels (measured both by the availability rate for rental requests at the pick-up rental stations and coverage of the origins and destinations). The optimal design of this system requires an integrated view of the travel costs of users, bicycle inventory costs and facility costs of bicycle stations and bicycle lanes as well as service levels. The purpose of this study is to create a formal model that provides such an integrated view, and to develop methods for obtaining solutions for the design variables in practical situations. The complexity of the problem precludes the exact solution of the optimization problem for instances of realistic size, and so we propose a heuristic method for efficiently finding near-optimal solutions. In the test problem for which enumeration is possible, the heuristic solution is within 2% optimal. Finally, a numerical example is created to illustrate the model and proposed solution algorithm.

1. Introduction

The bicycle sharing system uses bicycles as one form of public transportation in urban areas. Bicycle sharing focuses on the first mile and/or last mile of the user's commute. It also provides a connection to other modes of transit. The idea is that the commuters can take the bicycles whenever they need them and leave them behind when they reach their destinations. The city can benefit from the reduction in pollution and traffic congestion and less infrastructural investment than in the case of other transportation services. Since the first introduction of a bicycle sharing system in Amsterdam in the Netherlands in the 1960s, bicycle sharing systems have been receiving increased attention in recent years around the world, such as in Paris, France; Barcelona, Spain; Berlin, Germany; Washington, DC, USA; Montreal, Canada. However, there is relatively little literature published on the strategic design of bicycle sharing systems. This encourages us to carry out this study. The purpose of this research is to formulate and analyze a strategic design model for bicycle sharing systems with service level and bicycle stock considerations. The key design decisions considered are: the number and locations of bicycle sharing stations in the system, the network structure of bicycle lanes between bicycle stations, the selection of paths of users between origins and destinations, and the inventory levels of shared bicycles to be held at the bicycle stations. The design decisions are made with consideration being given to both the total cost and service levels (measured by both the availability rate for pick-up requests at the pick-up bicycle stations and coverage of the origins and destinations). The locations of the origins and destinations are assumed to be known and fixed, and the travel demands of each origin–destination pair are also assumed to be known, with known parameters of a stochastic demand process. The concerns in this model are long-term decisions regarding facility investments, building bicycle lanes, inventory levels, and the selection of travel paths, rather than day-to-day operational decisions such as the redistribution of bicycles during a day.

Bicycle sharing systems in urban areas are designed to offer the user one-way, short-distance trips to integrate with other modes of transportation. The general structure of such a system

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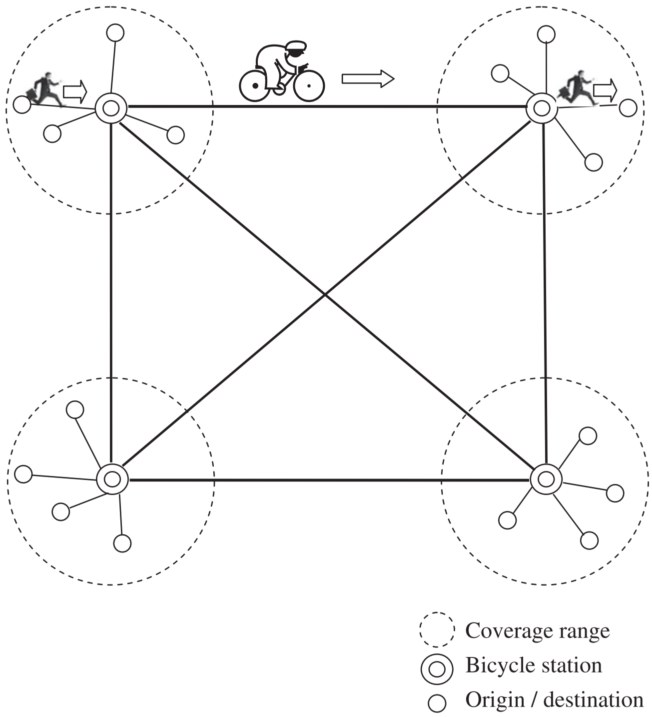

Fig. 1. Network structure of bicycle sharing systems.

is represented in Fig. 1. This structure represents the trips of users from origins to destinations. Each origin–destination trip consists of three links: (1) a user walks to pick up a bicycle at a station near his/her origin; (2) the user rides the bicycle to another station near his/her destination and checks in the bicycle; and (3) the user walks from the check-in station to the destination. Trips involving a single bicycle station are not possible, since users have to check out bicycles at a station somewhere else.

It is also crucial for the success of the system that users can easily find bicycle stations within a convenient walking distance. The systems need a sufficient number of bicycle stations to be set in the right locations for users to check out a bicycle near their origins and to check in a bicycle near their destinations. Existing examples show that the bicycle stations should not be located more than 300–500 m from important origins and destinations of traffic.

On the other hand, it is also important for the success of the system that the system guarantee the availability of bicycles. If users cannot find a bicycle at the check-out station, they may wait for an available bicycle, try to check-out another bicycle at a nearby station, or walk off the station. All of these actions cannot but frustrate the users. Each bicycle station must carry enough bicycles to increase the possibility that users can find a bicycle and thereby avoid the resulting frustration when users cannot find a bicycle. Therefore, measures of service quality in the system include both the availability rate (i.e., the proportion of rental requests at a bicycle station that are filled out of the bicycle stock on hand) and the coverage level (the fraction of total demand at origins and destinations that is within some specified time or distance from the nearest bicycle station). Having fewer bicycle stations allows total inventory costs to be lowered, but lower coverage of demand is provided as a result. Thus, there is a basic tradeoff in determining the number and locations of bicycle stations. More bicycle stations also allow shorter travel trips between origins/destinations and stations, and therefore potentially decreasing travel costs. However, additional costs for constructing and operating the bicycle stations will be incurred.

The optimal design of this system requires an integrated view of the travel costs of users, the bicycle inventory costs and facility costs of rental stations and bicycle lanes as well as service levels. The purpose of this study is to create a novel model that provides such an integrated view, and to develop methods for obtaining solutions for the design variables in practical situations.

2. Literature review

This research reviews the literature in four related directions: bicycle sharing system-related studies, hub location problems, maximal covering models, and joint location inventory problems.

Bicycle sharing systems have attracted a great deal of attention in recent years, having been used as a new inner-city transportation mode that can be integrated with existing public transit systems in many cities. Although the growth of the system has been rapid following the development of better tracking technology and the impact of the bicycle sharing systems has been profound (DeMaio, 2009), most of the studies related to bicycle systems in the literature have focused on the history and development of bicycle sharing systems (DeMaio, 2009; Shaheen, Guzman, & Zhang, 2010), promotional policies and safety issues (Martens, 2007; Aultman-Hall & Kaltenecker, 1999), repositioning of bicycles among bicycle stations (Vogel & Mattfeld, 2010; Nair & Miller-Hooks, 2011) and analyzing bicycle temporal and geographical usage patterns (Kaltenbrunner, Meza, Grivolla, Codina, & Banchs, 2010). There are, however, relatively few studies addressing the network and facility location design problem for bicycle sharing systems from the perspective of strategic planning. Lin and Yang (2011) address the design of the bicycle sharing system, formulate the problem as a mathematical model and solve it with a commercial solver. However, the proposed model involves a non-linear mixed integer program whose solution becomes intractable when applied to a real-world problem.

The bicycle sharing system addressed in this study can be viewed as a hub location inventory model that takes the coverage level into consideration. We briefly review the studies related to the hub location problem. The hub location problem has been one of the important classic facility location problems due to the use of hub and spoke networks in transportation and telecommunications systems. Hub and spoke network systems serve every origin/destination demand via a smaller set of links between origins/destinations and hubs and between pairs of hubs, rather than serving demand with direct links. The hub location problem involves determining the hub facilities and determining the links to connect origins, destinations and hubs. Since the early work by O'Kelly (1986), the hub location problem has been extended for various applications including models with flow-dependent cost discounts on arcs (O'Kelly & Bryan, 1998; Podnar, Skorin-Kapov, & Skorin-Kapov, 2002), models that consider capacity constraints on hub facilities (Yang, 2008; Rodriguez, Alvarez, & Barcos, 2007), models with stochastic demand (Yang, 2010), and models that locate hub arcs (Campbell, Ernst, & Krishnamoorthy, 2005a, 2005b). Since the hub location problems are difficult to solve exactly, there are a larger number of heuristics to tackle the many types of hub location problem proposed in the literature for various applications. Klincewicz (1992) applies tabu search and the GRASP heuristic to a P-hub location problem and Klincewicz (2002) develops a heuristic for the hub location problem with economies of scale. For an extensive review of applications and solution procedures for the hub location problem, we refer to Campbell, Ernst, and Krishnamoorthy (2002).

An important concept proposed in this study is the service level measured by the coverage range of the origins and destinations. A demand location is "covered" if a facility is located within a given distance of the location. The maximal covering model was first formulated by Church and ReVelle (1974) and has been used in a variety of applications. Mirchandani and Francis (1990) and Daskin (1995) provided surveys of applications and solution procedures for the maximal covering problem.

To ensure that the hub network can effectively handle the traffic through hubs and uncertain demands, several performance constraints may be included in the hub location models. However, these constraints are most common in telecommunications systems and logistics systems. In logistics systems design application, each open facility location needs to carry enough product inventories to ensure the desired service level while determining the optimal facility locations. Joint location-inventory models applied to logistics systems design include the work by Cole (1995), Nozick and Turnquist (1998, 2000, 2001), Daskin, Coullard, and Shen (2002), Shen, Coullard, and Daskin (2003), Miranda and Garrido (2004), Shu, Teo, and Shen (2005), Lin, Nozick, and Turnquist (2006). Although there is an extensive literature on joint location-inventory models, there is relatively little literature published on the hub location inventory problem. This also encourages us to carry out this study.

3. Model formulation

3.1. Problem definition

The problem can be summarized as follows. Given a set of origins, destinations, potential bicycle stations and the travel demands from origins to destinations with specific demand processes, we would like to know where to locate the bicycle stations, where to build the bicycle lanes, and what paths should be used for users from each origin to each destination and the required inventory level for sharing bicycles at each station to ensure the desired availability. As shown in Fig. 1, each origin–destination path consists of three links: a walking trip from a user's origin to the first station to check out a bicycle, a cycling trip from the first station to the second station where the bicycle is checked in, and a walking trip from the second station to the user's destination. Since the bicycles always need to be returned and checked in, paths involving a single bicycle station are not possible.

Service quality in the system is represented by two measures: the availability rate for sharing bicycles at each station, and the total amount of demand covered within a specified distance or travel time by at least one station. The designated availability rates and the desired coverage distance/travel-time are both input parameters specified by the system designer. If a higher availability rate is specified, it creates a need for an additional safety stock in the station inventory, and shifts the overall system cost calculation. If coverage standards are changed, the computation of penalty costs (for uncovered demand) in the model is affected, and can cause location decisions to be changed.

3.2. Mathematical model

To formulate this problem, the following symbols, variables, and parameters are first introduced.

Subscripts and sets

$i \in I$	denotes the origins
$j \in J$	denotes the destinations
$k, l \in K$	denotes the potential bicycle stations

Input parameters

λ_{ij}	is the yearly mean travel demand from origin i to destination j
d_{ik}	is the distance from origin i to station k
d_{kl}	is the distance from station k to station l
d_{kj}	is the distance from station k to destination j
f_k	is the fixed cost of locating a bicycle station at k
T	is the number of days per year (used to convert daily demand)
τ_k	is the replenishment lead time of bicycles at station k in days
α_k	is the desired service level at station k
c_{kl}	is the construction cost of constructing a bicycle lane from station k to station l; it is equal to 0 if there already exists a bicycle lane between station k and station l
h	is the inventory holding cost for a bicycle
q_{ik}	equals 1 if a station located at candidate site k cannot cover demand at origin i, and is 0 otherwise
q_{jl}	equals 1 if a station located at candidate site l cannot cover demand at destination j, and is 0 otherwise
γ_{ik}	is the unit traveling cost on links from origin i to station k for a user
β_{kl}	is the unit traveling cost on links from check-out station k to check-in station l for a user
γ_{lj}	is the unit traveling cost on links from station l to destination j for a user
δ	is the unit penalty cost for uncovered demands at origins and destinations

Decision variables

X_k	equals 1 if station k is opened and 0 otherwise
Y_{iklj}	is the fraction of the travel demand from origin i to destination j that is routed through station k and destination l in sequence; $0 \leqslant Y_{iklj} \leqslant 1$
Z_{kl}	equals 1 if a bicycle lane is required to be connected between station k and destination l; and 0 otherwise
S_k	the bicycle stock level at station k

Based on the notation, the following mathematical model can be formulated:

$$\begin{aligned}\min \sum_{i\in I}\sum_{k\in K}\gamma_{ik}d_{ik}\sum_{l\in K}\sum_{j\in J}Y_{iklj}\lambda_{ij} + \sum_{k\in K}\sum_{l\in K}\beta_{kl}d_{kl}\sum_{i\in I}\sum_{j\in J}Y_{iklj}\lambda_{ij} \\ + \sum_{l\in K}\sum_{j\in J}\gamma_{lj}d_{lj}\sum_{i\in I}\sum_{k\in K}Y_{iklj}\lambda_{ij} + \sum_{k\in K}f_kX_k + \sum_{k\in K}\sum_{l\in K}c_{kl}Z_{kl} + h\sum_{k\in K}S_k \\ + \delta\left(\sum_{i\in I}\sum_{k\in K}q_{ik}\sum_{l\in K}\sum_{j\in J}Y_{iklj}\lambda_{ij} + \sum_{j\in J}\sum_{l\in K}q_{jl}\sum_{k\in K}\sum_{i\in I}Y_{iklj}\lambda_{ij}\right)\end{aligned} \tag{1}$$

such that

$$\sum_{k\in K}\sum_{l\in K \neq k}Y_{iklj} = 1 \quad \forall i \in I,\ \forall j \in J \tag{2}$$

$$2Z_{kl} \leqslant X_k + X_l \quad \forall k \in K,\ \forall l \neq k \in K \tag{3}$$

$$Y_{iklj} \leqslant Z_{kl} \quad \forall i \in I,\ \forall k \in K,\ \forall l \neq k \in K,\ \forall j \in J \tag{4}$$

$$\Lambda_k = \frac{1}{T}\sum_{i\in I}\sum_{l\in K\neq k}\sum_{j\in J}Y_{iklj}\lambda_{ij} \quad \forall k \in K \tag{5}$$

$$s_k = \left\{ \min_s \sum_{q=0}^{s-1} \frac{e^{-\Lambda_k \tau_k} (\Lambda_k \tau_k)^q}{q!} \geqslant \alpha_k \right\} \quad \forall k \in K \tag{6}$$

$$Y_{iklj} \geqslant 0 \quad \forall i \in i,\ \forall k \in K,\ \forall l \neq k \in K,\ \forall j \in J \tag{7}$$

$$s_k \geqslant 0 \quad \text{and integer} \quad \forall k \in K \tag{8}$$

$$X_k = \{0, 1\} \quad \forall k \in K \tag{9}$$

The objective is to minimize the sum of traveling costs on links from origins to check-out stations, traveling costs between check-out and check-in stations, and traveling costs from check-in stations to destinations, fixed costs of the stations, bicycle inventory costs at stations, and penalty costs for uncovered demands. Eq. (2) ensure that all the demand is satisfied. Eq. (3) ensure that only the pair of bicycle stations that are both opened need to have a bicycle lane connecting them. Eq. (4) ensure that only the bicycle lanes that are opened are used in the commuter paths. Eqs. (5),(6) and (8) define the computation of the minimum inventory of bicycles at station k to meet the required service level, α_k. Eqs. (7), (8) ensure that the routings and inventory variables are nonnegative. Eq. (9) are integrality requirements for the location variables.

We assume that the yearly mean travel demand from origins to destinations follows a Poisson distribution with rate λ_{ij}. Therefore, the requested daily demand for rental bicycles at each station follows a Poisson distribution with rate Λ_k, defined by (5). The rental station must carry enough inventories to ensure a low probability $(1 - \alpha_k)$ of stocking out of bicycles during the replenishment lead time, τ_k. By Palm's theorem, the number of units in re-supply follows a Poisson distribution with parameter $\lambda_k \tau_k$. (Feeney & Sherbrooke, 1966). The inventory level needed at each station is then the minimum value of s such that:

$$\sum_{q=0}^{s-1} \frac{e^{-\Lambda_k \tau_k} (\Lambda_k \tau_k)^q}{q!} \geqslant \alpha_k \tag{10}$$

Unfortunately, this formulation is not computationally tractable. The key difficulties lie in the location variables and non-linear inventory costs. Since the related hub location problem is a *NP*-hard problem (Campbell et al., 2002) and the formulation provided above is at least as computationally complex as the hub allocation problem, the formulation provided should be a *NP*-hard problem.

4. Solution procedure

The solution procedure is developed based on a greedy heuristic. The core idea is that if a set of opened bicycle lanes has been determined, we can use them to identify a reasonable set of bicycle stations to open. Likewise, if a set of opened bicycle stations has been determined, we can use those locations to identify a reasonable set of bicycle lanes to be built. The greedy-drop heuristic iterates between locating bicycle stations given a collection of bicycle lanes, and locating bicycle lanes given a set of bicycle stations. More specifically, the algorithm can be represented as follows:

- Step 0. Initialization: mark all stations as open.
- Step 1. Mark all bicycle lanes connecting all open stations as open.
- Step 2. Identify the currently open station, which if closed, would result in the largest total cost reduction.
- Step 3. If a potential cost reduction is possible, mark the stations from Step 2 as closed and return to Step 2. If no cost reduction is possible, go to Step 4.
- Step 4. Mark all bicycle lanes used by users as open.
- Step 5. Identify a currently open bicycle lane, which if closed, would result in the largest total cost reduction.
- Step 6. If a cost reduction is possible, mark that bicycle lane as closed and return to Step 5. If no cost reduction is possible, go to Step 7.
- Step 7. Iterate Steps 1–6 until the maximum number of iterations is reached or the stations and bicycle lanes stop changing locations.

In Steps 2 and 5 of the above algorithm, currently opened stations and bicycle lanes are tested one at a time, to find the best single elimination, if any. This evaluation requires computing a new travel pattern of users across the network (and associated costs) with one additional station or bicycle lane closed (given previously opened sites). So within the overall algorithm described above there is an "inner" algorithm to compute the travel pattern (and associated cost) on a network where the set of opened stations and bicycle lanes is specified. This flow determination algorithm can be described as follows.

Given a set of opened stations and bicycle lanes locations, it is reasonable to assume that each user chooses the shortest traveling cost path from an origin to a destination. To evaluate the cost of opening a particular set of stations and bicycle lanes, we must calculate: (1) the traveling cost, (2) the inventory cost, (3) the facility costs of locating stations, (4) the construction cost of bicycle lanes, and (5) the penalty cost for uncovered demand. The computation is quite straightforward. The facility costs of stations and bicycle lanes are obvious, since we know the set of open stations and bicycle lanes. The traveling costs from origins to destinations can be easily calculated if the shortest path is identified for each origin/destination pair. Given a set of opened stations and a set of opened bicycle lanes, the shortest paths can be easily identified by Dijkstra's algorithm. Once the path choices are identified, the required bicycle inventory level at each station can be easily calculated by Eqs. (5) and (6) of the model in Section 3.2. Furthermore, the calculation of the penalty cost for uncovered demand is also clear. If there are any stations and bicycle lanes that are not used at all, we will eliminate them in the solution and revise the associated costs.

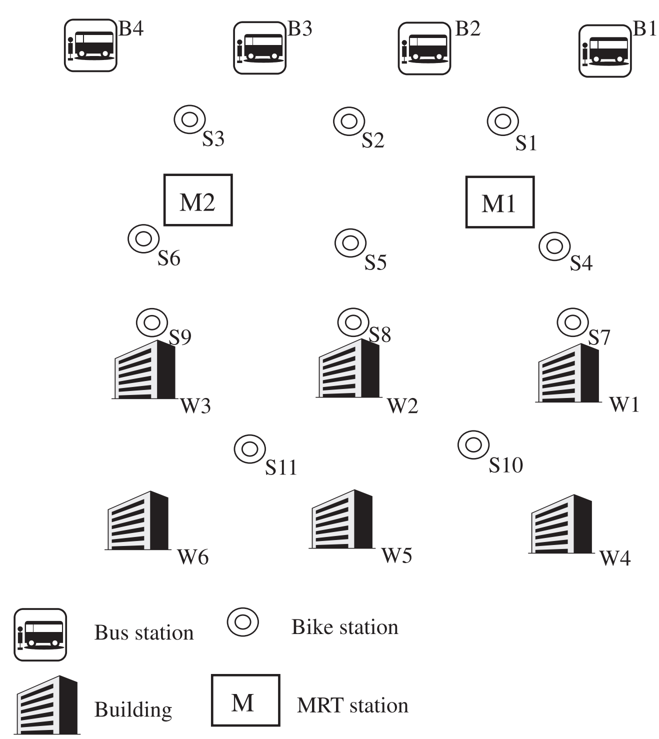

Fig. 2. Location sites for the illustrative example.

Lin et al. (2006) studied an uncapacitated fixed-charge network design problem in distribution systems design with economies of scale in transportation, and developed an iterative greedy-add algorithm for determining which distribution centers and consolidation centers to include in the network. There is a general similarity between their approach and the algorithm described here, although the context and specific implementation is quite different.

5. An illustrative example

5.1. Data settings

An illustrative example, as shown in Fig. 2, was created to illustrate the proposed model and algorithm. The network consists of four bus stations (node B1, B2, B3 and B4), two mass rapid transit (MRT) stations (node M1 and M2), and six office buildings (node W1, W2, W3, W4, W5 and W6). Eleven candidate sites are considered for the bicycle stations (node S1 to S11). Among them, there are six sites near the bus/MRT stations (node S1 to node S6) and five sites near the office buildings (node S7–S11). The bicycle sharing system is designed to integrate the public transportation systems and provide access to final destinations. A commuter walks from one of the bus/MRT stations (the origin) to the nearest bicycle station and checks out a bicycle. Then, he/she rides the bicycle to the second station and returns (checks in) the bicycle. Finally, he/she walks from the second station to the office building (the destination). The reverse direction, from the office buildings to the bus/MRT stations, also derives travel demand. Since the bicycle sharing system under study focuses on the first mile and/or last mile of the user's commute and it provides a connection to other modes of transit, the travel demands are only derived from the bus/MRT stations to office buildings and from office buildings to the bus/MRT stations. Therefore, there are 72 pairs of travel demands where there are 36 pairs from 6 bus/MRT stations to 6 office buildings and 36 pairs from six office buildings to 6 bus/MRT stations. The origin/destination travel demand matrix is shown in Table 1 and the distance matrices are shown in Tables 2 and 3. The coverage

Table 1
The yearly mean travel demands.

	B1	B2	B3	B4	M1	M2	W1	W2	W3	W4	W5	W6
B1	0	0	0	0	0	0	10,000	15,000	10,000	15,000	10,000	15,000
B2	0	0	0	0	0	0	20,000	25,000	20,000	25,000	20,000	25,000
B3	0	0	0	0	0	0	10,000	15,000	10,000	15,000	10,000	15,000
B4	0	0	0	0	0	0	20,000	25,000	20,000	25,000	20,000	25,000
M1	0	0	0	0	0	0	30,000	30,000	35,000	30,000	35,000	30,000
M2	0	0	0	0	0	0	40,000	40,000	45,000	40,000	45,000	40,000
W1	10,000	20,000	10,000	20,000	30,000	40,000	0	0	0	0	0	0
W2	15,000	25,000	15,000	25,000	30,000	40,000	0	0	0	0	0	0
W3	10,000	20,000	10,000	20,000	35,000	45,000	0	0	0	0	0	0
W4	15,000	25,000	15,000	25,000	30,000	40,000	0	0	0	0	0	0
W5	10,000	20,000	10,000	20,000	35,000	45,000	0	0	0	0	0	0
W6	15,000	25,000	15,000	25,000	30,000	40,000	0	0	0	0	0	0

Table 2
The distance matrix from origins to stations or from stations to origins (in meters).

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
B1	200	300	600	300	500	700	2400	2500	2600	2500	2600
B2	100	200	500	300	400	600	2400	2450	2550	2450	2550
B3	500	200	100	600	400	300	2550	2450	2400	2550	2450
B4	600	300	200	700	500	300	2600	2500	2400	2600	2500
M1	200	300	500	5	400	600	2002	2202	2502	2202	2502
M2	500	300	200	600	400	5	2502	2202	2002	2502	2202
W1	2302	2402	2702	2002	2202	2502	5	300	600	300	600
W2	2402	2302	2402	2202	2002	2202	300	5	300	300	300
W3	2702	2402	2302	2502	2202	2002	600	300	5	600	300
W4	2800	2800	2900	2400	2600	2700	350	400	700	300	600
W5	2800	2750	2800	2400	2550	2400	400	350	400	300	300
W6	2900	2800	2800	2700	2600	2400	700	400	350	600	300

Table 3
The distance matrix from stations to stations (in meters).

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
S1	0	150	300	150	200	400	2300	2400	2700	2600	2700
S2	150	0	150	210	150	210	2400	2300	2400	2600	2600
S3	300	150	0	400	200	150	2700	2400	2300	2700	2600
S4	150	210	400	0	155	310	2000	2200	2500	2200	2500
S5	200	150	200	155	0	155	2200	2000	2200	2400	2400
S6	400	210	150	310	155	0	2500	2200	2000	2500	2200
S7	2300	2400	2700	2000	2200	2500	0	160	320	160	300
S8	2400	2300	2400	2200	2000	2200	160	0	160	160	160
S9	2700	2400	2300	2500	2200	2000	320	160	0	300	160
S10	2600	2600	2700	2200	2400	2500	160	160	300	0	160
S11	2700	2600	2600	2500	2400	2200	300	160	160	160	0

Table 4
The cost parameters.

Input parameters	Values
λ_{ij}	Shown on Table 1
d_{ik}	Shown on Table 2
d_{kl}	Shown on Table 3
d_{kj}	Shown on Table 2
f_k	NTD 4.1 million to NTD 8.2 million per year
T	365
τ_k	1
α_k	0.95
c_{kl}	NTD 100 $*$ d_{kl}
h	NTD 2000 per year
q_{ik}	Using a coverage radius of 300 m to convert
q_{jl}	Using a coverage radius of 300 m to convert
γ_{ik}	NTD 0.2 per meter per trip
β_{kl}	NTD 0.04 per meter per trip
γ_{lj}	NTD 0.2 per meter per trip
δ	NTD 100 per trip

distance is assumed to be a radius of 300 m. A trip is covered if both its origin and destination is within the coverage distance from the near open bicycle station. The unit penalty cost for uncovered demand is assumed to be NTD 100 per trip. The unit waking costs and the unit bicycle riding costs may vary by links to reflect the up or down grade on sloping links. However, for the sake of simplicity, we assume that the unit walking cost is NTD 0.2 per meter per trip and the unit bicycle riding cost is NTD 0.04 per meter per trip. The fixed cost of building a bicycle lane is NTD 100/per meter times the distance between two stations. The fixed costs of bicycle stations range from NTD 4.1 million to NTD 8.2 million per year. We assume that the bicycle stocks are replenished every day. The unit bicycle holding cost is NTD 2000 per year. The availability rate is assumed to be 95%. The input parameters are listed in Table 4.

5.2. Test results

We implement the algorithm in C++ on a desktop computer (Intel 3.2 GHz Core i5 and 1.89 GB of memory) with a Microsoft Windows XP operating system. The solution is generated within two seconds. First, we open all 11 stations (S1–S11) and all bicycle lanes lying between them. The total cost of this design is NTD 330.542 million. Second, assuming that all bicycle lanes lying between open stations opened, we close one currently open bicycle station at a time to identify which currently open station closed will result in the largest cost reduction. The eleven total cost evaluations, based on closing a single bicycle station at S1, S2, S3, S4, S5, S6, S7, S8, S9, S10 or S11, are NTD 332.280 million, 326.442 million, 331.154 million, 339.158 million, 322.342 million, 345.234 million, 338.884 million, 340.506 million, 340.422 million, 352.624 million and 352.740 million, respectively. Therefore, if we wanted to close only one station (or to open 10 stations) to result in the largest total cost reduction, we would close station S5, as shown in Fig. 3. We then repeat the cost calculations, to evaluate the change in total cost for dropping each of the remaining ten open station locations, considered one at a time. We find that the next bicycle station to close is at S2, reducing the total cost to NTD 318.242 million. Fig. 4 illustrates the solution. Continuing this process yields a decision to close an additional station at S3 thereby creating a total cost of about NTD 318.234 million. Fig. 5 illustrates the solution. If we were to also close one more station, the total cost would rise rather than decline; hence the algorithm decides not to do that.

Together with the cost calculations, we need to determine the user travel paths. Given the eight stations open at S1, S4, S6, S7, S8, S9, S10 and S11, we first assume that all bicycle lanes lying between them are open. Then we find the shortest weighted path for each O/D pair. After all O/D pairs are routed on their corresponding

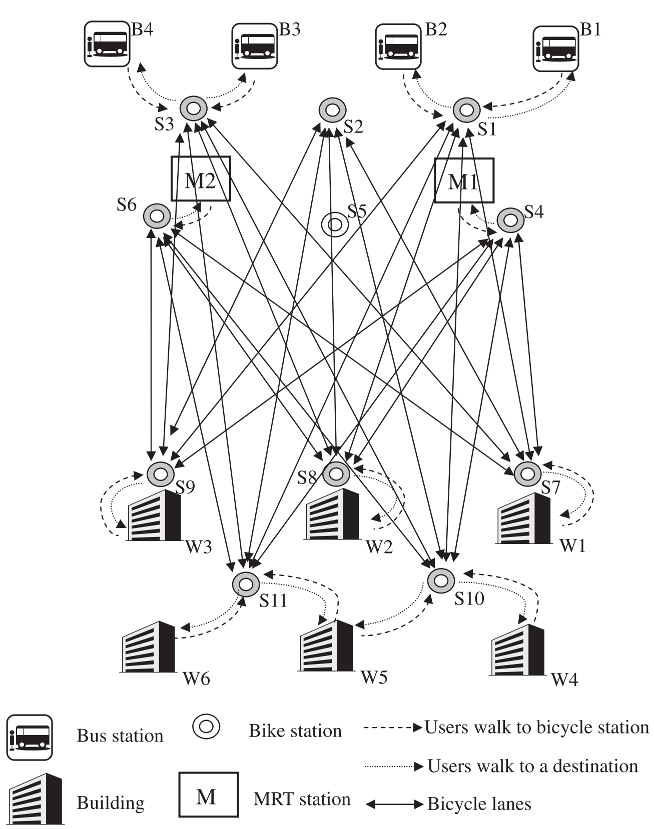

Fig. 3. Network design while closing one station.

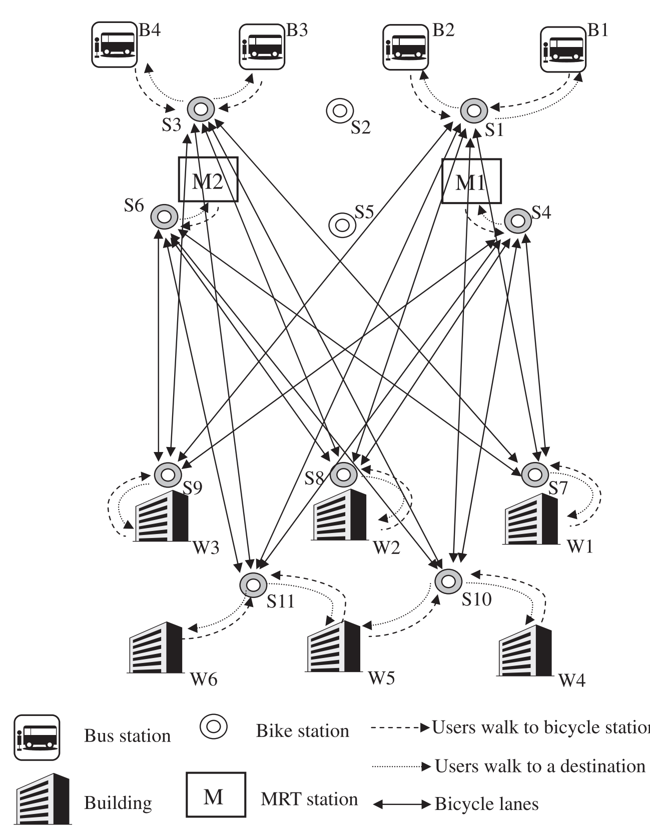

Fig. 4. Network design while closing two stations.

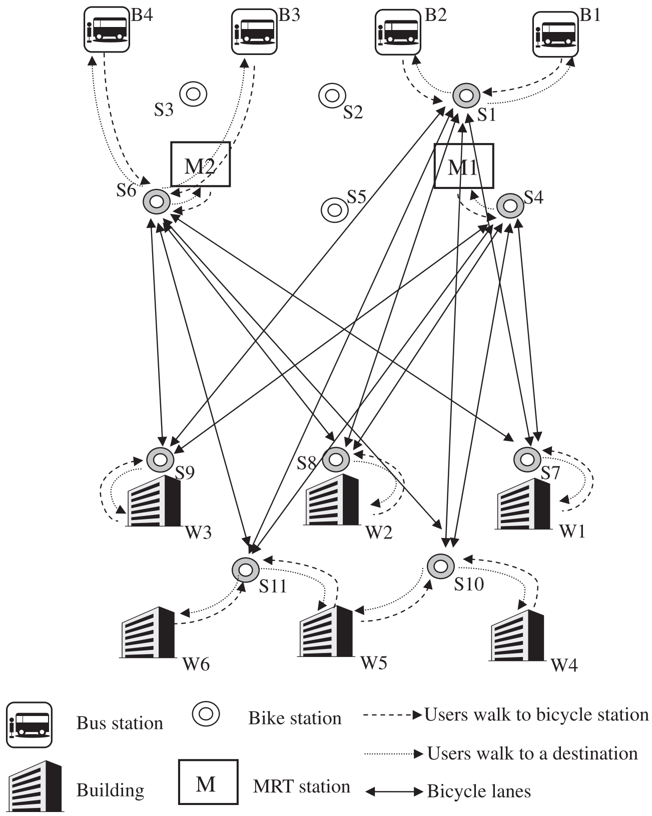

Fig. 5. Network design and routing choices for the illustrative example.

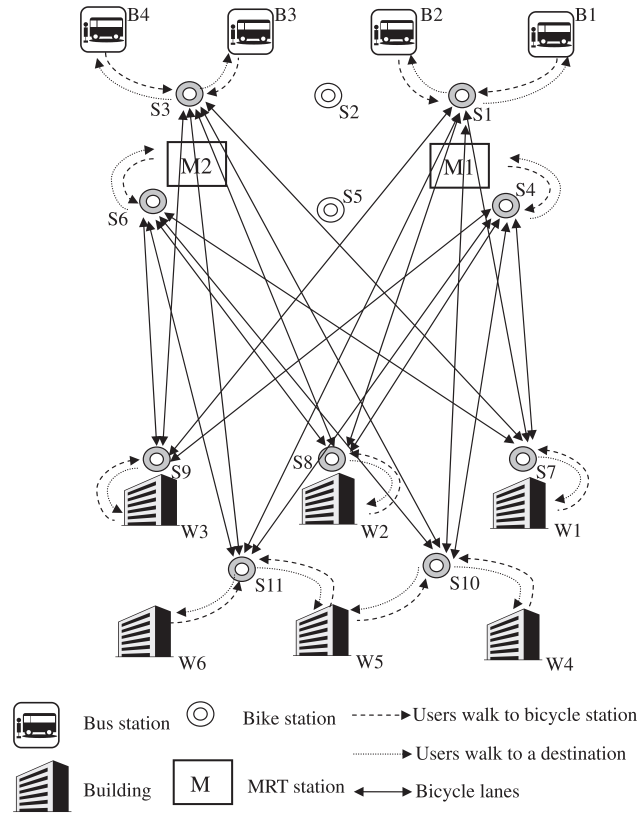

Fig. 6. Network design and routing choices while setting lower availability rates.

shortest paths, we find that only 30 bicycle lanes are used. There are bicycle lanes mutually between the set of stations S1, S4 and S6 and the set of stations S7, S8, S9, S10 and S11, and those bicycle lanes lying in the reverse direction. Other candidate bicycle lanes are not used at all. We eliminate them in the solution.

Given these eight stations and the 30 bicycle lanes that are open, the algorithm decides which bicycle lanes to close. Dropping one of 30 opened bicycle lanes increases total costs, so no additional bicycle lanes are closed. In the second iteration, given the 30 opened bicycle lanes, the stations are opened at S1, S4, S6, S7, S8, S9, S10 and S11. Because the chosen set of station locations is the same as in the first iteration, the selection of bicycle lanes will again result in the same 30 opened bicycle lanes being chosen. Since the locations have not changed in two consecutive iterations, the algorithm terminates.

Fig. 5 illustrates the solution. The system design suggests a solution of eight bicycle stations located at S1, S4, S6, S7, S8, S9, S10, and S11, and 30 bicycle lanes lying in between these stations. The bicycle stocks held at stations S1, S4, S6, S7, S8, S9, S10, and S11 are 615, 558, 1319, 387, 445, 416, 629 and 658, respectively. All travel demands are routed on the corresponding shortest travel cost path, given the open stations and bicycle lanes. Those users from the six bus/MRT stations merely walk to the nearest open bicycle station to check out a bicycle, ride to the open station nearest to their respective destinations, check in the bicycle at the station, and then walk to their destinations. For example, the users traveling from origin B1 to destination W1 walk to station S1 to check out a bicycle, ride to station S7, check in the bicycle at station S7, and then walk to destination W1. Those users from the six bus/MRT stations who travel to destination W5, however, may use different check-in stations. The users from the upper-right three origins (namely, B1, B2 and M1) check in bicycles at station S10 and then walk to destination W5. The users from the upper-left three origins (namely, B3, B4 and M2) check in bicycles at station S11 and then walk to destination W5. For those users who travel from the six office buildings to the bus/MRT stations, the routing choices are identical to those of the users who travel from the bus/MRT stations, except that they move in the reverse direction.

5.3. Sensitivity analyses

The proposed model described in Section 3 provides several parameters that are significant levers affecting the decisions on the bicycle stocks held at stations and the solution:

1. the availability rate of bicycles requested at stations (α);
2. the inventory holding cost for a bicycle (h);
3. the replenishment lead time of bicycles at stations (τ);
4. the fixed cost of locating a bicycle station (f).

To illustrate how these parameters affect the solution, we first change the availability rate of bicycles requested at stations to a lower service level to identify a network design with more bicycle stations and bicycle lanes. Second, we cut down the value of the inventory holding cost for a bicycle to a half to observe the change in routing choices and the network design. Third, we assume that the bicycle stocks are replenished with different frequencies to see the change in the network design and bicycle stocks held at stations. Fourth, we first change the fixed facility costs to higher values to identify a network design with fewer bicycle stations and to lower values to identify a network design with more bicycle stations.

Fig. 6 illustrates the network design and routing choices while setting the availability rate of bicycles requested at stations to 80%. In comparison with the network design of the above illustrative example, the solution yields a bicycle sharing network with 9

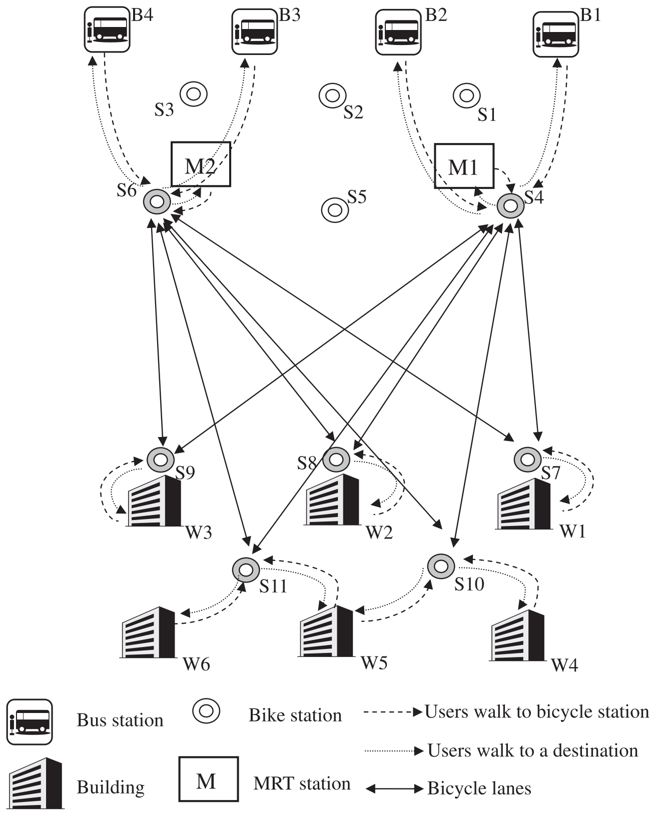

Fig. 7. Network design and routing choices while setting extremely high fixed station costs.

bicycle stations located at S1, S3, S4, S6, S7, S8, S9, S10 and S11, and 40 bicycle lanes situated between these stations. As bicycle station S3 is opened, the users from origins B3 and B4 pick up bicycles at station S3 instead of picking up bicycles at station M2 and their route choices change. For example, the users traveling from B3 to W1 pick up a bicycle at station S3 instead of S6 and switch from routes B3, S6, S7 and W1 to routes B3, S3, S7 and W1 since station S3 is now available. The bicycle stocks held at stations S1, S3, S4, S6, S7, S8, S9, S10 and S11 are 595, 595, 540, 707, 372, 428, 400, 609 and 637, respectively. The lower the availability rate, the fewer the bicycles that need to be stocked at stations. The bicycle stocks held at each open station are much lower than those based on the results presented in Section 5.2. This implies a lower total cost of opening a bicycle station (the sum of facility cost of opening the bicycle station plus the bicycle inventory holding costs at the station) and results in a network with more bicycle stations and bicycle lanes open. This is similar to a setup with lower fixed station costs.

When the model setup involves a lower inventory holding cost for a bicycle (with a value of NTD 1000), the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes in comparison with those of the results presented in Section 5.2. The network design and routing choices are identical to those for setting up lower availability rates as shown in Fig. 6. However, the bicycle inventory levels held at bicycle stations are different since the availability rates are not the same. The bicycle stocks held at stations S1, S3, S4, S6, S7, S8, S9, S10 and S11 are 615, 615, 558, 728, 387, 445, 416, 629 and 658, respectively. The lower the inventory holding cost for a bicycle, the lower the total inventory holding costs at open stations. This implies a lower total cost of opening a bicycle station, and results in a network with more bicycle stations and bicycle lanes open.

If we can replenish the bicycle inventory at bicycle stations more frequently, the total inventory will drop off dramatically. When we assume that the bicycle stocks are replenished twice a day (once after the morning peak period and once after the evening peak period), the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes in comparison with the network design of the illustrative example. The network design and routing choices are identical to those for setting up lower availability rates as shown in Fig. 6. The bicycle stocks held at stations S1, S3, S4, S6, S7, S8, S9, S10 and S11 are 316, 316, 287, 373, 200, 229, 215, 323 and 337, respectively. Even if we open more bicycle stations than as shown in the results presented in Section 5.2, the total inventory level will drop off dramatically. By contrast, if we can replenish the bicycle inventory at bicycle stations less frequently, the total inventory will increase dramatically. When we assume that the bicycle stocks are replenished once every two days, the solution yields a bicycle sharing network design and routing choices that are identical to those of the results presented in Section 5.2. However, the bicycle stocks held at stations increase dramatically. The bicycle stocks held at stations S1, S4, S6, S7, S8, S9, S10 and S11 are 1207, 1094, 2603, 757, 869, 813, 1235 and 1291, respectively.

Fig. 7 illustrates the network design and routing choices when the costs of setting up involve extremely high fixed costs ranging

Table 5
Cost compositions of illustrative examples with different parameters (NTD mill).

Test problem	Description	Facility costs	Bicycle links costs	Inventory costs	User costs + uncovered penalty	Total costs
I-1	Illustrative example	49.20	7.10	10.05	251.88	318.23
I-2	Lower availability rate	53.30	9.64	9.77	245.20	317.91
I-3	Lower inventory holding cost	53.30	9.64	5.05	245.20	313.19
I-4	Replenish inventory more frequently	53.30	9.64	5.19	245.20	313.33
I-5	Replenish inventory less frequently	49.20	7.10	19.74	251.88	327.92
I-6	High fixed costs	90.20	4.56	10.01	260.96	365.73
I-7	Lower fixed costs	26.65	9.64	10.10	245.20	291.59

Table 6
Test problems.

Test problem	Number of origins	Number of destinations	Candidate bicycle stations
Illustrative Example	12	12	11
Test Problem 1	30	30	14
Test Problem 2	30	30	18

Table 7
Solution quality and computational performance.

Test Problems	Solution procedures		Enumeration		Cost gap (%)
	Costs (NTD Mill)	Time (s)	Costs (NTD Mill)	Time (s)	
I-1	318.2	1.0	318.2	60.1	0.0
I-2	317.9	0.7	317.9	59.4	0.0
I-3	313.2	0.7	313.2	60.3	0.0
I-4	313.3	0.7	313.3	60.1	0.0
I-5	327.9	0.9	327.9	59.1	0.0
I-6	365.7	1.1	365.7	59.9	0.0
I-7	291.6	0.7	291.6	59.7	0.0
1-1	32.5	19.1	32.5	4715.7	0.0
1-2	35.6	21.7	35.6	4725.7	0.0
1-3	40.9	24.2	40.7	4731.6	0.5
1-4	32.4	19.2	32.4	4629.1	0.0
1-5	35.5	21.8	35.5	4716.6	0.0
1-6	32.0	18.9	32.0	4728.2	0.0
1-7	32.1	22.8	32.1	4724.6	0.0
2-1	31.5	71.3	31.5	150530.0	0.0
2-2	35.0	71.3	35.0	150119.0	0.0
2-3	40.9	93.1	40.7	150652.0	0.5
2-4	53.7	101.9	52.7	148552.0	1.9
2-5	40.8	83.8	40.6	148833.0	0.5
2-6	40.5	83.6	40.3	150392.0	0.5
2-7	41.7	83.4	41.5	149671.0	0.5

from NTD 8.2 million to NTD 16.4 million. In comparison with the network design of the above illustrative example, the solution yields a bicycle sharing network with only seven bicycle stations located at S4, S6, S7, S8, S9, S10 and S11, and 20 bicycle lanes situated between these stations. Although the number of open stations decreases, all users still pick up and drop off bicycles at the open stations that are within the coverage distance. As bicycle station S1 is closed, the users from origins B1 and B2 pick up bicycles at station S4 instead and their route choices change. For example, the users traveling from B1 to W3 pick up a bicycle at station S4 instead of S1 and switch from routes B1, S1, S9 and W3 to routes B1, S4, S9 and W3 since station S1 is no longer available.

Fig. 6 illustrates the network design and routing choices when the costs of setting up involve lower fixed costs that range from NTD 2.05 million to NTD 4.1 million. In comparison with those based on the results presented in Section 5.2, the solution yields a bicycle sharing network with more bicycle stations and more bicycle lanes. The network design, routing choices and bicycle stocks held at stations are identical to those for setting a lower inventory holding cost for a bicycle (with a value of NTD 1000).

Table 5 shows the cost composition of the solution results discussed above.

5.4. Solution quality

In order to investigate the computational performance and solution quality of the heuristic, two sets of additional test problems were created which are similar to the illustrative example, but with different numbers of origins/destinations and candidate bicycle stations. Table 6 describes each of the test problems. To better study the influence of cost parameters on the proposed solution procedure, several test problems are created which are similar to test problems 1 and 2, but with different fixed costs of locating bicycle stations, inventory holding costs, availability rates and replenishment lead times. Table 7 presents the solutions from the procedure and computation times. Note that for all the problems the enumeration is feasible within 2 days, and the solution procedure identifies a solution that is extremely close to the true optimal solutions by enumeration. The differences between the found costs and the optimal ones are less than 2% (less than 0.2% on average). The computation time required is much shorter than that by enumeration. Computationally, the solution procedure is quite good and holds substantial promise for the solution to large problems.

6. Conclusions

Bicycle sharing system design requires an integrated view of transportation, inventory and facility costs as well as service quality. This paper has developed a hub location inventory model that provides an integrated view of the various costs and service quality concerns, as well as a computationally feasible method for obtaining solutions in realistic situations.

The solution procedure developed consists of two core elements. The first element is an iterative use of a greedy-drop heuristic for locating bicycle stations and bicycle lanes. The second is a heuristic to cost out the objective function for a given set of bicycle stations and bicycle lanes. This procedure to cost out the objective function for a given set of locations identifies which travel path each O/D pair should use.

An illustrative network is created to illustrate the proposed model and solution algorithm. The sensitivity analyses were conducted to test how several important inventory-related parameters affect the inventory holding decisions, the network design, and routing choices.

Future enhancements would be useful in at least the following directions. First, the calculation of the bicycle inventory at the stations is conservative since the check-in bicycles are not accounted for as being available for reuse. To make the best use of the available bicycles, the check-in bicycles should be accounted for as being available to be reused. However the calculation is more complicated because the bicycle trip riding durations should be taken into account in determining the number of bicycles available for reuse. It would be useful to develop a more accurate estimate. Second, it might be useful to develop a solution algorithm to simultaneously adjust the bicycle stations and bicycle lanes to improve the feasible solution.

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