Application of Weighted Principal Component Analysis in Comprehensive Evaluation for Power Quality

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Abstract*-In order to improve comprehensive evaluation of power quality to be more objectively and scientifically, based on weighted Principal Component Analysis, a method of Power Quality comprehensive evaluation for power quality. The method improves the traditional principal component analysis, combining PCA with AHP organically. First, use the improved AHP to determine the weights of original indexes; then, instead of the original indicators of the comprehensive principal component index, use the variance contribution rate of the principal component as the principal component weight; finally, use the linear weighted value of the principal components as a comprehensive assessment. Examples shows that the weighted principal component analysis can evaluate power quality simple, rapidly and accurately.

***Keywords*- PCA; power quality; improved AHP; comprehensive evaluation**

I. INTRODUCTION

With the continuous development of the electricity market, power has been transformed into a special commodity [1]. Users are free to choose the power company; the scientific evaluation of power quality, achieving power price by power quality and realizing a two-way selection are the inherent requirements of power market law [2]. Six existing power quality standards of our country are the voltage deviation, frequency deviation, voltage phase unbalance, voltage fluctuation and flicker, harmonics public, temporary over-voltage and transient over-voltage [3].

There are many kinds of power quality evaluation methods [1, 2, 4], such as, fuzzy method, hierarchical analysis method, principal component analysis (PCA) method, combination weighting method. Most of these methods are committed to use a variety of mathematical methods, integrate more indicators into a single indicator as objective as possible, and further to get the results of comprehensive assessment for power quality, it makes power quality comprehensive evaluation to be a useful exploration. However, these methods need to determine the weight of multi-dimensional index, and it costs large amount of calculation time.

Therefore, this paper present a comprehensive evaluation method for power quality based on Weighted Principal Component Analysis, PCA can reduce data dimensions, eliminate duplication of information between indicators ,make the variance contribution rate of each principal component as the weights. Traditional PCA determines the weight of the principal components only base on survey data, it is too objectivity. Weighted PCA uses analytic hierarchy process （AHP） to determine the weight of each evaluation index, in PCA each principal component is a linear combination of the assessment indicators, regard the linear weighted value of the main components as the result of comprehensive evaluation. Thus, the weighted PCA is the organic combination of objective analysis method and subjective analysis method, it can evaluate power quality scientifically, simple and accurately.

II. DETERMINING THE WEIGHT

A. AHP

1） The basic principle of AHP

Analytical Hierarchy Process (AHP) is a qualitative analysis and quantitative analysis of multi-objective decision analysis methods, created by T.L.Satty who is Operations Research in the United States in the 20th century home of 70 years; it is essentially a method of analysis which can make human judge principled. According to the inter-related and affiliation of factors, we can divide question into different levels, so as to constitute a multi-level system structure model. The steps of using AHP to solve problem are: first, create a hierarchical structure of the problem; secondly, determining matrix structural pairwise comparison; then, judging the relative weight by the Matrix elements; finally, calculate the combination weight of each layer element, and do the consistency test.

2）Improve the scale determining

AHP commonly use 1-9 scale, but the 1-9 score scale usually inconsistent with the language, lack of coordination may lead to errors between evaluation findings and consistency test. Many experts provide different scales. The following uses method in the literature[4] to create a new index scale, dividing the determine class into six levels :equally important, somewhat important, important, obviously important, strong important and extremely important .Supposing the score of

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equally important is 1,the score of somewhat important is λ ,the score of important is λ^2 ,the score of obviously important is λ^4, the score of strong important is λ^6, the score of vital important is λ^8 .The new index scale can meet the transitive. Because the limit on Judgments is 9, λ^8 =9 , λ=1.3161 (Note: The values above λ^8 are denoted by 9) [4]. The index scale obtained from the above principle is shown in Table Ⅰ.

TABLE I. EXPONENTIAL SCALE AND ITS MEANING

scale	Scale meaning
a_{ij} =1	u_i is equally important to u_j
a_{ij} =1.3161	u_i is somewhat important than u_j
a_{ij} =1.7321	u_i is important than u_j
a_{ij} =3	u_i is obviously important than u_j
a_{ij} =5.1966	u_i is strong important than u_j
a_{ij} =9	u_i is vital important than u_j

3) Construction judgment matrix

According to national standards and status of power quality, power quality sub-indicators to be assessed are voltage deviation, frequency deviation, three phase imbalance, voltage total harmonic distortion, short-time flicker, sub-indicators to be assessed and the variable names are shown in Table Ⅱ. Suppose there are n indicators to be assessed, establish the judgment matrix by pairwise comparison according to Table Ⅰ.

TABLE II. EVALUATION INDEXES

Evaluation indexes	Variable
voltage deviation	X_1
frequency deviation	X_2
Three phase imbalance	X_3
voltage total harmonic distortion	X_4
short-term flicker	X_5

4) Consistency test

Under normal circumstance, we use consistency indicator $C.R.$ to measure the advantages and disadvantages of the scale.

Suppose

$$C.I. = \frac{\lambda_{\max} - n}{n-1} \tag{1}$$

$C.I.$ should be as small as possible.

Saaty gives the value of average consistency test $R.I.$. Repeat the trial 1000 times, calculating average value of the largest eigenvalue for random matrix A to get the average random consistency test indicator, it is shown in Table Ⅲ.

TABLE III. RANDOM INDEX OF N-ORDER MATRIX

order	1	2	3	4	5
$R.I.$	0	0	0.52	0.89	1.12
order	**6**	**7**	**8**	**9**	**10**
$R.I.$	1.26	1.36	1.41	1.46	1.49

Suppose

$$C.R. = \frac{C.I.}{R.I.} \tag{2}$$

When $C.R. < 0.1$, the consistent of comparison matrix A can be accepted.

When $C.R. > 0.1$, AHP is no longer applicable, we should change the hierarchical structure or revalue matrix A .

5) calculating the weights determine by AHP

When judgment Matrix meets the consistency test conditions, the "normalization" eigenvectors of A corresponding to $\lambda_{\max}$, can give a mean weight w of n indexes $x_1, x_2, \cdots, x_n$, $w = (w_1, w_2, \cdots, w_5)$.

B. PCA

Supposes the original variable is $x_1, x_2, \cdots, x_n$, after principal components analysis, the new variables is $z_1, z_2, \cdots z_m$ (m<n) , they are the liner combination of $x_1, x_2, \cdots, x_n$. New variable $z_1, z_2, \cdots z_m$ is the original form of the coordinate system and orthogonal coordinate system rotated by the translation obtained. The space which is formed by $z_1, z_2, \cdots z_m$ is called m-dimensional main plane space. In the hyperplane main space, the first principal component F_1 corresponds to the largest data variation (contribution rate e_1) direction, the relationship between the contribution rate corresponding to the principal component $z_2, \cdots z_m$ may be $e_2 \geq \cdots \geq e_m$. Therefore, variable $z_1, z_2, \cdots z_m$ carry the most information of original data, m-dimensional hyperplane space is m-dimensional sub-space which retains the largest information of original data [5].

The main step of PCA is shown as follows:

1) The mathematical model of PCA

Set a certain random variable x which contains P indicators, $x = (x_1, x_2 \cdots x_p)$. Sampling N points of variables and get the original data matrix:

$$x = (x_1, x_2 \cdots x_p) = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & & \vdots \\ x_{n1} & \cdots x_{n2} & \cdots & x_{np} \end{bmatrix} = (x_{ij})_{n \times p}$$

Where x_{ij} on behalf of the value of index j in sampling point i.

PCA simplify the complex relationships among variables for analysis, under the principle of trying to secure a minimum loss of data information, it can best integrated simplify the cross-section data tables of multi-variable, that is, it can reduce the dimensional of the high-dimensional variable space.

2) Data Standardization

In order to eliminate the influence which various brings from the difference of targets magnitude and the dimension, using the formula (3) to do the standardized process.

$$X_{ij} = (x_{ij} - \bar{x}_i) / \sigma_i \tag{3}$$

In the formula: x_{ij} is the primary data of index i in sampling point j; $\bar{x}_i$、σ_i are respectively on behalf of sample mean and the standard deviation of index i. After data standardization, obtains matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1p} \\ X_{21} & X_{22} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{np} \end{bmatrix} = (X_1, X_2, \cdots, X_p) ,$$

And X_i is the ith row vector of X, $i = 1, \cdots, p$.

3) Calculate the correlation matrix R of matrix X.

4) Calculate the eigenvalues of correlation matrix R : $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_5$, as well as the orthonormal eigenvectors $u_1, u_2, \cdots u_5$ which are corresponded to the characteristic values.

5) Find the principal components: $z_k = \sum_{j=1}^{5} u_{kj} X_j$, k=1,2,…,5, Where X_j is the standardized indicator j.

6) Calculate cumulative variance contribution rate $E = \sum_{k=1}^{m} \lambda_k / \sum_{j=1}^{5} \lambda_j$,the value of E is on behalf of the percentage of the first m principal components' information responds to the total information. m generally takes the minimum m which satisfy to $E \geq 85\%$.

7) Using the variance contribution rate of each principal component to be its weight. The comprehensive evaluation index value is $Z = \sum_{k=1}^{m} \left[\lambda_k / \sum_{j=1}^{5} \lambda_j \right] * \left[\sum_{j=1}^{5} u_{kj} X_j \right]$. We can sort the comprehensive evaluation results according to the value of Z.

C. The realize principle of weighted PCA

Weighted standardized index is: $X_j^* = w_j X_j$, j=1,2,3,4,5, where w_j is on behalf of the important weight of index i determined by AHP. So the new principal component is $z_k = \sum_{j=1}^{5} u_{kj} X_j^*$, k=1, 2,3,4,5.

Comprehensive evaluate value which is determined by PCA is $Z = \sum_{k=1}^{m} [\lambda_k / \sum_{j=1}^{5} \lambda_j] * [\sum_{j=1}^{5} u_{kj} X_j^*]$.

III. MODEL SOLUTION

A. The evaluation system of power quality

Comprehensive evaluation of power quality [6] is necessary to consider the services index, but also consider the technical indicators, and power quality can be measured from the voltage deviation, voltage fluctuation and flicker, voltage waveform distortion, voltage unbalance, and phase-frequency over-voltage, etc. Frequency quality can be measured in terms of frequency deviation. For convenience, this article only use the voltage quality and the frequency quality to evaluate power quality, the main technical indicators are shown in Table 2.

B. The Application of Weighted PCA

According to the evaluation system established, using the weighted PCA to comprehensive evaluate the six national technical standards of the all monitoring points in countries. Assessment object is the voltage quality and the frequency quality of 10 monitoring points for a distribution network, the index data is shown in Table Ⅳ [7].

TABLE IV. VALUES OF POWER QUALITY INDICATORS

monitoring points	voltage deviation （%）	frequency deviation （%）	Three-phase unbalance degree（%）	Voltage total harmonic distortion （%）	Short-time flicker （%）
A	7.24	1.36	0.7976	4.0539	0.2883
B	6.64	2.04	0.8312	4.0061	0.3546
C	8.39	1.89	0.8244	4.0187	0.1685
D	7.69	1.23	0.7675	4.0428	0.2398
E	7.58	1.17	0.7992	4.0356	0.2659
F	7.08	1.76	0.8106	4.0057	0.3107
G	6.69	1.98	0.8019	4.0126	0.3308
H	7.31	1.31	0.7981	4.0359	0.2734
I	7.06	1.72	0.8004	4.0279	0.3103
J	6.94	1.92	0.7998	4.0103	0.3504

TABLE V. THE RESULTS OF PCA

principal component	eigenvalue	variance contribution rate （%）	cumulative variance contribution rate （%）
F1	3.0286	60.57	60.57
F2	1.467	29.34	89.91
F3	0.34423	6.88	96.80
F4	0.1384	2.77	99.57
F5	0.021741	0.43	100

In accordance with the steps of weighted PCA, the eigenvalue, variance contribution rate and cumulative variance contribution rate of the principal components are calculated by the matlab programming, as shown in Table Ⅴ.

We can know from Table Ⅴ that $(\lambda_1 + \lambda_2) / \sum_{j=1}^{5} \lambda_j = 0.8991 > 85\%$. Therefore, the first two principal components can approximately replace other indicators to do the comprehensive evaluation.

Using Matrix A of AHP:

$$A=\begin{bmatrix} 1 & 0.5773 & 1 & 1 & 0.7598 \\ 1.7321 & 1 & 1 & 1.3161 & 1.3161 \\ 1 & 1 & 1 & 1 & 0.7598 \\ 1 & 0.7598 & 1 & 1 & 0.7598 \\ 1.3161 & 0.7598 & 1.3161 & 1.3161 & 1 \end{bmatrix}$$

Calculated: the largest eigenvalue of determining matrix A is $\lambda \max = 5.0334$, $C.R. = \frac{C.I.}{R.I.}$ =0.0075<0.1, it can meet the consistency test conditions. W= （-0.37007， -0.54826， -0.41612， -0.38995， -0.48707）.

The first two principal components relation by the standardized eigenvectors corresponding to λ_1, λ_2 is:

$$z_1 = -0.706X_1^* + 0.101X_2^* + 0.126X_3^* + 0.542X_4^* + 0.427X_5^* \quad (4)$$

$$z_2 = -0.10X_1^* - 0.643X_2^* + 0.509X_3^* + 0.263X_4^* - 0.498X_5^* \quad (5)$$

$$Z = 0.6057z_1 + 0.2934z_2 \quad (6)$$

According to the principle that is mentioned above, the comprehensive evaluation result of power quality monitoring points are shown in TableVI.

TABLE VI. THE COMPREHENSIVE EVALUATION RESULT OF ALL POINTS(A-J)

component	z1	z2	Z
A	0.858	0.233	0.588
B	0.658	0.465	0.535
C	1.160	0.432	0.829
D	0.997	0.200	0.663
E	0.966	0.175	0.636
F	0.799	0.376	0.594
G	0.680	0.445	0.542
H	0.886	0.217	0.600
I	0.792	0.361	0.586
J	0.745	0.439	0.580

IV. RESULT ANALYSIS

From the expression of first principal component we can see, voltage deviation, voltage total harmonic distortion and short-time flicker have greatest impact on the first principal component. From the expression of the second principal components we can see, frequency deviation, phase imbalance, short-time flicker have a larger influence on the second principal components. Therefore, the controlling of voltage deviations can improve the power quality evaluation result significantly.

V. CONCLUSION

The weighted principal component analysis of power quality comprehensive evaluation method, its principle is clear, simple, determining the weight of each index by the improved AHP and using the variance contribution rate of the main components as the weight of each main component, it can increase the objective and scientific of weight effectively. Compared with other comprehensive evaluation methods, weighted PCA has played a significant role in reducing dimension, greatly reducing the amount of calculation and retains most information of the original data. Experiments show that using the weighted PCA can obtain the result of power quality comprehensive evaluation intuitively and scientifically.

Shortcomings:

1) The PCA which is used in this article only retain 89.91% of the total amount information, a large amount of information loss, we can improve the PCA, reduction dimensionality at the same time loss the minimum amount of information.

2) The difference between the evaluation results is too small , it is not easy to distinguish.

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