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Strategic planning for value-based management

An empirical examination

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Abstract *This paper developed a model to identify empirically the critical strategic variables for value-based management. Since “value” is abstract in its nature, managers need to know more concrete and clear target measures that derive the value of their business. In this model common financial variables and their variations are used as input variables to synthesize the market value added (MVA). The model used an artificial intelligence technique that is non-parametric by nature and intended to capture a dynamic relationship between input and output variables. The study results show that this model is able to identify a set of strategic variables that are linked directly to the MVA of companies involved.*

Introduction

Financial managers and top executives are aware of the importance of “value-based management”. They have been under constant pressure from market participants who are actively involved in company operations through leveraged buyouts, hostile take-over, and proxy contests. Top executives have also been increasingly engaged in the financial markets through mergers and acquisitions, restructuring, leveraged buyouts and share repurchases (Copeland *et al.*, 2000). CFOs of large public companies are very much familiar with diverse value-based management methods, with the majority of those utilizing a method developing it internally. However, they admitted that there is room for improvement in the design and implementation of value-based management systems (Ryan and Trahan, 1999). We have seen the consequences of ignoring value-based management in corporate strategies and governmental policies: hostile takeovers in the US in 1980s, financial crises in Southeast Asian countries in 1998, and a collapse of Internet companies in 2000. When management focuses on building shareholder value, it creates much healthier companies, which are not only good for shareholders and other stakeholders, but also for the economy.

Recent surveys indicated that many public and private companies have strong interests in managing for value. Ramirez *et al.* (1991) surveyed Fortune 500 CFOs and managers and showed concerns about the prevailing myopic view of Wall Street, which concentrate on short-term performance while ignoring long-term benefits. Trahan and Gitman (1995) surveyed Fortune 500 and Forbes 200 small company CFOs and found out that CFOs have strong desires to know more about the impact of financial decisions on stock value. In addition, small company CFOs wanted to know more about impacts of cash flow, institutional and managerial ownership,



and short-sighted management on stock price. The survey by Davis (1996) also showed that executives would like to focus on cash flow and performance measures. All these surveys indicated that executives are strongly interested in creating shareholder value by generating earnings and cash flow above their cost of capital used.

In order to implement value-based management, managers must have concrete and clear target measures, since “value” is abstract and vague in its nature. They need to know specifically the strategic variables that derive the value of their business so that they can develop strategic planning and track its progress. Managers have been using return on assets, profit margin (PM), return on shareholders’ equity, earnings-per-share growth rate, and return on total capital (Ramanujam *et al.*, 1986; Schendel and Patton, 1978; Singh, 1986; Varadarajan and Ramanujam, 1987). Many corporations adopted discounted cash flow (DCF) approach publicized by LEK/Alcar Consulting Group (Fisher, 1995; Lieber, 1996), cash flow return on investment (CFROI) popularized by BCG-HOLT (Peterson and Peterson, 1996), and return on invested capital (ROIC) (Copeland *et al.*, 2000; Rappaport, 1986). Myers (1996, 1997) and Stewart (1994) illustrated diverse models used for value-based management and discussed which models work better. Although these measures could be coincidentally related to stock prices, they are not necessarily considered to be primary movers of market values. Security price movements (Lubatkin and Shrieves, 1986) have also been used to measure corporate performance. However, security price movements are considered inadequate to gauge the capability of an organization’s managers, and these movements cannot cope with unsystematic risk such as barriers to entry and other competitive factors within an industry. These traditional performance measures have long been criticized primarily because they do not provide proper guidance for strategic decisions and value management of corporations. In recent years, two measures of corporate performance, market value added (MVA) and economic value added (EVA) (Peterson and Peterson, 1996; Stewart, 1991), have been attracting much attention from both investors and corporate managers.

The purpose of the paper is to identify empirically the variables that contribute significantly to market value determination. Since value is abstract in nature, managers need to have a specific operational guide for implementing value management. Managers and CEOs have a strong affinity for common accounting and financial measures and have been using them as guidance for value management because these measures are very intuitive, operational, and easy to understand. However, these practices have been confusing at best because these common measures are known to be unrelated to the value of corporation.

In this paper, a model is built to examine if common financial measures are related to the value of corporations. More specifically, this paper is concerned with identifying strategic variables that provide operational guidance for value management. Once these common measures are identified, they can be used to estimate the value of operating efficiency of a corporation, represented by the MVA (Stewart, 1994). This model could ultimately provide ways of developing corporate planning with a focus and control on value management.

Multivariate statistical techniques are not considered to be appropriate in this research because majority of input variables are correlated among themselves in their nature, and the relationship between individual financial measures and MVA would likely be dynamic and non-linear. Instead, an adaptive learning network (ALN)

approach, an artificial intelligence (AI) technique, is used to develop a model primarily, because it is nonparametric and captures dynamic relationships between input and output variables. In the next section, MVA and EVA are briefly discussed and essential characteristics of the ALN approach are presented.

MVA and EVA revisited

The Stern Stewart (SS) & Company publishes “The Stern Stewart Performance 1000”, (Walbert, 1994) in which the 1,000 largest publicly-owned US industrial and non-financial service companies are ranked according to the MVA. The SS company defines MVA as the difference between a company’s total market value of both debt and equity of the firm and the amount that investors have contributed to produce that value (or its book value). MVA is considered as the amount of wealth a firm’s management creates from the capital that investors have entrusted to management. It is also viewed as the market value assessed in the security market of the company’s internal operating efficiency (Walbert, 1994) and, therefore, can be used as a single comprehensive measure for assessing the value of the management’s performance. A positive MVA, for example, represents the amount of wealth the company has created, while a negative MVA shows the amount of capital which management has dissolved. MVA is consistent with shareholders’ wealth maximization in which both the risk and the expected net cash flows in the future are reflected.

The SS Company’s report also presents EVA as the amount of wealth a firm creates for its shareholders in a given year. EVA is defined as a firm’s aftertax net operating profit in a given year minus its cost of capital that year. An economic book value is different from its accounting book value; it includes items such as bad debt reserves and deferred income taxes, and capitalizes R&D spending, amortizing the costs over the five years. Capital cost consists of the costs of debt and equity, applied to total capital at the beginning of the year. Unlike traditional accounting measures of performance, EVA is viewed as value that firms create or dissolve from the capital entrusted to management for that year (Lehn and Hakhija, 1996). Proponents of these two measures contend that managers should use EVA as their key measure of internal performance in a given year and as the driver of their business decisions. They also assert that EVA drives MVA and is more closely correlated with MVA.

Adaptive learning network

Barron *et al.* (1984) developed the ALN approach, as a refinement of the group method of data handling (GMDH) algorithm of the Ivakhnenko and Ivakhnenko (1974), from using advanced statistics, expert systems, and AI research including neural networks. ALN performs a traditional task of fitting model coefficients to bases of observed data in a network form. The network structure of the model resembles the neurons and synapses of a human brain, and uses mathematical functions that represent numeric knowledge on each processing unit (nodes). The network consists of a number of processing units and interconnections between the units, and each node is represented by a polynomial of n variables in which all cross products and combinations of the variables to different degrees are included.

Montgomery (1989) developed an effective computer-based algorithm, called the abductive induction mechanism (AIM™), from which the final solution is synthesized in the form of a network. The final model is a layered network of feed-forward

functional elements in which the coefficients, number and types of network elements, and the connectivity are learned inductively and automatically. Each processing unit (node) has a unique equation of multi-variable configurations: singles, doubles, triples, normalizers, white elements, unitizers, and wire elements (Montgomery, 1989). Normalizers transform the original input variables into standardized normal variables with a mean of zero and a variance of one. The white element is a linear combination of all inputs to the current layer. Unitizers convert the normalized data back into the original data to assess the output values.

The algebraic forms of singles, doubles, and triples (Appendix 1) are homogeneous multinomials of degree 3 in one, two, and three variables and allow interaction among input variables. It is well known that a suitably high degree multinomial – a polynomial of n variables in which all cross products and combinations of the variables to differing degrees appear – can approximate arbitrary functions of many variables accurately (Barron *et al.*, 1984). However, all these terms in the equation may not always appear in a node since non-contributing terms to output are automatically thrown out. The output of elements in one layer will then feed into subsequent layers together with other original input variables. Networks are synthesized from layer-to-layer until the model ceases to improve based on predicted squared error (PSE) criterion.

The objective of the ALN algorithm is to train and identify the final model that minimizes the PSE, the errors on as yet unforeseen data without overfitting the data (Barron, 1981, 1984). Assuming that the “covariance structure” of training observations is nearly the same as for future observations, the PSE is defined as the sum of the training squared error (TSE) and overfit penalty as shown in the following:

$$\text{PSE} = \text{TSE} + 2\sigma_p^2 \frac{K}{N}$$

where TSE is the average squared error of the model on the training sample observations, K is the number of coefficients that are estimated to minimize TSE, σ_p^2 is the priori estimate of true error variance on the validating observations, and N is the size of the training sample observation. The minimum PSE is always attainable because as additional coefficients are added to the model, TSE decreases at a decreasing rate while the overfit penalty increases linearly. If the adaptive model is obtained by minimizing TSE alone, the model will perform well on the training data set, but it can perform poorly on test samples. When the model has an overly complex structure and many coefficients, it will give a poor estimate of error on the test data set. By adding a term for overfit penalty, the minimum expected squared difference between the estimated model and the true model on the future data set can be obtained (Barron, 1984).

The ALN technique is a powerful supervised inductive learning tool, which can reveal subtle relationships that are not otherwise apparent within the framework of a multivariate statistical technique. The power of the network lies in its ability to decompose complex problems into much smaller and simpler ones, and to solve them. The network structure makes decision-making much easier because the number of factors to consider and the alternatives to evaluate become much smaller. The use and maintenance of the model is also easy because the best network structure, node types, and coefficients are automatically selected depending upon the data set used.

Model and data description

MVA is the value of a firm's operations (Brigham and Ehrhardt, 2002, p. 475; Copland *et al.*, 2000, p. 144), which take into account both incurred expenses and the opportunity cost of capital employed in the business, and is used as the output (dependent) variable in this study. The major contributing variables to MVA are the return on investment (ROI), weighted average cost of capital (WACC) and the size of the invested capital (CAPITAL), which constitute economic profit (Copland *et al.*, 2000, p. 143). Since the value of a company is defined as sum of the invested capital and the additional value of management that has been added, it must depend primarily on the spread between ROI and WACC. If a company earns more than its WACC, management is adding value and, consequently, the company is worth more than its capital invested, and vice versa. Operating margin (OM), PM, sales growth, and return on capital (ROC) could also contribute to the MVA determination and, therefore, were included as input variables in the model. BETA was included as a risk measure for the model. Additional variables, including market-to-book value ratio (MB), price-earnings ratio (PE), and EVA, are also used as input variables to examine if they contribute to the value of the firm.

The input data for these variables were collected primarily from the SS Company's Performance 1000 data and COMPUSTAT database for 1999. The variables, CAPITAL, MB, PE and WACC were the average figures over the past three years in order to assess long-term impacts of input variables on MVA. In this research three-year averages are used because the prediction outcomes turned out to be much better than those when used four- and five-year average data as input variables. The standard deviation of these input variables was also used as inputs to the model. The data set started initially with 1,000 of the largest publicly owned US companies that are included in the Stern Stewart Performance 1000. The companies in the financial service industry were deleted from this sample since they have entirely different asset and financial structure. The observations which contain several missing data were also eliminated. The final sample of 608 observations was obtained and used for this study. Seventy five percent of the total sample was randomly selected and used for training because a large sample is required to obtain an accurate training of the final model. Twenty five percent of the total sample was used for testing the model.

Empirical results

The final ALN model is synthesized from the training data set to minimize the PSE and shown in Figure 1. It is a layered network of feed-forward functional elements, which contains the best network structure, node types, coefficients, and connectivity to minimize the PSE without overfitting the data. The final model uses five different input variables that contribute significantly to the MVA determination with using the EVA twice in the network. The ALN model includes nodes such as singles, doubles, and triples as a part of the network, and each node is represented by an equation with estimated coefficients.

The equations in Appendix 2 show the final ALN in polynomial equation forms, and each equation number represents the node number of the ALN in Figure 1. In Figure 1, the five input variables with one repeat are first transformed into standardized normal variables with the mean of zero and a variance of one using Normalizers in the Appendix 2. These standardized variables are next fed into the first layer to generate

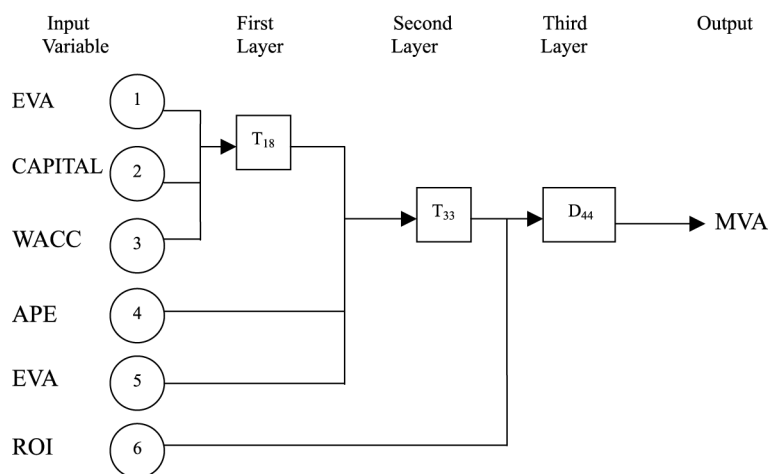


Figure 1.
Adaptive network model

a series of intermediate output values. For example, the node T₁₈ in Figure 1 was synthesized using normalized values of EVA, CAPITAL, and WACC, and then fed into the node T₃₃ with two other inputs: average PE ratio (APE) and EVA. The value of the node T₃₃ with ROI are again fed into the subsequent node D₄₄ in the third layer, and finally converted back to MVA with the mean and variance of the original output variables. This final ALN model currently becomes a knowledge base from which a series of MVA values can be estimated using the five input variables.

The prediction result of the final ALN together with multiple regression models is shown in Table I. The result of the multiple regression approach is shown only for comparison purpose in this exhibit. The results indicate that the ALN model outperformed the linear regression approach for predicting MVA of firms. The *R*-square (the squared value of the Pearson Correlation) between the actual and predicted values by the ALN model is 87.87 percent, which is three times higher than that of the linear regression; the *R*-square of the linear regression is only 28.77 percent. All of the prediction statistics in Table I clearly indicate that the ALN model estimates MVA much more accurately and consistently than the regression model is able to estimate.

The ALN model selected WACC, ROI, APE, EVA, and CAPITAL for synthesizing the final model as shown in Table II. In order to create values from operating activities, the company's ROI must be larger than WACC. In addition, the size of capital investment is very important because it will determine the scale of the value creation,

	ALN	Regression
Sample	152	152
Average absolute error	4,578.7	10,337
Error standard deviation	10,560	19,237
Average squared error	131,755,010	474,467,606
Squared error standard deviation	18,215,159	415,577,099
<i>R</i> -squared	0.8787	0.2877

Table I.
Prediction statistics by the ALN and regression approaches

Table II.
Variable sensitivity and
importance analysis

Selected variables	ALN model		Linear regression	
	Sensitivity	Importance	Sensitivity	Importance
WACC	0.9604	0.9564	0.5626	0.2913
ROI	(0.0320)	0.0301		
APE	(0.0048)	0.0007		
AEVA	0.0025	0.0042		
CAPITAL	0.0002	0.0086		
BETA			0.2199	0.5884
AMB			0.0563	0.0481
AOM			0.0458	0.0111
APM			(0.0405)	0.0128
AROI			0.0281	0.0062

Note: () indicates negative impact on the predicted variable

given the spread between ROI and WACC. These three variables are essential components of economic profit and are considered to be critical variables in corporate value management. Kim (2002) also supports the above findings such that WACC and CAPITAL are primary contributing variables to MVA determination. Average PE (APE) ratio and average EVA were also selected to construct the final ALN model, but these variables turned out to have little impact on MVA as they varied.

In Table II the sensitivity and importance analysis for each selected input variable is presented. The sensitivity value indicates the sensitivity of the model output value to changes in each input value at a mid point of the predicted variable. The importance value indicates the expected overall contribution of each input variable to the predicted output value changes, which is standardized by the total output changes possible. For example, sensitivity values in Table II indicate that WACC is the dominant variable that influences the determination of MVA; WACC explained over 96 percent of total variation of MVA at a mid point of the predicted value. The importance value also showed the dominance of WACC. It indicates that WACC explained over 96 percent of total MVA variations while only 3.2 percent of MVA changes are explained by the ROI variation. The rest of the variables, APE, AEVA and CAPITAL, turned out to be almost negligible in their contributions as they varied. Conversely, the results of the linear regression are quite different. According to the Sensitivity analysis, WACC explained about 56 percent of the variation of MVA while BETA explained 22 percent. As to the overall importance, BETA supersedes all other variables in its influence since BETA explains 59 percent of total MVA variation possible. The next influential variable is WACC that determined 29 percent of the predicted variable.

The results of the study provide important implications for corporate strategic planning. The fact that WACC is the most dominant strategic variables among six important strategic variables selected indicates that corporate managers and executives should find ways of enhancing the value of the company's WACC in order to increase its MVA. Its positive relationship with MVA at a midpoint of the predicted value indicates that there exist value-creating opportunities when companies undertake risky projects. When a company undertakes risky project(s) and is successful in generating large excess profits above its WACC, its EVA automatically will go up. Then, capital markets should react positively to the firms' operating results and will provide rewards to the firms. As a result, both the MVA and average PE ratios

of the company will rise automatically, *ceteris paribus*. This relationship allows management to use EVA and APE ratios as tracking variables to monitor the progress of the value-making strategy by using WACC. This finding is contrary to the general belief that high MVA is attributed to a low WACC since the value of a company depends largely on the spread between ROI and WACC.

Both ROI and the size of capital investment (CAPITAL) were also selected to synthesize the final ALN model, but their impacts were very limited. ROI was able to explain only about 3 percent of the output variation, while CAPITAL variable could only explain far less than 1 percent of the output variation. Consequently, these variables should not be used as target variables for strategic management. Growth rate in sales could have a positive impact on value creation, provided the company is profitable, but it turned out that it had no effect at all on MVA.

The final ALN model identified six important strategic variables that contribute to the value of the firm, and their functional relationships with the value of the firm involved. Practitioners can currently use this final ALN model as an expert systems model to assess the company's MVA and to develop a strategic planning for value creation. While using this model does not require having any knowledge about the ALN model, users can simply plug in six input variables into this final ALN model to assess the value of the firm. They can currently examine how sensitive the company's MVA is to the change in each input variable at this juncture and to develop a feasible strategic plan for the value creation using sensitivity of each variable involved. Users should also find out what it takes to change individual input variables in order to develop a feasible and optimal strategic plan(s).

Concluding remarks

This study has shown that the component variables of economic profit are linked directly to the value of a firm and could be used to estimate the operating efficiency of corporations, represented by MVA. Furthermore, critical strategic variables that drive the value of corporations have been identified. The study results indicate that companies with high WACC were able to create more values from operations, but it is only possible if a company maintains appropriate excess return beyond its WACC in order to build up values from operations. The size of CAPITAL investment cannot be ignored because it is also directly associated with the scale of the value creation. These variables are concrete and directly manageable by managers and can be used when establishing strategic planning for value management. Although economic profit is known to be a short-term performance measure, it can be used to assess an intrinsic value of a corporation, which is normally driven by the long-term cash flow generating ability of the company.

This study used a sample of 608 observations, which encompassed a wide range of industries in manufacturing. However, each industry has somewhat different asset and financial structures, and consequently it could be worthwhile to examine the prediction outcome of MVA when the final model is trained and tested within the confines of particular industry or similar industries. Since individual industries have their own common characteristics in financial structure, we can safely conjecture that the prediction results would be significantly improved if the sample size is large enough for both training and testing.

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Appendix 1. Algebraic form of equations

$$\text{Single} = W_0 + W_1 \cdot X_1 + W_2 \cdot X_1^2 + W_3 \cdot X_1^3$$

$$\text{Double} = W_0 + W_1 \cdot X_1 + W_2 \cdot X_2 + W_3 \cdot X_1^2 + W_4 \cdot X_2^2 + W_5 \cdot X_1 \cdot X_2 + W_6 \cdot X_1^3 + W_7 \cdot X_2^3$$

$$\begin{aligned} \text{Triple} = & W_0 + W_1 \cdot X_1 + W_2 \cdot X_2 + W_3 \cdot X_3 + W_4 \cdot X_1^2 + W_5 \cdot X_2^2 + W_6 \cdot X_3^2, \\ & + W_7 \cdot X_1 \cdot X_2 + W_8 \cdot X_1 \cdot X_3 + W_9 \cdot X_2 \cdot X_3 + W_{10} \cdot X_1 \cdot X_2 \cdot X_3 \\ & + W_{11} \cdot X_1^3 + W_{12} \cdot X_2^3 + W_{13} \cdot X_3^3 \end{aligned}$$

where X_i and W_i denote input variables and coefficients, respectively.

Appendix 2. Adaptive network equations

Normalizers:

1. AEVA = $-0.0241 + 0.0013X_1$
2. CAPITAL = $-0.4369 + 1.0E-4X_1$
3. WACC = $-4.2504 + 0.4466X_1$
4. APE = $-0.2221 + 0.0157X_1$
5. ROI = $-0.6071 + 0.0664X_1$

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Triples:

$$\begin{aligned} 18. \text{ TRIPLE} = & -0.939 + 0.4612X_1 + 0.2191X_1^2 - 0.0313X_1^3 + 0.5721X_2 \\ & + 0.04X_1X_2 - 0.0182X_1^2X_2 - 0.0053X_2^2 + 0.0124X_1X_2^2 \\ & - 0.0083X_2^3 + 0.549X_3 + 0.1572X_1X_3 - 0.0983X_1^2X_3 \\ & + 1.1579X_2X_3 + 0.2262X_1X_2X_3 - 0.1927X_2^2X_3 \\ & + 0.2543X_3^2 + 0.0548X_1X_3^2 + 0.5175X_2X_3^2 - 0.0155X_3^3 \end{aligned}$$

$$\begin{aligned} 33. \text{ TRIPLE} = & -0.0611 + 0.7558X_1 + 0.0925X_1^2 - 0.0129X_1^3 - 0.1666X_1^2X_2 \\ & + 0.048X_3 + 0.0131X_1^2X_3 + 0.2011X_1X_2X_3 + 0.04X_3^2 \\ & - 0.0171X_1X_3^2 + 0.0944X_2X_3^2 - 0.0043X_3^3 \end{aligned}$$

Doubles:

$$\begin{aligned} 44. \text{ DOUBLE} = & 1.0383X_1 + 0.0955X_2 + 0.2693X_1X_2 - 0.0346X_1^2X_2 - 0.0325X_2^2 \\ & - 0.1315X_1X_2^2 \end{aligned}$$

Unitizers:

$$16. \text{ MVA} = 11442.2407 + 31852.2926X_1$$

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