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An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon

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Abstract

An inventory model for a deteriorating item (seasonal product) with linearly displayed stock dependent demand is developed in imprecise environment (involving both fuzzy and random parameters) under inflation and time value of money. It is assumed that time horizon, i.e., period of business is random and follows exponential distribution with a known mean. The resultant effect of inflation and time value of money is assumed as fuzzy in nature. The particular case, when resultant effect of inflation and time value is crisp in nature, is also analyzed. A genetic algorithm (GA) is developed with roulette wheel selection, arithmetic crossover, random mutation. For crisp inflation effect, the total expected profit for the planning horizon is maximized using the above GA to derive optimal inventory decision. On the other hand when inflationary effect is fuzzy then the above expected profit is fuzzy in nature too. Since optimization of fuzzy objective is not well defined, the optimistic/pessimistic return of the expected profit is obtained using possibility/necessity measure of fuzzy event. Fuzzy simulation process is proposed to determine this optimistic/pessimistic return. Finally a fuzzy simulation based GA is developed and is used to maximize the above optimistic/pessimistic return to get optimal decision. The models are illustrated with some numerical examples and some sensitivity analyses have been presented.

Keywords: Time value of money; Stochastic planning horizon; Possibility; Necessity

1. Introduction

In the present competitive market, the inventory/stock is decoratively displayed through electronic media to attract the customers and to push the sale. Levin et al. [1], Schary and Becker [2] and Wolfe [3] established the impact of product availability for stimulating demand. Mandal and Phaujder [4,5], Datta and Pal [6] and

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others considered linear form of stock dependent demand, i.e., $D = c + dq$, where D, q represent demand and stock level respectively, c, d are two constants, so chosen to fit the demand function best, where as Giri et al. [7], Mandal and Maiti [8], Maiti and Maiti [9] and others took the demand of the form $D = dq^{\beta}$, where d, β are suitable constants.

Effect of inflation and time value of money in inventory problems is well established. The initial attempt in this direction was made by Buzacott [10]. He dealt with an EOQ model under inflation subject to different types of pricing policies. In the subsequent year, Bierman et al. [11] showed that the inflation rate does not affect the optimal order quantity perse; rather, the difference between the inflation rate and the discount rate affects on it. Due to increasing complexities of the world economy, it is very difficult to estimate this difference precisely. So, for real life inventory problems it is better to estimate this difference as a fuzzy quantity. Though a considerable number of research work has been done in this area (cf. Misra [12], Padmanabhan and Vrat [13], Hariga and Ben-Daya [14], Chen [15], Dey et al. [16], Moon and Lee [17], etc.), only few of them have considered these as fuzzy quantities.

Classical inventory models are usually developed over infinite planning horizon. According to Gurnani [18], Chung and Kim [19], the assumption of an infinite planning horizon is not realistic due to several reasons such as variation of inventory costs, changes in product specifications and designs, technological changes, etc. Moreover, for seasonal products like fruits, vegetables, warm garments, etc., business period is not infinite. There are some models (cf. Datta and Pal (1992), Bhunia and Maiti [20], Mahapatra and Maiti [21], etc.) in which time horizon has been considered as finite. For seasonal products, the planning horizon varies over years and may be considered as random with a distribution. Moon and Yun [22] developed an EOQ model with a random planning horizon. Recently Moon and Lee [17] presented an EOQ model under inflation and discounting with a random product life cycle. Till now, none has developed inventory models incorporating random planning horizon, stock dependent demand, imprecise effect due to inflation and discounting.

When some inventory parameters are fuzzy in nature the resultant objective function also becomes fuzzy. After the introduction of fuzzy set theory in 1965 by Zadeh, extensive research work has been done on defuzzification of fuzzy numbers. Among these techniques centroid method [23], weighted average method [24], graded mean value method [25], nearest interval approximation method [26], graded mean integration value [27], etc., have drawn more attention. All these techniques replace the fuzzy parameters by their nearest crisp number/interval and the reduced crisp objective function is optimized. To deal with fuzzy objective function, Liu and Iwamura [28] proposed a method where an optimistic return of the objective function is optimized. They used possibility measure of fuzzy event to transform the fuzzy objective function to an equivalent crisp objective and also proposes a fuzzy simulation method to determine the value of this crisp equivalent for complicated situations. Maiti and Maiti [9] extended this work where pessimistic return of the objective function is optimized using necessity measure of fuzzy event and they used it to solve a two-warehouse fuzzy inventory model.

In this paper, an inventory model for a deteriorating item is formulated with displayed stock dependent demand over a planning horizon. It is assumed that the planning horizon is uncertain, random in nature and follows exponential distribution with a known mean. Here the inflation and time value of money are considered and the resultant effect of these two is taken into account. The problem has been solved with both crisp and imprecise resultant effect. For crisp model expected profit is maximized using a GA with roulette wheel selection, arithmetic crossover and random mutation. In the case of fuzzy model, the problem with fuzzy objective function is converted to a chance constrained programming using possibility/necessity measure of fuzzy event, where optimistic/pessimistic return of the objective function with some degree of optimism/pessimism is optimized. Following Liu and Iwamura [28], a fuzzy simulation process is proposed to maximize the optimistic/pessimistic return and a fuzzy simulation based genetic algorithm with above mentioned GA operators is developed to solve the model. The models are illustrated with some numerical data. Some sensitivity analyses on expected profit are presented.

2. Assumptions and notations

The mathematical model in this paper is developed on the basis of following assumptions and notations:

Assumptions:
1. Demand rate is assumed here to vary with the displayed inventory level. But this psychological influence must have an upper limit, i.e., above some level, demand will remain unchanged.
2. The time horizon (a random variable) is finite.
3. The time horizon fully accommodates first N cycles and end during $(N+1)$th cycle.
4. Lead time is negligible.
5. Replenishment rate is infinite but replenishment size is finite.
6. Shortages are not allowed.

Notations:
1. $T =$ Duration of a complete cycle.
2. $q(t) =$ On hand inventory of a cycle in time t, $(j-1)T \leqslant t \leqslant jT$ $(j = 1, 2, \ldots, N)$.
3. $q_0 =$ Inventory level above which demand becomes constant.
4. $t_1 =$ Time in the first cycle when inventory level reaches q_0, i.e. $q(t_1) = q_0$ for $0 < t_1 < T$.
5. $D(q) =$ The demand rate, where

$$D(q) = \begin{cases} \alpha + \beta q_0, \ \alpha, \beta \geqslant 0, \ (q(t) \geqslant q_0) \text{ during } (0 \leqslant t \leqslant t_1), \\ \alpha + \beta q(t), \ \alpha, \beta \geqslant 0, \ (0 \leqslant q(t) \leqslant q_0) \text{ during } (t_1 \leqslant t \leqslant T). \end{cases}$$

6. $\theta =$ Constant deterioration rate on the on hand inventory at time t.
7. $C_1 =$ Holding cost per unit item per unit time.
8. $C_3 =$ Ordering cost per replenishment cycle.
9. $s =$ Selling price of one unit.
10. $c =$ Purchasing cost of one unit.
11. $H =$ Total time horizon(a random variable) and h is real time horizon.
12. $N =$ Number of fully accommodated cycles to be made during the real time horizon h and time horizon ends during $N + 1$th cycle.
13. $Q =$ Total ordered quantity in a cycle.
14. $i =$ Inflation rate.
15. $r =$ Discount rate.
16. $R = r - i$, may be crisp or fuzzy.
17. $P(N, T) =$ Total profit after completing N fully accommodated cycles.
18. $\mathrm{HC_L} =$ Holding cost in last cycle.
19. $\mathrm{SR_L} =$ Sales revenue in last cycle.
20. $s_1 =$ Reduced selling price in last cycle.
21. $E\{\mathrm{TP_L}(T)\} =$ Expected total profit from last cycle.
22. $E(\mathrm{TP}) =$ Expected total profit from the planning horizon.

3. Mathematical formulation

In the development of the model, we assume that there are N full cycles during the real time horizon h and the planning horizon ends during $(N+1)$th cycle, i.e., within $t = NT$ and $t = (N+1)T$. At the beginning of every jth $(j = 1, 2, \ldots, N+1)$ cycle, company purchases an amount Q units of the item and when inventory level reaches zero, then again order for the next cycle is placed (cf. Figs. 1a, 1b). For the last cycle some amount may be left after the end of planning horizon. This amount is sold at a reduced price in a lot.

3.1. Formulation for jth $(1 \leqslant j \leqslant N)$ cycle

The differential equations describing the inventory level $q(t)$ in the interval $(j-1)T \leqslant t \leqslant jT (1 \leqslant j \leqslant N)$ are given by

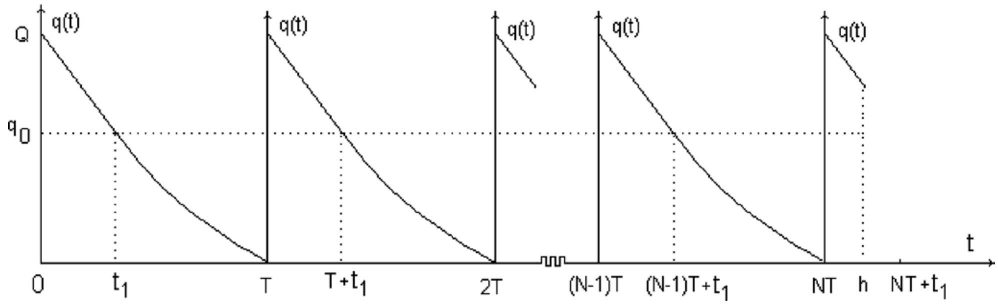

Fig. 1a. Inventory levels at different situations for $NT < h \leqslant NT + t_1$.

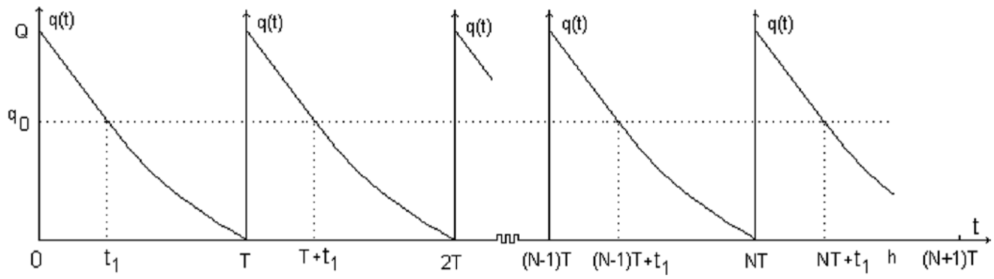

Fig. 1b. Inventory levels at different situations for $NT + t_1 < h \leqslant (N+1)T$.

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = -\alpha - \beta q_0 - \theta q(t), \quad (j-1)T \leqslant t \leqslant (j-1)T + t_1, \tag{1}$$

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = -\alpha - \beta q(t) - \theta q(t), \quad (j-1)T + t_1 \leqslant t \leqslant jT, \tag{2}$$

where $\alpha, \beta, \theta > 0$ and $0 < t_1 < T$, subject to the conditions that,

$$q(t) = Q \ \text{at}\ t = (j-1)T, \quad q(t) = q_0 \ \text{at}\ t = (j-1)T + t_1 \quad \text{and} \quad q(t) = 0 \ \text{at}\ t = jT.$$

The solutions of the differential Eqs. (1) and (2) are given by

$$q(t) = \begin{cases} -\frac{(\alpha+\beta q_0)}{\theta} + \left\{q_0 + \frac{\alpha+\beta q_0}{\theta}\right\}\mathrm{e}^{\theta\{(j-1)T+t_1-t\}}, & (j-1)T \leqslant t \leqslant (j-1)T + t_1 \\ \frac{\alpha}{(\beta+\theta)}\{\mathrm{e}^{(\beta+\theta)(jT-t)} - 1\}, & (j-1)T + t_1 \leqslant t \leqslant jT \end{cases} \tag{3}$$

So,

$$q\{(j-1)T + t_1\} = q_0 \quad \text{gives, } t_1 = T - \frac{1}{\beta+\theta}\log\left|1 + \frac{q_0(\beta+\theta)}{\alpha}\right|. \tag{4}$$

So, order quantity in a cycle is given by

$$Q = -\frac{(\alpha + \beta q_0)}{\theta} + \left\{q_0 + \frac{(\alpha+\beta q_0)}{\theta}\right\}\mathrm{e}^{\theta t_1}.$$

Present value of holding cost of the inventory for the jth $(1 \leqslant j \leqslant N)$ cycle is given by

$$HC_j = C_1\int_{(j-1)T}^{(j-1)T+t_1} q(t)e^{-Rt}dt + C_1\int_{(j-1)T+t_1}^{jT} q(t)e^{-Rt}dt$$
$$= \frac{C_1(\alpha+\beta q_0)}{R\theta}(e^{-Rt_1}-1)e^{-R(j-1)T} - \frac{C_1}{(R+\theta)}\left\{q_0+\frac{(\alpha+\beta q_0)}{\theta}\right\}(e^{-Rt_1}-e^{\theta t_1})e^{-R(j-1)T}$$
$$+\frac{C_1\alpha e^{-RjT}}{(\beta+\theta)(R+\beta+\theta)}\{e^{(R+\beta+\theta)(T-t_1)}-1\}+\frac{C_1\alpha}{R(\beta+\theta)}\{e^{-RjT}-e^{-Rt_1}\cdot e^{-R(j-1)T}\}. \tag{5}$$

Present value of purchasing cost for the jth ($1 \leqslant j \leqslant N$) cycle is given by

$$PC_j = c\left[-\frac{(\alpha+\beta q_0)}{\theta}+\left\{q_0+\frac{(\alpha+\beta q_0)}{\theta}\right\}e^{\theta t_1}\right]e^{-R(j-1)T}. \tag{6}$$

Present value of ordering cost for the jth ($1 \leqslant j \leqslant N$) cycle is given by

$$C_3^j = C_3 e^{-R(j-1)T}. \tag{7}$$

Present value of sales revenue for the jth ($1 \leqslant j \leqslant N$) cycle is given by

$$\mathrm{SR}_j = s\int_{(j-1)T}^{(j-1)T+t_1}\{\alpha+\beta q_0\}e^{-Rt}\,dt + s\int_{(j-1)T+t_1}^{jT}\{\alpha+\beta q(t)\}e^{-Rt}\,dt$$
$$= s\left[\frac{\alpha+\beta q_0}{R}\{e^{-RjT}-e^{-R(j-1)T+t_1}\}+\frac{\alpha}{R}\{e^{-R(j-1)T+t_1}-e^{-RjT}\}\right]$$
$$-s\left[\frac{\alpha\beta}{(\beta+\theta)(R+\beta+\theta)}\{e^{-RjT}-e^{(\beta+\theta)T-(R+\beta+\theta)t_1}\cdot e^{-R(j-1)T}\}\right.$$
$$\left.-\frac{\alpha\beta}{R(\beta+\theta)}\{e^{-RjT}-e^{-Rt_1}\cdot e^{-R(j-1)T}\}\right]. \tag{8}$$

So,

$$P(N,T) = \sum_{j=1}^{N} SR_j - \sum_{j=1}^{N}(C_3^j + \mathrm{HC}_j + \mathrm{PC}_j). \tag{9}$$

Now,

$$\sum_{j=1}^{N} e^{-R(j-1)T} = \left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right). \tag{10}$$

So,

$$P(N,T) = \left[s\frac{(\alpha+\beta q_0)}{R}(1-e^{-Rt_1})+\left\{\frac{s\alpha}{R}-\frac{s\alpha\beta}{R(\beta+\theta)}\right\}(e^{-Rt_1}-e^{-RT})-\frac{s\alpha\beta}{(\beta+\theta)(R+\beta+\theta)}\right.$$
$$\left.\times\{e^{-RT}-e^{(\beta+\theta)T-(R+\beta+\theta)t_1}\}\right]\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)-C_1\left[\frac{(\alpha+\beta q_0)}{R\theta}(e^{-Rt_1}-1)-\frac{1}{(R+\theta)}\left\{q_0+\frac{(\alpha+\beta q_0)}{\theta}\right\}\right.$$
$$\times\{e^{-Rt_1}-e^{\theta t_1}\}-\frac{\alpha}{(\beta+\theta)(R+\beta+\theta)}\{e^{-RT}-e^{(\beta+\theta)T-(R+\beta+\theta)t_1}\}+\frac{\alpha}{R(\beta+\theta)}$$
$$\left.\times\{e^{-RT}-e^{-Rt_1}\}\right]\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)-(c\cdot Q+C_3)\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right). \tag{11}$$

Here, we consider that the planning horizon H is a random variable and follows exponential distribution with p.d.f. as

$$f(h) = \begin{cases}\lambda e^{-\lambda h}, & h \geqslant 0,\\ 0, & \text{otherwise,}\end{cases} \tag{12}$$

where h is real planning horizon.

Since the planning horizon H has a p.d.f. $f(h)$, the present value of expected total profit from N complete cycles is given by

$$\begin{aligned}E\{P(N,T)\} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}P(N,T)\cdot f(h)\mathrm{d}h = \left[\frac{s(\alpha+\beta q_0)}{R}(1-\mathrm{e}^{-Rt_1})+\left\{\frac{s\alpha}{R}-\frac{s\alpha\beta}{R(\beta+\theta)}\right\}(\mathrm{e}^{-Rt_1}-\mathrm{e}^{-RT})\right.\\
&\left.+\frac{s\alpha\beta}{(\beta+\theta)(R+\beta+\theta)}\{\mathrm{e}^{(\beta+\theta)T-(R+\beta+\theta)t_1}-\mathrm{e}^{-RT}\}\right]\left[\frac{\mathrm{e}^{-\lambda T}-\mathrm{e}^{-(R+\lambda)T}}{(1-\mathrm{e}^{-RT})(1-\mathrm{e}^{-(R+\lambda)T})}\right]\\
&-C_1\left[\frac{(\alpha+\beta q_0)}{R\theta}(\mathrm{e}^{-Rt_1}-1)-\left\{q_0+\frac{(\alpha+\beta q_0)}{\theta}\right\}\frac{\{\mathrm{e}^{-Rt_1}-\mathrm{e}^{\theta t_1}\}}{(R+\theta)}-\frac{\alpha\{\mathrm{e}^{-RT}-\mathrm{e}^{(\beta+\theta)T-(R+\beta+\theta)t_1}\}}{(\beta+\theta)(R+\beta+\theta)}\right.\\
&\left.+\frac{\alpha}{R(\beta+\theta)}\{\mathrm{e}^{-RT}-\mathrm{e}^{-Rt_1}\}\right]\times\left[\frac{\mathrm{e}^{-\lambda T}-\mathrm{e}^{-(R+\lambda)T}}{(1-\mathrm{e}^{-RT})(1-\mathrm{e}^{-(R+\lambda)T})}\right]\\
&-(c\cdot Q+C_3)\left[\frac{\mathrm{e}^{-\lambda T}-\mathrm{e}^{-(R+\lambda)T}}{(1-\mathrm{e}^{-RT})(1-\mathrm{e}^{-(R+\lambda)T})}\right]. \qquad (13)\end{aligned}$$

3.2. Formulation for last cycle

The differential equations describing the inventory level $q(t)$ in the interval $NT < t$ are given by

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = -\alpha-\beta q_0-\theta q(t), \quad NT \leqslant t \leqslant NT+t_1, \qquad (14)$$

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = -\alpha-\beta q(t)-\theta q(t), \quad NT+t_1 \leqslant t, \qquad (15)$$

where $\alpha, \beta, \theta > 0$, subject to the conditions that,

$$q(NT) = Q \quad \text{and} \quad q(t) = q_0 \quad \text{at } t = NT+t_1.$$

The solutions of the differential Eqs. (14) and (15) are given by

$$q(t) = \begin{cases}-\frac{(\alpha+\beta q_0)}{\theta}+\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\mathrm{e}^{\theta(NT-t)}, & NT \leqslant t \leqslant NT+t_1,\\ -\frac{\alpha}{(\beta+\theta)}+\{q_0+\frac{\alpha}{\beta+\theta}\}\mathrm{e}^{(\beta+\theta)(NT+t_1-t)}, & NT+t_1 \leqslant t.\end{cases} \qquad (16)$$

In the last cycle, we consider two cases depending upon the cycle length. Let h be the real value corresponding to the random variable H.

3.2.1. Case-I $(NT < h \leqslant NT+t_1)$

Present value of holding cost of the inventory for the last cycle is given by

$$\begin{aligned}HC_{L1} &= C_1\int_{NT}^{h}q(t)\mathrm{e}^{-Rt}\mathrm{d}t\\
&= \frac{C_1(\alpha+\beta q_0)}{(R\theta)}\{\mathrm{e}^{-Rh}-\mathrm{e}^{-RNT}\}-\frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\mathrm{e}^{\theta NT}\cdot\{\mathrm{e}^{-(R+\theta)h}-\mathrm{e}^{-(R+\theta)NT}\} \qquad (17)\end{aligned}$$

Present value of purchasing cost $= cQ\cdot\mathrm{e}^{-RNT}$.
Present value of ordering cost $= C_3\mathrm{e}^{-NRT}$.
Present value of sales revenue is given by

$$SR_{L1} = s\int_{NT}^{h}\{\alpha+\beta q_0\}\mathrm{e}^{-Rt}\,\mathrm{d}t = \frac{s(\alpha+\beta q_0)}{R}\{\mathrm{e}^{-RNT}-\mathrm{e}^{-Rh}\}. \qquad (18)$$

3.2.2. Case-II $(NT+t_1 < h \leqslant (N+1)T)$

Present value of holding cost of the inventory for the last cycle is given by

$$\begin{aligned}HC_{L2} &= C_1\int_{NT}^{NT+t_1} q(t)\mathrm{e}^{-Rt}\,\mathrm{d}t + C_1\int_{NT+t_1}^{h} q(t)\mathrm{e}^{-Rt}\,\mathrm{d}t\\ &= \frac{C_1(\alpha+\beta q_0)}{R\theta}\{\mathrm{e}^{-R(NT+t_1)}-\mathrm{e}^{-RNT}\} - \frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\{\mathrm{e}^{-(R+\theta)t_1-RNT}-\mathrm{e}^{-RNT}\}\\ &\quad+\frac{C_1\alpha}{R(\beta+\theta)}\{\mathrm{e}^{-Rh}-\mathrm{e}^{-R(NT+t_1)}\}+\frac{C_1}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\{\mathrm{e}^{-R(NT+t_1)}-\mathrm{e}^{(\beta+\theta)(NT+t_1)-(R+\beta+\theta)h}\}.\end{aligned} \tag{19}$$

Present value of purchasing cost $= cQ\cdot\mathrm{e}^{-RNT}$.
Present value of ordering cost $= C_3\mathrm{e}^{-NRT}$.
Present value of sales revenue is given by

$$\begin{aligned}SR_{L2} &= s\int_{NT}^{NT+t_1}\{\alpha+\beta q_0\}\mathrm{e}^{-Rt}\,\mathrm{d}t + s\int_{NT+t_1}^{h}\{\alpha+\beta q(t)\}\mathrm{e}^{-Rt}\,\mathrm{d}t\\ &= \frac{s(\alpha+\beta q_0)}{R}\{\mathrm{e}^{-RNT}-\mathrm{e}^{-R(NT+t_1)}\}+\frac{s\alpha}{R}\{\mathrm{e}^{-R(NT+t_1)}-\mathrm{e}^{-Rh}\}+\frac{s\alpha\beta}{R(\beta+\theta)}\{\mathrm{e}^{Rh}-\mathrm{e}^{-R(NT+t_1)}\}\\ &\quad+\frac{s\beta}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\{\mathrm{e}^{-R(NT+t_1)}-\mathrm{e}^{(\beta+\theta)(NT+t_1)-(R+\beta+\theta)h}\}.\end{aligned} \tag{20}$$

So, expected holding cost for the last cycle is given by

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} HC_L\cdot f(h)\mathrm{d}h = \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} HC_{L1}\cdot f(h)\mathrm{d}h + \sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} HC_{L2}\cdot f(h)\mathrm{d}h. \tag{21}$$

Expected sales revenue from the last cycle is given by

$$\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} SR_L\cdot f(h)\mathrm{d}h = \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1} SR_{L1}\cdot f(h)\mathrm{d}h + \sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T} SR_{L2}\cdot f(h)\mathrm{d}h. \tag{22}$$

Expected sales revenue due to the sale at a reduced price of the leftover, if any, during the last cycle is given by

$$s_1\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{e}^{-Rh}q(h)\cdot f(h)\mathrm{d}h = s_1\sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{e}^{-Rh}q(h)\cdot f(h)\mathrm{d}h + s_1\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{e}^{-Rh}q(h)\cdot f(h)\mathrm{d}h. \tag{23}$$

So, expected total profit from last cycle is given by

$$\begin{aligned}E\{TP_L(T)\} &= \left\{\frac{s(\alpha+\beta q_0)}{R}+\frac{c_1(\alpha+\beta q_0)}{R\theta}\right\}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{\mathrm{e}^{-(R+\lambda)t_1}-1}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}+\left\{\frac{1-\mathrm{e}^{-\lambda t_1}}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}\right]\\ &\quad+\frac{s(\alpha+\beta q_0)}{R}\left[\frac{\mathrm{e}^{-\lambda t_1}-\mathrm{e}^{-\lambda T}-\mathrm{e}^{-(R+\lambda)t_1}+\mathrm{e}^{-Rt_1-\lambda T}}{1-\mathrm{e}^{-(R+\lambda)T}}\right]\\ &\quad+\left\{\frac{s\alpha}{R}-\frac{s\alpha\beta}{R(\beta+\theta)}+\frac{C_1\alpha}{R(\beta+\theta)}\right\}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{\mathrm{e}^{-(R+\lambda)T}-\mathrm{e}^{-(R+\lambda)t_1}}{(1-\mathrm{e}^{-(R+\lambda)T})}\right\}\right.\\ &\quad\left.+\left\{\frac{\mathrm{e}^{-(R+\lambda)t_1}-\mathrm{e}^{-Rt_1-\lambda T}}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}\right]+\frac{s\beta}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{\mathrm{e}^{-(R+\lambda)t_1}-\mathrm{e}^{-Rt_1-\lambda T}}{(1-\mathrm{e}^{-(R+\lambda)T})}\right\}\\ &\quad+\frac{\lambda s\beta}{(R+\beta+\theta)(R+\beta+\theta+\lambda)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{\mathrm{e}^{(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}-\mathrm{e}^{-(R+\lambda)t_1}}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}\\ &\quad+\frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\left[\frac{\lambda}{(R+\theta+\lambda)}\left\{\frac{1-\mathrm{e}^{-(R+\theta+\lambda)t_1}}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}+\left\{\frac{\mathrm{e}^{-\lambda t_1}-1}{1-\mathrm{e}^{-(R+\lambda)T}}\right\}\right]\end{aligned}$$

$$
\begin{aligned}
&-\frac{c_1(\alpha+\beta q_0)}{R\theta}\left[\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}-\left\{\frac{e^{-\lambda t_1}-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\left[\left\{\frac{e^{-(R+\lambda+\theta)t_1}-e^{-(R+\theta)t_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}-\left\{\frac{e^{-\lambda t_1}-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&-\frac{C_1}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left[\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{(1-e^{-(R+\lambda)T})}\right\}\right.\\
&\left.+\frac{\lambda}{(R+\beta+\theta+\lambda)}\left\{\frac{e^{-(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}-e^{-(R+\lambda)t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]-(c\cdot Q+C_3)\left\{\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\\
&+\frac{s_1\lambda}{(R+\lambda)}\left(\frac{\alpha+\beta q_0}{\theta}\right)\left[\frac{e^{-(R+\lambda)t_1}-1}{1-e^{-(R+\lambda)T}}\right]+\frac{s_1\lambda}{(R+\theta+\lambda)}\left\{Q+\frac{\alpha+\beta q_0}{\theta}\right\}\times\left[\frac{1-e^{-(R+\lambda+\theta)t_1}}{1-e^{-(R+\lambda)T}}\right]\\
&+\frac{s_1\alpha}{(\beta+\theta)}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{e^{-(R+\lambda)T}-e^{-(R+\lambda)t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{s_1\lambda}{(R+\lambda+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{e^{-(R+\lambda)t_1}-e^{(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}}{1-e^{-(R+\lambda)T}}\right\}.
\end{aligned}\tag{24}
$$

3.3. Total profit from the system

Now, total expected profit from the complete time horizon is given by

$$E(TP)=E(P(N,T))+E\{TP_L(T)\}.$$

So

$$
\begin{aligned}
E(TP)=&\left[\frac{s(\alpha+\beta q_0)}{R}(1-e^{-Rt_1})+\left\{\frac{s\alpha}{R}-\frac{s\alpha\beta}{R(\beta+\theta)}\right\}(e^{-Rt_1}-e^{-RT})\right.\\
&\left.+\frac{s\alpha\beta}{(\beta+\theta)(R+\beta+\theta)}\{e^{(\beta+\theta)T-(R+\beta+\theta)t_1}-e^{-RT}\}\right]\left[\frac{e^{-\lambda T}-e^{-(R+\lambda)T}}{(1-e^{-RT})(1-e^{-(R+\lambda)T})}\right]\\
&-C_1\left[\frac{(\alpha+\beta q_0)}{R\theta}(e^{-Rt_1}-1)-\left\{q_0\frac{(\alpha+\beta q_0)}{\theta}\right\}\frac{\{e^{-Rt_1}-e^{\theta t_1}\}}{(R+\theta)}-\frac{\alpha\{e^{-RT}-e^{(\beta+\theta)T-(R+\beta+\theta)t_1}\}}{(\beta+\theta)(R+\beta+\theta)}\right.\\
&\left.+\frac{\alpha}{R(\beta+\theta)}\{e^{-RT}-e^{-Rt_1}\}\right]\left[\frac{e^{-\lambda T}-e^{-(R+\lambda)T}}{(1-e^{-RT})(1-e^{-(R+\lambda)T})}\right]-(c\cdot Q+C_3)\left[\frac{e^{-\lambda T}-e^{-(R+\lambda)T}}{(1-e^{-RT})(1-e^{-(R+\lambda)T})}\right]\\
&+\frac{s(\alpha+\beta q_0)}{R}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{e^{-(R+\lambda)T-1}}{1-e^{-(R+\lambda)T}}\right\}+\left\{\frac{1-e^{-\lambda t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\left\{\frac{s(\alpha+\beta q_0)}{R}+\frac{c_1(\alpha+\beta q_0)}{R\theta}\right\}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{e^{-(R+\lambda)t_1}-1}{1-e^{-(R+\lambda)T}}\right\}+\left\{\frac{1-e^{-\lambda t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{s(\alpha+\beta q_0)}{R}\left[\frac{e^{-\lambda t_1}-e^{-\lambda T}-e^{-(R+\lambda)t_1}+e^{-Rt_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right]\\
&+\left\{\frac{s\alpha}{R}-\frac{s\alpha\beta}{R(\beta+\theta)}+\frac{C_1\alpha}{R(\beta+\theta)}\right\}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{e^{-(R+\lambda)T}-e^{-(R+\lambda)t_1}}{(1-e^{-(R+\lambda)T})}\right\}+\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{s\beta}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{(1-e^{-(R+\lambda)T})}\right\}\\
&+\frac{\lambda s\beta}{(R+\beta+\theta)(R+\beta+\theta+\lambda)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{e^{(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}-e^{-(R+\lambda)t_1}}{(1-e^{-(R+\lambda)T})}\right\}\\
&+\frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\left[\frac{\lambda}{(R+\theta+\lambda)}\left\{\frac{1-e^{-(R+\theta+\lambda)t_1}}{1-e^{-(R+\lambda)T}}\right\}+\left\{\frac{e^{-\lambda t_1}-1}{1-e^{-(R+\lambda)T}}\right\}\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{c_1(\alpha+\beta q_0)}{R\theta}\left[\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}-\left\{\frac{e^{-\lambda t_1}-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{C_1}{(R+\theta)}\left\{Q+\frac{(\alpha+\beta q_0)}{\theta}\right\}\left[\left\{\frac{e^{-(R+\lambda+\theta)t_1}-e^{-(R+\theta)t_1-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}-\left\{\frac{e^{-\lambda t_1}-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&-\frac{C_1}{(R+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left[\left\{\frac{e^{-(R+\lambda)t_1}-e^{-Rt_1-\lambda T}}{(1-e^{-(R+\lambda)T})}\right\}\right.\\
&\left.+\frac{\lambda}{(R+\beta+\theta+\lambda)}\left\{\frac{e^{-(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}-e^{-(R+\lambda)t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&-(c\cdot Q+C_3)\left\{\frac{1-e^{-\lambda T}}{1-e^{-(R+\lambda)T}}\right\}+\frac{s_1\lambda}{(R+\lambda)}\left(\frac{\alpha+\beta q_0}{\theta}\right)\left[\frac{e^{-(R+\lambda)t_1}-1}{1-e^{-(R+\lambda)T}}\right]\\
&+\frac{s_1\lambda}{(R+\theta+\lambda)}\left\{Q+\frac{\alpha+\beta q_0}{\theta}\right\}\left[\frac{1-e^{-(R+\lambda+\theta)t_1}}{1-e^{-(R+\lambda)T}}\right]+\frac{s_1\alpha}{(\beta+\theta)}\left[\frac{\lambda}{(R+\lambda)}\left\{\frac{e^{-(R+\lambda)T}-e^{-(R+\lambda)t_1}}{1-e^{-(R+\lambda)T}}\right\}\right]\\
&+\frac{s_1\lambda}{(R+\lambda+\beta+\theta)}\left\{q_0+\frac{\alpha}{(\beta+\theta)}\right\}\left\{\frac{e^{-(R+\lambda)t_1}-e^{(\beta+\theta)t_1-(R+\beta+\theta+\lambda)T}}{1-e^{-(R+\lambda)T}}\right\}.
\end{aligned}
\tag{25}
$$

3.4. Stochastic model (model-1)

When the resultant effect of inflation and discounting (R) is crisp in nature, then our problem is to determine T to

$$\operatorname{Max} E(TP). \tag{26}$$

3.5. Fuzzy stochastic model (model-2)

In the real world, resultant effect of inflation and time value of money (R) is imprecise, i.e. vaguely defined in some situations. So we take R as fuzzy number, i.e. as $\widetilde{R}$. Then, due to this assumption, our objective function $E(TP)$ becomes $E(\widetilde{TP})$. Since optimization of a fuzzy objective is not well defined, so instead of $E(\widetilde{TP})$ one can optimize its equivalent optimistic or pessimistic return of the objective as proposed by Maiti and Maiti [9]. Using this method the problem can be reduced to an equivalent crisp problem as discussed below.

If $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy subsets of real numbers $\Re$ with membership functions $\mu_{\widetilde{A}}$ and $\mu_{\widetilde{B}}$ respectively, then taking degree of uncertainty as the semantics of fuzzy number, according to Liu and Iwamura [28], Dubois Prade [29,30] and Zimmermann [31]:

$$\operatorname{Pos}(\widetilde{A}\star\widetilde{B})=\sup\{\min(\mu_{\widetilde{A}}(x),\mu_{\widetilde{B}}(y)),\ x,y\in\Re,\ x\star y\}, \tag{27}$$

where the abbreviation Pos represent possibility and $\star$ is any one of the relations $>, <, =, \leqslant, \geqslant$.

On the other hand necessity measure of an event $\widetilde{A}\star\widetilde{B}$ is a dual of possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as

$$\operatorname{Nes}(\widetilde{A}\star\widetilde{B})=1-\operatorname{Pos}(\overline{\widetilde{A}\star\widetilde{B}}), \tag{28}$$

where the abbreviation Nes represents necessity measure and $\overline{\widetilde{A}\star\widetilde{B}}$ represents complement of the event $\widetilde{A}\star\widetilde{B}$.

So for the fuzzy stochastic model one can maximize the crisp variable z such that necessity/possibility measure of the event $\{E(\widetilde{TP})>z\}$ exceeds some predefined level according to decision maker in pessimistic/optimistic sense. Accordingly the problem reduces to the following two models:

Model-2a: When decision maker prefers to optimize the optimistic equivalent of $E(\widetilde{TP})$, the problem reduces to determine T to,

$$
\begin{aligned}
&\text{maximize} \quad z\\
&\text{subject to} \quad \operatorname{pos}\{E(\widetilde{TP})\geqslant z\}\geqslant\alpha_1
\end{aligned}
\tag{29}
$$

Model-2b: On the other hand when the decision maker desires to optimize the pessimistic equivalent of $E(\widetilde{TP})$, the problem is reduced to determine T to,

$$\begin{aligned}&\text{maximize} \quad z\\ &\text{subject to} \quad \mathrm{nes}\{E(\widetilde{TP}) \geqslant z\} \geqslant \alpha_2,\\ &\text{i.e.,} \quad \mathrm{pos}\{E(\widetilde{TP}) \leqslant z\} < 1-\alpha_2.\end{aligned} \tag{30}$$

4. Solution methodology

To solve the stochastic model (model-1) GA is used. The basic technique to deal problems (29) or (30) is to convert the possibility/necessity constraint to its deterministic equivalent. However, the procedure is usually very hard and successful in some particular cases (cf. [9]). Following Liu and Iwamura [28], Maiti and Maiti [9], here two simulation algorithms are proposed to determine z in (29) and (30) respectively for a feasible T.

Algorithm 1. Algorithm to determine a feasible T to evaluate z for the problem (29):

To determine z for a feasible T, roughly find a point R_0 from fuzzy number $\widetilde{R}$, which approximately minimizes z. Let this value be z_0 and set $z = z_0$ (for simplicity one can take $z_0 = 0$). Then R_0 is randomly generated in α_1-cut set of $\widetilde{R}$ and let z_0 = value of $E(TP)$ for $R = R_0$ and if $z < z_0$ replace z with z_0. This step is repeated a finite number of times and final value is taken as the value of z. This phenomenon is used to develop the algorithm.

1. Set $z = z_0$.
2. Generate R_0 uniformly from the α_1 cut set of fuzzy number $\widetilde{R}$.
3. Set z_0 = value of $E(TP)$ for $R = R_0$.
4. If $z < z_0$ then set $z = z_0$.
5. Repeat steps 2, 3 and 4, N_1 times, where N_1 is a sufficiently large positive integer.
6. Return z.
7. End algorithm.

Algorithm 2. Algorithm to determine a feasible T to evaluate z for the problem (30):

We know that $\mathrm{nes}\{E(\widetilde{TP}) \geqslant z\} \geqslant \alpha_2 \Rightarrow \mathrm{pos}\{E(\widetilde{TP}) < z\} \leqslant 1-\alpha_2$. Now roughly find a point R_0 from fuzzy number $\widetilde{R}$, which approximately minimizes $E(TP)$. Let this value be z_0 (for simplicity one can take $z_0 = 0$ also) and ε be a positive number. Set $z = z_0 - \varepsilon$ and if $\mathrm{pos}\{E(\widetilde{TP}) < z\} \leqslant 1-\alpha_2$ then increase z with ε. Again check $\mathrm{pos}\{E(\widetilde{TP}) < z\} \leqslant 1-\alpha_2$ and it continues until $\mathrm{pos}\{E(\widetilde{TP}) < z\} > 1-\alpha_2$. At this stage decrease value of ε and again try to improve z. When ε becomes sufficiently small then we stop and final value of z is taken as the value of z. Using this criterion, required algorithm is developed as below. In the algorithm the variable F_0 is used to store initial assumed value of z and F is used to store value of z in each iteration.

1. Set $z = z_0 - \varepsilon$, $F = z_0 - \varepsilon$, $F_0 = z_0 - \varepsilon$, tol = 0.0001.
2. Generate R_0 uniformly from the $1-\alpha_2$ cut set of fuzzy number $\widetilde{R}$.
3. Set z_0= value of $E(TP)$ for $R = R_0$.
4. If $z_0 < z$.
5.   then go to step 11.
6. End If
7. Repeat step-2 to step-6 N_2 times.
8. Set $F = z$.
9. Set $z = z + \varepsilon$.
10. Go to step-2.
11. If $(z = F_0)$ // In this case optimum value of $z < z_0 - \varepsilon$
12.   Set $z = F_0 - \varepsilon$, $F = F - \varepsilon$, $F_0 = F_0 - \varepsilon$.

13. Go to step-2
14. End If
15. If ($\varepsilon <$ tol)
16. go to step-21
17. End If
18. $\varepsilon = \varepsilon/10$
19. $z = F + \varepsilon$
20. Go to step-2.
21. Output F.
22. End algorithm.

So for a feasible value of T, we determine z using the above algorithms and to optimize z we use GA. GA used to solve model-1 is presented below. When fuzzy simulation algorithm is used to determine z in the algorithm, this GA is named as fuzzy simulation based genetic algorithm (FSGA). This is used to determine fuzzy objective function values.

4.1. Genetic algorithm (GA)/fuzzy simulation based genetic algorithm (FSGA)

Genetic algorithm is a class of adaptive search technique based on the principle of population genetics. In natural genesis. we know that chromosomes are the main carriers of the hereditary information from parents to offsprings and that genes, which carry hereditary factors, are lined up in chromosomes. At the time of reproduction, crossover and mutation take place among the chromosomes of parents. In this way, hereditary factors of parents are mixed up and carried over to their offsprings. Darwinian principle states that only the fittest animals can survive in nature. So a pair of fittest parents normally reproduce better offspring.

The above mentioned phenomenon is followed to create a genetic algorithm for an optimization problem. Here potential solution of the problem are analogous with the chromosomes and chromosome of better offspring with the better solution of the problem. Crossover and mutation are performed among a set of potential solutions and a new set of solutions are obtained. It continues until terminating conditions are encountered. Michalewicz [32] proposed a genetic algorithm named the contractive mapping genetic algorithm (CMGA) and proved the asymptotic convergence of the algorithm by the Banach fixed-point theorem. In CMGA, movement from an old population to a new population takes place only when the average fitness of a new population is better than the old one. This algorithm is modified with the help of a fuzzy simulation process to solve the fuzzy stochastic models of this paper. The algorithm is named FSGA and this presented below. In the algorithm, p_c, p_m are probabilities of the crossover and the probability of mutation, respectively, I is the iteration counter, and $P(I)$ is the population of potential solutions for iteration I. The $(P(I))$ function initializes the population $P(I)$ at the time of initialization. The $(P(I))$ function evaluates the fitness of each member of $P(I)$ and at this stage an objective function value due to each solution is evaluated via the fuzzy simulation process (using Algorithm 1 or Algorithm 2). In case of stochastic model (model-1) objective function is evaluated directly without using simulation algorithms. So in that case this GA is named ordinary GA. M is iteration counter in each generation to improve $P(I)$ and M_0 is upper limit of M.

4.2. GA/FSGA algorithm

1. Set $I = 0$, $M = 0$, $M_0 = 50$.
2. Initialize p_c, p_m.
3. Initialize $(P(I))$ and let N' be its size.
4. Evaluate $(P(I))$.
5. While $(M < M_0)$
6. Select N' solutions from $P(I)$ for mating pool using roulette-wheel selection process [32]. Let this set be $P_1(I)$.

Select solutions from $P_1(I)$ for crossover depending on p_c.
Perform crossover on selected solutions to obtain population $P_1(I)$.
Select solutions from $P_1(I)$ for mutation depending on p_m.
Perform mutation on selected solutions to obtain new population $P(I+1)$.
Evaluate $(P(I+1))$.
  Set $M = M + 1$.
  If average fitness of $P(I+1) >$ average fitness of $P(I)$ then
    Set $I = I + 1$.
    Set $M = 0$.
  End If.
End While.
Output: Best solution of $P(I)$.
End algorithm.

4.3. GA/FSGA procedures

(a) *Representation:* A 'n dimensional real vector' $X = (x_1, x_2, \ldots, x_n)$ is used to represent a solution, where $x_1, x_2, \ldots, x_n$ represent n decision variables of the problem.
(b) *Initialization:* N' such solutions $X_1, X_2, X_3, \ldots, X_{N'}$ are randomly generated by random number generator. This solution set is taken as initial population $P(1)$. Here we take $N' = 50$, $p_c = 0.3$, $p_m = 0.2$, $I = 1$. These parametric values are assumed as these give better convergence of the algorithm for the model.
(c) *Fitness value:* Value of the objective function due to the solution X, is taken as fitness of X. Let it be $f(X)$. Objective function is evaluated via fuzzy simulation process (using Algorithm 1 or Algorithm 2) for Model-2.
(d) *Selection process for mating pool:* The following steps are followed for this purpose
  (i) Find total fitness of the population $F = \sum_{i=1}^{N'} f(X_i)$.
  (ii) Calculate the probability of selection p_i of each solution X_i by the formula $p_i = f(X_i)/F$.
  (iii) Calculate the cumulative probability q_i for each solution X_i by the formula $q_i = \sum_{j=1}^{i} p_j$.
  (iv) Generate a random number 'r' from the range $[0, 1]$.
  (v) If $r < q_1$ then select X_1: Otherwise select X_i $(2 \leqslant i \leqslant N)$, where $q_{i-1} \leqslant r \leqslant q_i$.
  (vi) Repeat step (iv) and (v) N' times to select N' solutions from old population. Clearly one solution may be selected more than once.
  (vii) Selected solution set is denoted by $P_1(I)$ in the proposed GA/FSGA algorithm.
(c) *Crossover:*
  (i) *Selection for crossover:* For each solution of $P(I)$ generate a random number r from the range $[0, 1]$. If $r < p_c$ then the solution is taken for crossover, where p_c is the probability of crossover.
  (ii) *Crossover process:* Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the range $[0, 1]$ and their offsprings Y_{11} and Y_{21} are obtained by the formula:

$$Y_{11} = cY_1 + (1 - c)Y_2, \quad Y_{21} = cY_2 + (1 - c)Y_1.$$

(d) *Mutation:*
  (i) *Selection for mutation:* For each solution of $P(I)$ generate a random number r from the range $[0, 1]$. If $r < p_m$ then the solution is taken for mutation, where p_m is the probability of mutation.
  (ii) *Mutation process:* To mutate a solution $X = (x_1, x_2, \ldots, x_n)$ select a random integer r in the range $[1, n]$. Then replace x_r by randomly generated value within the boundary of rth component of X.

5. Numerical illustration

5.1. Stochastic model

To illustrate the models we consider the following numerical data.

$$C_3 = \$50,\quad c = \$5,\quad s = \$9,\quad C_1 = \$1.0,\quad \alpha = 180,\quad q_0 = 100,\quad \lambda = 0.05,\quad s_1 = \$4,\quad r = 0.15,\quad i = 0.05,$$

i.e. $R = 0.1$ in appropriate units.

The optimal values of T along with maximum expected total profit have been calculated for different values of θ and β and the results are displayed in Table 1.

It is observed that for fixed value of θ, as β increases, expected profit increases. And for fixed value of β, as θ (deterioration rate) increases, expected profit decreases. All these observations agree with the reality.

5.2. Fuzzy stochastic model

Here, the resultant inflationary effect is considered as a triangular fuzzy number i.e. $\widetilde{R} = \tilde{r} - \tilde{i} = (0.11, 0.15, 0.19) - (0.04, 0.05, 0.06) = (0.05, 0.1, 0.15)$ and assume $\alpha_1 = 0.9$, $\alpha_2 = 0.05$, $\varepsilon = 10$ and all other data remain same as in stochastic model. The maximum optimistic/pessimistic return from expression (29) and (30) has been calculated for different θ and β, and results are displayed in Table 2.

In this case also same trend of result as in the case of stochastic model is observed.

Table 1
Results for stochastic model

β	θ	T	$E(TP)$
0.50	0.100	0.5887	6296.69
0.55	0.100	0.6072	6364.63
0.60	0.100	0.6229	6442.63
0.50	0.125	0.5888	6128.27
0.55	0.125	0.6034	6205.14
0.60	0.125	0.6156	6289.79
0.50	0.150	0.5824	5992.24
0.55	0.150	0.5949	6073.87
0.60	0.150	0.6061	6161.96

Table 2
Results for fuzzy stochastic model

β	θ	Optimistic return ($\alpha_1 = 0.9$)	Pessimistic return ($\alpha_2 = 0.05$)
0.50	0.100	6621.77	6145.35
0.55	0.100	6694.51	6211.06
0.60	0.100	6777.61	6286.71
0.50	0.125	6445.28	5980.66
0.55	0.125	6527.22	6055.19
0.60	0.125	6617.18	6137.41
0.50	0.150	6302.54	5847.76
0.55	0.150	6389.36	5927.00
0.60	0.150	6482.83	6012.61

Table 3
Sensitivity analysis with respect to present inflation rate for stochastic model

R	Percentage change in R	Percentage change in R expected total profit ($\theta = 0.1$ and $\beta = 0.50$)	Percentage change in expected total profit ($\theta = 0.15$ and $\beta = 0.5$)	Percentage change in expected total profit ($\theta = 0.1$ and $\beta = 0.6$)	Percentage change in expected total profit ($\theta = 0.15$ and $\beta = 0.6$)
0.06	−40	+62.99	+63.18	+63.58	+63.66
0.07	−30	+41.08	+41.21	+41.43	+41.49
0.08	−20	+24.22	+24.30	+24.42	+24.45
0.09	−10	+10.86	+10.89	+10.94	+10.95
0.10	00	0.0 (*6296.69)	0.0 (*5992.23)	0.0 (*6442.63)	0.0 (*6161.96)
0.11	+10	−08.99	−09.02	−09.05	−09.06
0.12	+20	−16.56	−16.61	−16.67	−16.69
0.13	+30	−23.02	−23.09	−23.16	−23.20
0.14	+40	−28.60	−28.69	−28.77	−28.81

Table 4
Sensitivity analysis with respect to the parameter λ for stochastic model

λ	Percentage change in λ	Percentage change in expected total profit ($\theta = 0.1$ and $\beta = 0.5$)	Percentage change in expected total profit ($\theta = 0.15$ and $\beta = 0.5$)	Percentage change in expected total profit ($\theta = 0.1$ and $\beta = 0.6$)	Percentage change in expected total profit ($\theta = 0.15$ and $\beta = 0.6$)
0.030	−40	+19.22	+19.53	+19.42	+19.57
0.035	−30	+14.42	+14.58	+14.51	+14.62
0.040	−20	+09.60	+09.69	+09.61	+09.73
0.045	−10	+04.81	+04.82	+04.81	+04.83
0.050	00	0.0 (*6296.69)	0.0 (*5992.23)	0.0 (*6442.63)	0.0 (*6161.96)
0.055	+10	−04.71	−04.74	−04.73	−4.76
0.060	+20	−09.41	−09.45	−09.43	−09.49
0.065	+30	−14.02	−14.12	−14.04	−14.18
0.070	+40	−18.60	−18.69	−18.63	−18.83

5.3. Sensitivity analysis

A sensitivity analysis is performed for stochastic model with respect to different resultant inflationary effect (R) for crisp inflation and results are presented in Table 3. It is observed that as R increases, profit decreases which agrees with reality.

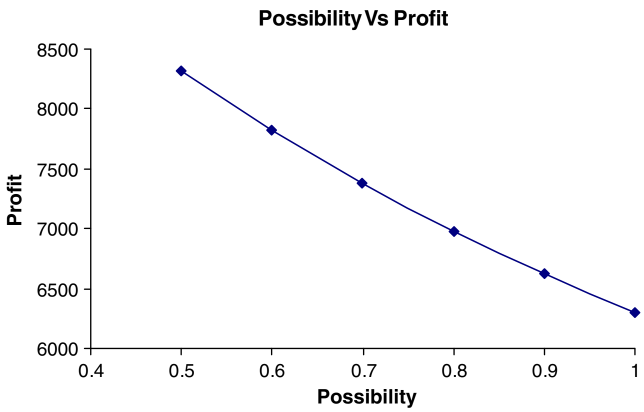

Fig. 2. Possibility vs profit for $\beta = 0.5$ and $\theta = 0.1$.

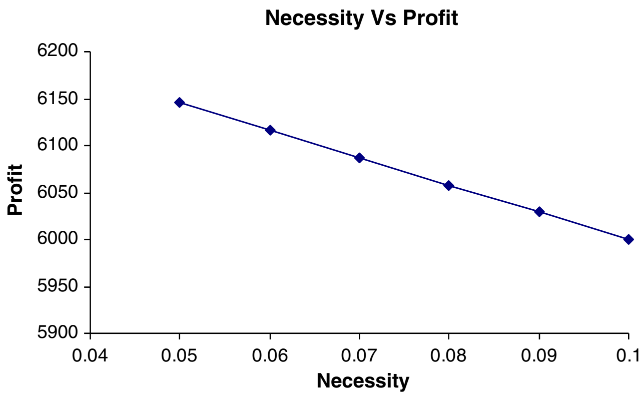

Fig. 3. Necessity vs profit for $\beta = 0.5$ and $\theta = 0.1$.

A sensitivity analysis is performed for the maximum expected total profit with respect to the different values of parameter λ for stochastic model and presented in Table 4. It is observed that as λ decreases, profit increases. This happens because as λ decreases, expected time horizon increases which increases the total expected profit.

Results due to different values of confidence levels α_1 and α_2 for Models-2a and 2b are calculated and plotted in Figs. 2 and 3. In both cases, as expected, profit decreases with the increase of confidence levels.

6. Conclusion

Stock dependent inventory models are normally developed in a finite or infinite time horizon. But for seasonal goods where time horizon is finite but imprecise in nature, it can be estimated as a fuzzy or stochastic parameter. In this paper, for the first time inventory model of a deteriorating item with displayed stock dependent demand has been considered under inflation and time discounting over a stochastic time horizon. Again inflation and discount rate of money are also assumed as imprecise in nature. As a result we consider resultant effect of inflation and time value of money (R) as a fuzzy parameter $\widetilde{R}$. A methodology is suggested for optimization of a fuzzy objective, where instead of the objective function, the optimistic/pessimistic return of the objective is optimized. The methodology presented here is quite general and can be applied to the inventory problems with dynamic demand, allowing shortages, etc.

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