CONCEPT AND MATHEMATICS OF ISLAMIC VALUATION AND FINANCIAL ENGINEERING

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ABSTRACT

We argue that Islamic principles, in particular the avoidance of *ribā* and *gharar* should be applied with respect to *real* economic value rather than to monetary value in terms of conventional currency.

In order to reconcile monetary value with economic value, we propose a reference currency linked to an appropriate commodity basket, reflecting the *common* economic realities and needs of the respective monetary union. Based on this currency, real economic value can be computed in analogy with conventional financial engineering methods, but on a different reference assets, which are commodity-linked rather than interest-based.

In order to reflect global economic needs and realities, a global reference currency should be linked to a basket of commodities including in particular the natural resources necessary to ensure both sustainable survival of mankind and a sustainable living standard above poverty.

Referring to the recent financial crisis of the European Union (EU), we argue that apart from the *common* economic realities and needs within a given socio-political union, such as the Organization of Islamic Cooperation (OIC) member countries, also the also the *different* realities and needs should be honoured appropriately. We propose a 3-level construction of reference currencies, reflecting the economic realities and needs globally, for each region, and for each country.

The previously defined multilevel currency on the basis of a basket of economic key-commodities is proposed as a suitable numéraire for asset-based valuation, particularly suitable in the context of Islamic financial engineering.

We compare conventional financial engineering, based on bank account and zero-bond numéraires as computed from fixed income forward contracts, with Islamic financial engineering based on numéraires computed from commodity-based *bay'u s-salam* or/and forward contracts.

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I. INTRODUCTION

Recent decades have witnessed an explosion in financial innovation and engineering of novel contracts. First of all, from the so-called 'conventional banking system' emerged increasing need to adapt to the challenges from the demand side. Secondly, particularly also within the so-called 'Islamic finance industry', the incentive to innovate around prohibited and disadvantaged transactions has been unfortunately high.

A clash between abused financial engineering and Islamic principles seems to prevail, not only since the renaissance of Islamic finance, and not in the Muslim world only. In medieval Europe canonical law prohibited usury, i.e. *ribā*. European merchants however used a combination of contracts called *contractum trinius* to circumvent prohibition of usury. The construction uses put-call parity to synthesize a conventional interest-bearing loan. During the recent renaissance of Islamic finance, this construction prevailed in practices of some so-called 'Islamic' banks, when they include into *murābaḥa* contracts a combination of conditions similar to *contractum trinius*, thus effectively ensuring and concealing a risk-free profit, which might also be called 'regulatory arbitrage'. Exceeding considerably the inflation rate, such a so-called 'risk-free return' is nothing else but usury, i.e. *ribā*. Such combination of contracts in order to circumvent Islamic principles may be considered as product structuring which is a part of conventional financial engineering.

Here and below, we will be concerned however not so much with product structuring but rather with quantitative valuation of financial instruments. Starting from fundamental Islamic principles, the question of whether a designed contract implies a clean *bay´* rather than *ribā* is a central one. We argue that *ribā* and *gharar* may easily arise through neglect of risk or inappropriate valuation methods for value and risk of assets and financial instruments. This possibility, in turn, strongly necessitates an estimation of expected forward values, market risk, and default risk which is consistent with Islamic principles of avoiding *ribā* and *gharar*. Based on consistent estimatation of risk and return the expected costs of risks should be quantified for all parties to the contract, in order to judge, whether a contract is free from usury (*ribā*) and evitable risk (*gharar*), and whether remaining inevitable risks are distributed consciously and fairly between the counter-parties, for example between the investor (*rabbu l-māl*) and the entrepreneur (*muḍārib*) within a *muḍāraba* contract. Therefore, an unbiased quantitative estimation of risks and returns is a pre-condition in order to make these Islamic principles of finance operative.

Careful computation of expected value, expected return, and expected risks is a prerequisite not only for risk management, but also already for thorough valuation of compliance of contracts with respect to regulatory requirements, including compliance with Islamic law.

For this purpose, we propose below that, design and valuation of financial contracts, and the corresponding risk management, should all be performed on the basis of an Islamic financial engineering, which is based on realistic measures of value. We propose below that real value should be based on a reference currency which reflects the surrounding economic needs and realities relevant for the counterparties of the contract.

What should be expected from such a suitable reference currency? A counter example to the suitability of a common currency system is provided by the euro. Therefore, a three-level construction of reference currencies will be discussed below. The proposed currency system is intended to avoid the pitfalls of hard monetary union, while reflecting the global and local needs and realties in a balanced manner through the prices of certain key assets, in particular commodities.

There are considerable benefits of a value-oriented risk management for social economy. On the Banque de France conference, March 4, 2011, Kenneth Rogoff of Harvard University commented[1]: *"Western policymakers and economists often portray Islamic financial systems, with their emphasis on shared risk and responsibility in lending, as less efficient than western systems that put no strictures on debt. Yet one can equally argue that Western financial intermediation is far too skewed towards debt, and as a consequence generate many unnecessary risks."*

Particularly for the member countries of the Organization of Islamic Cooperation (OIC) we argue in favour of a thorough Islamic financial engineering. It should be built on foundations of thorough quantitative valuation based on real measures of value. In this way it will enable Islamic risk management to make transparent expected return and risks of any project or transaction. Given this information, counterparties will become enabled to realize a truly fair distribution of values and risks in their contract. This also implies that risk management takes a much more preventive role, being involved already before the design of any contract.

Islamic financial industry following the guideline of such principles of Islamic financial engineering will be able to contribute to sustainable development by (i) more risk-(and-return)-consciousness reflected in the participatory structure of financial contracts, and thus, (ii) encouraging Islamic financial institutions to innovate financial products consistent with *real* implementation of Islamic principles, reflecting the *real* necessities of modern business and economy.

II. FINANCIAL ENGINEERING FOR SUSTAINABLE DEVELOPMENT

II.1. ON THE ROLE OF FINANCE AND FINANCIAL ENGINEERING

The role of financial institutions is to provide the capital for projects, in particular, for those projects which are useful or even vital for development of societies with sustainable infrastructures. Notwithstanding this fact, many financial institutions (e.g. in Europe), which had been shaken by the last financial crisis, have been observed to refuse credits or offer credits only with unbearably high risk premiums.[2] This attitude has severely threatened the existence of in particular local small and medium size family businesses and enterprises, since these suffer from additional discrimination by the traditional rating systems favouring large international players instead. As it is known, however, the economic and innovative power of the society is driven very much by locally rooted small and medium size enterprises, rather than the big transnational players. This is known to hold true in Europe, as well. And it is very likely to hold similarly also for the group of OIC member countries, which have for long been striving similarly for their sustainable development, while facing similar challenges along the way.

As a consequence, it is the task of each government to take care that the financial institutions follow up their duty of providing the required capital for small and medium size companies in order for them to remain operational and continue making essential contributions for an innovative and vital economy. Following to the financial crisis from 2007, new regulatory frameworks such as Basel III encourage credit institutes to impose more severe conditions and tougher ratings on entrepreneurs searching for credits. The regulatory requirements now became the pretence of credit institutes for harsher credit conditions, contradicting to

[1] See Rogoff (2011)

[2] Indeed, results of the European Central Bank (ECB) Bank Lending Survey (Jan. 2009) comprising over 100 European banks from the eurozone, revealed how credit standards applied to approval of loans and credit lines tightened considerably for both, enterprises and households; see ECB (2009).

generally decreasing interest rates in Europe, particularly in Germany. Government tried to intervene on this with several measures to enable and to push financial institutions to follow up their duty of providing liquidity for enterprises. However, by the time being, Europe's financial industry reacts only very reluctantly. In this aspect, governments of most OIC countries should be in a better position not only because the influence of governments on the domestic financial sector has traditionally been more powerful, but much more also because the basis of understanding between governments and financial industry is derived from the common rules of *sharī'a*. In this context, a very different culture of entrepreneurship has shaped the Islamic finance industry with the commonly accepted participatory means of financing, while fixed income products being obsolete.

In particular, conventionally agreed interest loans are commonly objected for the fact that they avoid a sharing participation in the default risk related to the purpose of the loan. For the same reason, transfer of risk by selling it to another party is often objected.

Nevertheless, within the Islamic finance community, this sometimes appears to create a far spread impression that a passive attitude towards risk would be more ethical rather than an active management of the risks involved within a project and related contracts. We would like however to argue that both transparency about all risk involved and clear agreements about the way they are to be shared should be part of any contract. Also we believe that for a single agent, entrepreneur or investor, it should be legitimate to use a portfolio of contracts with different counterparts in order to minimize his own exposures to different types of risks. In fact, avoidance of evitable uncertainty - called *gharar* - is a fundamental request according to Islamic discourse. Hence, it is clear that, modern tools of risk management and hedging should be applied appropriately in order to reduce risk exposures, using all the knowledge we currently have.

Quantitative evaluation of risks involved is a necessary precondition in order to enable the counterparties to first obtain transparency about the existing risks and, on the basis of this information, to reach a fair agreement about the mutual distribution of risks.

As far Islamic finance is concerned, we currently perceive commonly a lack of rigor in quantitative evaluation and related interpretation, culminating in avoidance of mathematics as being too sophisticated. However we'd like to remind that striving for knowledge is any Muslim's duty. In Section III below, we will sketch some essential mathematical aspects of Islamic finance, which enable conclusive financial engineering methods, sharing the rigor of mathematical finance.

II.2. THE MISLEADING MYTH OF RISK-FREE INTEREST AND FIXED INCOME

One historically grown reason for the adverse attitude of some Muslim scholars towards financial mathematics and, in particular, the fair value approach comes from the conventionally common practices of discounting future cash flows with so-called "risk-free" interest rates often derived from forward rates of interbank markets and certain government bonds. Indeed these rates are deceptive. Firstly, neither interbank markets nor governments are really "risk-free" as their name suggests. Secondly, the level of the rates charged is considerably higher than sustainable. In interbank markets, one cause is their agents' bias towards short payment frequencies (yielding basis spread). For government bonds, their default risk (yielding credit spread) is rated by agencies which are supposed to be neutral, but in fact have proven again and again to be strongly biased, in particular in times of economic strain. Also, the same conventional financial institutions who used to be strong risk avoiders when it comes to credits for enterprises, in particular smaller ones, turned out to be strong speculators in terms of high-interest bonds. The resulting high level of interest rates badly

affects public and private sector, and also contributes to push inflation rates up. Hence, conventional discount curves related to aforementioned interest rates are legitimately to be objected, and likewise the current system of biased ratings and credit spreads. Below we will argue indeed for a more flexible and adequate construction of discount curves. We will also point out that the mathematical framework of fair value – based on relative prices with respect to some reference asset – does not require at all that the reference asset should be given by an artificial zero-coupon bond linked to "risk-free" discount rates, which, in turn, relate to fixed interest loans and the conventional interbank markets for forward rates. The mathematical framework of fair value works perfectly also with a universal commodity-linked currency, or even equity as a reference asset.

In fact, the notion of a risk-free fixed income is a myth kept up by the conventional banking sector, rather than an economic reality. Corresponding rates are set mainly by the agreement of an interbank market among conventional creditors. To calibrate a current credit contract to such rates may be questionable particularly for the situations where one or more counterparties of the contract have limited or no access to this interbank market.

After we have seen that fixed interest rates by themselves are not suitable as a basis for a risk-neutral measure, we conclude that a new basis for such a measure must be sought. This should be done by taking into account the position of the agent in the market, i.e. by carefully investigating her exposure to the various risks of the relevant markets and economies involved.

Furthermore, credit spreads should be considered more flexible, i.e. not constant, and not necessarily always positive. A negative credit spread for some period would reflect the possibility that the credibility of the considered counterparty is in fact better than that of the reference.[3]

It is one main issue in Islamic finance that a supposed risk-free interest rate should be close to zero. The reference rates of some countries like Switzerland and Japan have already come very close to this since many years. Also in Europe, after the financial crisis of late 2000s, interest rates dropped drastically. This indicates that the economic reality in interest-based financial markets might essentially honour – sooner or later – the fact that there is *no free lunch* in any market and that the assumption of a risk free return is a myth.

The leverage obtained by interest-based loans and fractional reserve has led to a debt-based accounting and finance which has nourished conventional finance industry on one hand, but also entailed an exploding price of public debt and private credit on the other. The latter has led to recent crises, challenging the conventional debt-based economic system as a whole.

II.3. INFLATION OF CURRENCIES: TIME-VALUE VERSUS TIME-LESS VALUE

To the extent that inflation of any currency is inevitable, a time-value which compensates the expected inflation should be disputable. If we decide that full participation in the inflation risk is not bearable to the investor, hence at least a partial compensation of hedging against inflation risk should be admitted. On the other hand, one might argue that investor and entrepreneur perhaps share some common risks, such as the risks of everyday life, and

[3] Recently, in December 2011, Denmark placed 3-month government bonds, which had a negative effective rate of return, due to a negative spread rate as a security premium to be paid by the investors as a compensation for the relative security of these bonds.

accordingly, the inflation risk – as it is inevitable to both parties – should also be shared among them rather than being put on one party's shoulders.

Real contracts usually involve several cash flows and/or depend on asset values at different points of time in future. Hence, a fair contract evaluation requires the ability to compare cash flows and/or asset values at different times. The nominal value of assets is usually measured in units of a certain nominal currency. However, the real economic value of this currency itself may change with time, e.g due to inflation. For our fair valuation, we are nevertheless interested in the intrinsic real value of currency, and of our assets. E.g. in the face of inflation, this real value of assets might be measured by inflation-adjusted prices. These are obtained by adjusting future cash flows by discount factors accounting just for the expected inflation of the currency.

Viewed superficially, discount factors might be objected for introducing apparent undesirable time-value to cash flows. However, if they are chosen just such to compensate for the time-dependency of the nominal value of the currency, they in fact may yield in fact just the desired time-less real value standard. We would like to emphasize that, timelessness of money value as suggested by Islam was requested for a (money) currency expressing a real measure of value rather than just a nominal one. Islam advocates in fact the use time-independent measures for value. At the times of the Prophet (ﷺ) gold exploration and speculative trade were much more limited such that, gold as a currency was more rare and less volatile than today. It was a fairly good inflation-free currency. Its value was stable and hardly to be influenced. Only exceptional political events such as conquests and sieges could trigger sudden regional gains and losses of huge gold treasures, which then could indeed change the local real value. Except for such extreme events, the real value of gold was quite stable. Today however, developments of mining industry, demands from high technology, and different markets pressure almost continuously on the value of gold in different directions. Although gold might be still more stable than most paper money, its real value has become much more volatile than at times of the Prophet (ﷺ).

Hence the challenge is to find the reference asset which represents a timeless stable standard of value, similarly as gold in previous times.

II.4. THE 21TH CENTURY GOLD: BACK TO COMMODITY-BASED CURRENCY

From very early times of Islam up to the 20th century, the commodities of gold and silver have played an important role in defining the modern currencies. The histories of the GBP and USD in this respect and, in particular, their successive decoupling from their reference commodities simultaneously with their devaluation is described e.g. in El Diwany (2010).

Gold has been commonly considered as the most stable physical the commodity, with the least volatility of its price. For centuries, before the rise of leveraged finance, it was chosen as *the* traditional asset-based currency.

Within the EU, after some period of fixed cross currency rates, the euro was introduced. Less known however is that, an early predecessor of the euro was defined already in early 1930s. The universal European currency intended as a "currency for peace" was called "l'Europa" (see Le Fédériste (1933) for reference). It was defined as a basket of several valuable commodities. Similarly, and much later, Lietaer (2001) introduced a global Trade Reference Currency (TRC), dubbed also as "Terra", linked to a basket of a dozen internationally circulated commodities, including gold.

One advantage of the general concept of a basket of commodities underlying to a reference currency is the increased stability.[4] With its currency linked to a basket of commodities and its monetary authority backed 100 percent by a sufficiently large reserve of these commodities, artificial depreciation (appreciation) of a country's local currency (domestic prices) would come to an end, together with an improved immunization of its economy against adverse monetary and fiscal side effects.

An inevitable effect of any commodity-based currency is the increase of efforts for production of or mining for the underlying commodities. Taking into account the challenges of 21st century and beyond, previous choices of baskets did not yet account sufficiently for the aspect of sustainability and desirability of the production of the commodities chosen for the basket.

According to Islam, at times of the Prophet (ﷺ), exploration and production of gold was still desirable as the most precious commodity known at that time. As stated earlier, it was not yet challenged by an excessive mining industry with all its social and ecological problems. From our current modern point view, in our opinion, the negative effects related to the intensive industrial production, such as ecological damages and exploitation of workers, cannot be ignored. In essence, what is true for gold and other metals is also true for oil and gas as well as agricultural commodities. For any commodity, consideration should me made whether and to which extend its production is desirable. So from an Islamic perspective, we would like to argue in favour of a historically conscious reading that the "gold" mentioned in the Holy Qur'an should be read just as synonymous for the "most precious commodity". If we ask ourselves – from the perspective of the global challenges ahead of us – what are these "most precious commodities", we might identify such commodities as cleanly produced renewable energy, clean drinking water, and agricultural products produced according to ecological standards.

In parlance of mathematical finance, the search for a real-economy linked reference asset i.e. currency, corresponds to finding a numéraire with respect to which relative prices of tradable assets from the corresponding market are martingales. One might ask, why to restrict to commodities when composing the reference currency. Why not consider e.g. equity?

A first answer comes from mathematical finance. Per definition a numéraire is required not only to be tradable, but also to be void of self-generated cash flows, such as dividends. Therefore equity is highly inappropriate as numéraire. Remember that, the value of a share in a company is coupled to the value of the cash flows this company generates. Furthermore such assets with a value coupled to a complex structure of their underlying economic entity could hardly taken as an economic reference without introducing some auto-referential circularities into the valuation problem. This is because the cash flows of an economic entity need valuation themselves in order to determine their value.

The second answer comes from the basic definitions of Islamic law[5]. Assets serving as money should in particular be fungible (*mithlī*), i.e. measurable in a general sense, and extensive in the physical sense. Islamic metric categories distinguish further the methods of volume (*makīl*), weight (*mawzūn*), and quantity (*ma'dūd mutaḳārib*). E.g. metals like gold and silver are *mawzūn*, while agricultural commodities like wheat and barley are traditionally *makīl*.

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[4] Variance of a sufficiently diversified basket reduces: $Var(A + B) \leq Var(A) + Var(B) \Leftrightarrow Cov(A, B) \leq 0$.

[5] See e.g. Chapter 19 in Schacht (1982).

At this point it is necessary to mention that, in our days we have at hand all the physical and technical methods to measure in particular the commodity-resources which are globally relevant. We not only can price drinking water, renewable energy, but also CO_2-emissions. If a global board decided to include the ecological footprint into the world reference currency, this would then could be included with a negative price, because its increase reduces the effective performance of the world economy.

II.5. A LESSON FROM THE EURO CRISIS: ECONOMIC DIVERSITY AND UNITY EFFECT THE CURRENCY SYSTEM

In Europe in particular, growing public debt of several European Union (EU) member countries has surfaced already in several financial crises, which escalated by salvation programs for financial institutions. According to IMF 2010, the public debt of Italy and Greece is significantly exceeding annual GDP, while several others (like Portugal, Ireland, and Belgium) are following with over 90% of GDP. Continuing conventional public loan policy, it is likely that European public debt will exceed all possible salvation funds in the long run. Comparing 2008 financial crisis of Iceland with 2010 debt crisis of Greece, some important difference appears: Iceland had its own currency which helped to soften the consequences, but Greece is currently trapped within the EUR monetary union. Recently enterprises and banks already simulate scenarios for the reintroduction of the drachma[6] in Greece. The point which was missed, when in 2001 Greece entered for the second time[7] into a monetary union, is that, a common currency can be sustainable only within a political region which is sufficiently in its economic realities and needs.

If some important factor such as economic productivity differs largely between the participating countries, an artificially introduced monetary union creates artificial fluxes and processes which may be in contradiction to the economic reality of some country. This recently became evident with Greece again. The idea, that a monetary union of a region like Europe is a priori going to strengthen its political union, has proven again to fail, because in the case of Greece, the euro did not reflect Greek economic reality, with economic mismatch finally resulting in political instability.

The euro currency was introduced well-intended, with the idea to enhance the political and economic solidarity between the countries involved, and with the ultimate goal to make them together more competitive on the world markets. However, the recent euro crisis has shown that, when economic foundations of the constituents are basically too different, the hard entry into a currency union may result in unexpected problems[8]. If productivity of the countries highly differs, the stronger countries have to subsidize the weaker ones in order to avoid a potential collapse in the common currency. On the other hand, the hard exit of one or more countries from a monetary union likewise poses a difficult problem.[9] As a result, the political disputes going along with this process not only put the common currency union at risk, but even worse, they may damage also the political union.

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[6] From 1831 to 2001 the Greek currency was the Drachma

[7] From 1869 to 1914 the Drachma was effectively coupled 100% to the *union monétaire latine*

[8] For example, the European Commission (EC) is currently split about whether to introduce eurobonds or not. The introduction of eurobonds would be consequent since there exists already a common currency. The opponents however argue in view of the different economic realities within the member countries. The dilemma is that, any currency has to reflect the economic realities, but the euro member countries now discover that they will not be able to sufficiently converge on their economic realities.

[9] Recently Simon Wolfson, a prominent euro-sceptic, offered a 250,000 GBP Economics Prize (the largest sum after the Nobel Prize) for the best plan to wind up the euro in an orderly way.

In our opinion, for a country belonging to a certain political entity (e.g. the EU), either to share a common currency 100 per cent or to withdraw from the monetary partnership may be an insufficient and inappropriate choice in the face of the current economic and political realities. The currently practiced "0 or 1" hard entries into a currency system apparently ignore the risk that a similar hard exit from it may endanger the entire monetary system.

Therefore alternatively, we propose a structured system of currencies. It starts on the global level with a reference currency, defined as a basket of selected commodities with individual prices $S_1 \ldots S_n$, yielding its price as

$$C_w = \sum_{i=1}^{n} w_i S_i \tag{1}$$

The commodities may be selected under the particular aspect of their relevance to the sustainable needs and realities of the global world.

Similarly, we may consider a basket of commodities, the production of which is desirable just within a certain region, e.g. the political entity of the OIC member countries. The basket then may define the currency with price C_r of the region. $\Delta C_r := C_r - C_w$ accounts for structural differences of the region from the rest of the world. It may also give additional (nonnegative) weight to some of the commodities in C_w, e.g. clean drinking water, which is already within the global world basket. It may also contain new commodities specific for the region. Hence

$$\Delta C_r = \sum_{i=1}^{n} w_i^{(r)} S_i + \sum_{i=n+1}^{m} w_i^{(r)} S_i \tag{2}$$

Finally, we admit an extra basket for individual countries in order to take into account their particular situation, differing from that of its region as well as the world as a whole.

$$\Delta C_c = \sum_{i=1}^{m} w_i^{(c)} S_i + \sum_{i=m+1}^{l} w_i^{(c)} S_i \tag{3}$$

The currency of any country with price C_c will be composed from a worldwide defined fraction β_w of the world currency, the regionally defined fraction β_r of its regional commodity basket, and the remaining local country fraction $(1- \beta_w -\beta_r)$ of its local commodity basket. Hence,

$$C_c = \beta_w C_w + \beta_r \Delta C_r + (1 - \beta_w - \beta_r) \Delta C_c \tag{4}$$

The fraction β_w should be negotiated on a world conference, common for all regions. It should be as high as possible, in order to meet global challenges. On the other hand, $\beta_w < 1$ is desirable in order to admit the differences according to the particular situation of different regions of the world.

Similarly, the fraction β_r should be negotiated on a regional conference, common for all countries. It should be as high as possible in order to meet common regional challenges. However, $\beta_r < 1$ is desirable in order to admit in this case the differences according to the particular situation of the different countries of the region.

By construction, the higher the world fraction β_w, the more strongly any two regional currencies of the world are correlated. The same is true for the currencies of the countries

within a region: the higher the value of $\beta_w+\beta_r$, the stronger the correlation between the local currencies of any two countries within any region will be. In particular, the negotiable weight β_r may give to a region, e.g. Europe, the flexibility to regulate a partial entry into a currency union with some realistic weight β_r , which should be chosen as to reflect as close as possible the real degree of economic unification.

III. VALUATION METHODS FOR CONVENTIONAL AND ISLAMIC FINANCE

III.1. CONVENTIONAL VALUATION

The value of an asset can equivalently be quoted in the form of a price or in the form of a rate which is relative to a reference price. Hence, a quoted rate per se does not yet imply *ribā* while, vice versa, quoted prices may conceal *ribā*. The form of the quotation, whether it is given price $S(t)$ or rate $R(t)$, is irrelevant to the question of usury. The equivalence relation is

$$\frac{S(t)}{S(0)} = 1 + R(t) \cdot t \tag{5}$$

When $S(t)$ is unknown and subject to risk, it will be stochastic and one has to consider its expected value $E_Q[S(t)]$ under a certain risk measure Q.

Towards the risk of changing $S(t)$ one can alternative take a risk-neutral position[10] or a risk-adjusted position[11], corresponding to different choices of the measure Q. The present value $S(0)$ is independent of this choice. In order to determine whether a certain quoted price or rate implies usury, it is rather important to have at hand a method to determine a fair price independently from actual market quotes. This independent method is provided by mathematical finance. Assuming a (stochastic) model for $S(t)$, the expected price should be computed under the chosen measure Q.

If we choose Q to be the t -forward risk-adjusted measure, briefly called forward measure, then the conventional present value (at time 0) of the stochastic variable S is given by

$$S(0) = \frac{1}{1 + tR(t)} E_Q[S(t)] \tag{6}$$

The spot rate $R(t)$ may equivalently be converted into a discount factor, corresponding to the initial value $P(0,t)$ of a zero-coupon bond with a nominal value of $P(t,t) = 1$ and time-to-maturity t, i.e.

$$P(0,t) = \frac{1}{1 + tR(t)} \tag{7}$$

Note that the zero(-coupon) bond price $P(0,t)$ is non-linear w.r.t. the spot rate $R(t)$.

With (7), the relation (6) above may be rewritten as

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[10] In conventional finance the martingale measure for a risk-neutral position is assumed to be defined by a so-called risk free bank account B as numéraire. The martingale is the relative price w.r.t. B.

[11] Conventionally, adjustment is made w.r.t. the forward price risk. The corresponding martingale is the simply-compounded forward rate. The martingale measure is defined by a zero bond P as numéraire.

$$\frac{S(0)}{P(0,t)} = \frac{E_Q[S(t)]}{P(t,t)} = E_Q\left[\frac{S(t)}{P(t,t)}\right] \tag{8}$$

which is simply the martingale expression for the relative price of S with respect to the zero-coupon bond $P(\cdot,t)$.

Conventional finance usually assumes depreciation of cash flows in a certain currency during time. It further assumes that the appropriate way to account for this, is to discount with a particular spot rate curve $T \mapsto R(T)$ derived from forward rates $F_i(t) = F(t;T_{i-1},T_i)$ corresponding to quoted interest rates from certain interbank markets (e.g. LIBOR).

From an Islamic finance perspective, the most questionable point here is the market from which the quotes are taken. The method of calibration of the expected spot value to the forward curve by itself agrees with the principle of fair value.

Even within the still very conventional setting of $R(t)$ and $P(\cdot,t)$ and relations (6) and (8), there is already the flexibility to adjust the rate $R(t)$ (and equivalently the zero-coupon bond) to the any expected time-dependent performance. It is not at all required that it is linked to the conventional interbank markets for deposits or swaps.

If it is linked to performance of equity, commodity, or (as we will consider below) to currency, it may be consistent with principles of Islamic finance. Also compensation of inflation of a currency may be considered as legitimate under circumstances that the counterparty cannot bear to be exposed to inflation risk, e.g. because it is too small and does not have the possibility to protect itself. In this case $R(t)$ could be linked to inflation rate.

Concerning relation (2) above, remarkably, in mathematical finance the meaningful quantities are not the absolute, but the relative prices. Price dynamics is investigated essentially as the dynamics of the relative price $\hat{S} := S / S_0$ with respect to the price S_0 of a reference asset. In (8) above it is a zero bond for maturity *t*, when *Q* is the *t*-forward measure.

Several desirable properties for the numéraire S_0 are usually postulated. In particular, it should be tradable (ideally with infinite liquidity), and it should be positive.

III.2. ISLAMIC VALUATION: THE REFLECTION OF REAL ECONOMY

In this section we focus on value in terms of real economy. We suppose that this concept of value is at the core of Islamic finance principles: The stability of real value in terms of real economy, as a key criterion. Accordingly, stability should also be a key feature of any sustainable measure of real economic value.

In order to achieve value-stability in terms of computed relative prices, the numéraire defining the valuation measure should by itself represent a stable value. The more this value is stable, the more the numéraire price S_0 is stable,[12] hence the lower is the volatility, i.e. the less is the risk of change for relative prices computed with respect to this numéraire.

[12] The price in turn should be expressed with respect to stable reference currency, which may be still conventional at this point of the discussion.

It has become common practice in conventional financial engineering to assume that, for the purpose of a risk-neutral valuation, or a corresponding forward risk-adjusted valuation, bank accounts, or respectively zero bonds, provide the suitable reference asset. It is further assumed that these assets are defined in terms of "risk-free" [13] zero (bond) rates, which in turn are constructed from quotes of certain interbank markets.

The implicit theoretical assumption of such procedure is that, the interbank interest markets are reflecting the economic reality appropriately for any of the nominal currencies. However, this expectation has proven to be unrealistic, particularly in situations of economic strain. Hence, the conventional valuation does not reflect a real value related to the real economy.

Furthermore, the conventionally constructed bank accounts and zero bonds are linked to a debt-based financial industry. From this point of view, it is very questionable to treat them on the same footing as real assets for the purpose of financial engineering. A numéraire asset should satisfy the criterion of tradability. But when zero bonds are traded liquidly as assets, this effectively is nothing but trade on interest. Even worse, traded interest in conventional finance industry is ultimately linked to leveraged debt, whence these supposed "assets" are in fact not backed by a real economic value. Hence, it amounts to speculation on debt-financed interest, which is *ribā* and *gharar*. Therefore, conventional interest is obsolete according to Islamic principles, due to its very construction. Islamic finance however is genuinely based on the value of real assets. Hence, the conventional numéraires are not suitable in the context of Islamic finance.

As the numéraires of conventional finance neither result in a real valuation reflecting real economic value, nor are they consistent with basic principles of Islamic finance, they should be replaced by suitable real assets representing economic value as real as possible. We propose the commodity-linked currency (4) as a natural risk-neutral reference asset for this purpose. With this choice, relation (8) will be replaced by

$$\frac{S(0)}{\mathrm{C_c}(0)} = \mathrm{E_Q}\left[\frac{S(t)}{\mathrm{C_c}(t)}\right] \tag{9}$$

Note that this relation holds, independent of the auxiliary currency units within which S and $\mathrm{C_c}$ are evaluated. This invariance is an advantage of relative prices. In general, the relative price in (9) will be stochastic. However, the relative price of $\mathrm{C_c}$ to itself becomes trivially constant equal to 1. The measure Q here is the risk-neutral (martingale) measure with respect to $\mathrm{C_c}$ as numéraire.

III.3. CONVENTIONAL VERSUS ISLAMIC VALUATION

Let us give first a simple description of a forward contract and it conventional valuation. A forward contract agrees a fixed price, the fair forward price F, to be paid at delivery time T in exchange for an asset S (say a commodity) having present value $S(0)$ and unknown value $S(T)$ at delivery. A fair value forward contract on some traded asset S should always

[13] Even conventional fixed income markets are in fact never risk-free. In particular they suffer from the risk of changing market interest rates, which frequently has been neglected in the common practice of conventional financal engineering.

have value 0 at time 0, the time of contraction. Using relative prices w.r.t. zero bonds having price $P(t,T)$, conventional financial engineering demands

$$0 = \frac{S(0)}{P(0,T)} - \frac{F}{P(T,T)} \text{ ,or equivalently } F = \frac{S(0)}{P(0,T)}. \tag{10}$$

Hence, $P(0,T)$ acts as a discount factor. Note that (10) implies a nominal value concept with respect to some conventional currency, but tries to compensate this by taking zero bonds as numéraire. Above, (10) is postulated directly, with deterministic $P(0,T)$. For independent unknown stochastic prices $S(t)$ of the asset and $B(t,T)$ of the zero bond, it follows that

$$\mathrm{E}_\mathrm{P}\left[\frac{S(t)}{B(t,T)}\right] = \frac{\mathrm{E}_\mathrm{P}[S(t)]}{\mathrm{E}_\mathrm{P}[B(t,T)]} = \frac{\mathrm{E}_\mathrm{P}[S(t)]}{P(t,T)}, \tag{11}$$

with deterministic prices $P(t,T) = \mathrm{E}_\mathrm{P}[B(t,T)]$. With $F = \mathrm{E}_\mathrm{P}[S(T)]$, equation (10) is recovered immediately. The independency assumption may hold for equity or commodity assets, when it appears plausible that such assets develop essentially independent from the interbank markets determining $B(t,T)$. However, when asset prices are non-trivially correlated with the stochastic discount factors, the above derivation of (10) has to be replaced by

$$F = \mathrm{E}_\mathrm{P}[S(T)] = \mathrm{E}_\mathrm{P}\left[\frac{S(T)}{B(T,T)}\right] = \mathrm{E}_\mathrm{P}\left[\frac{S(t)}{B(t,T)}\right] = \frac{S(0)}{P(0,T)}, \tag{12}$$

whence again the fair price of the future contract is simply $S(0)/P(0,T)$.

Let us now consider the valuation of a forward contract within an Islamic financial engineering framework relative to a reference currency C. According to (9) above, we replace (12) by

$$\mathrm{E}_\mathrm{Q}\left[\frac{S(T)}{C(T)}\right] = \mathrm{E}_\mathrm{Q}\left[\frac{S(t)}{C(t)}\right] = \frac{S(0)}{C(0)}. \tag{13}$$

Unlike (12) where by definition $B(T,T) = 1$ for a zero bond, here $C(T)$ is in general still unknown, stochastic. Furthermore the stochastic forward price $S(t)$ and the stochastic currency $C(t)$ are unlikely to become either independent or fully dependent at maturity. Then only value which can be estimated is their relative value. The fair relative value w.r.t. to the given currency is constant. For the future contract hence the fair price should be specified as relative to the reference currency, simply as $S(0)/C(0)$. Remarkably, the fair price is quite analogous to the conventional future price, which also can be viewed as the relative price w.r.t. a zero bond.

Important from the perspective of Islamic principles is that, in the conventional case the zero bond numéraire is a purely interest-based, and linked essentially to interbank markets, while in the Islamic case the numéraire is asset-linked. We propose to relate it to a commodity-linked currency. The defining asset here is a commodity basket supposed to reflect a more appropriate and relevant economic reality.

Now let us consider the special case of full dependence, $S(t) = a \cdot C(t)$, i.e. the traded asset is proportional to the reference currency (with constant factor a), and hence proportional to the commodity basket defining C. Then (13) yields

$$\mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right] = \frac{S(0)}{C(0)} = a = \frac{\mathrm{E}_{\mathrm{Q}}[S(T)]}{\mathrm{E}_{\mathrm{Q}}[C(T)]} = \frac{F}{\mathrm{E}_{\mathrm{Q}}[\mathrm{C(T)}]}, \tag{14}$$

and therefore $F = a \cdot \mathrm{E}_{\mathrm{Q}}[C(T)]$.

Comparing conventional and Islamic approach, a practically important difference is that, while zero bond prices are conventionally computed purely deterministically from available quotes of money market, future, and swap prices, for our reference currency and commodity basket, the relevant forward prices are currently not easily available from market quotes. Their estimation is however possible via calibrated stochastic processes. When the asset *S(T)* is non-trivially correlated with the reference *C(T)* , then a stochastic simulation of their relative value may be necessary in order to determine the expected value in (13). This shows the need of stochastic forward rate models for commodities in analogy to the conventional LIBOR market model.

We finally emphasize that, the numéraires in (12) and (14) are *not* equivalent numéraires, i.e. they are *not* resulting from transformation of each other according to the change of numéraire theorem[14]. They are rather the different choices for the primary numéraire, defining the martingale measure *P* respectively *Q*, i.e. defining what is meant by the fair value.

This fair value according to (14) becomes more than another arbitrary definition exactly when the reference currency C reflects a real value linked to the real economy. Only in this way the Islamic choice (14) becomes really superior to the conventional choice (13). Otherwise, 'value' would remain just a conventional concept. This is the case in the conventional case, since the interbank interest markets behind zero bond curves for conventional currencies do in most cases not really reflect the economic reality to which the counterparties are exposed. This becomes evident particularly in situations, when the finance industry meets some of their home-made crises. Then, the conventional zero curves tend to exhibit artefacts from the conventional banking sector. In particular the bias towards short maturities introduces basis spreads into the forward rates traded and quoted in the conventional sector. Since the 2007 financial crisis, it has been realized now within the conventional banking sector that, because of the basis spread included in quoted forward prices, the discount curve which is supposed to yield fair arbitrage-free prices, can no longer be equal to the (interbank) market forward curve, rather a 2-curve-approach becomes necessary[15]. The interpretation is that, the conventional interbank markets are biased, rather than risk-neutral.

III.4 SYNTHETIC *RIBĀ* DISABLED

We exemplify the consequences of our modified evaluation framework for the design of Islamic derivatives. It is commonly agreed that such derivatives should be permitted only for the purpose of hedging.

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[14] See e.g. Brigo and Mercurio (2007).
[15] See e.g. Bianchetti (2009) for the theoretical background, and Ametrano and Bianchetti (2009) for practical implementation of the 2-curve-approach.

Jobst (2010) has presented three examples of synthetic instruments from asset-based investment finance. Their valuation nevertheless has been based on conventional measure of value using interest-based discount factors. As a consequence, the call-put parity could be used to synthesize a conventional loan.

If the present value is computed using our proposed reference asset-based currency, the value of a certain amount of this currency would be constant, without any interest. With the corresponding valuation, call-put parity can no longer be abused to synthesize a conventional interest-based loan. In terms of real economic value, combinations of *ribā*-free contracts will fail to synthesize *ribā*, when all parts are valuated with respect to the unique relevant real value reference. Hence the motivation for such combinations will become void.

IV. VALUATION OF FORWARD CONTRACTS AND *BAY´U S- SALAM*

In conventional financial engineering, forward contracts constitute the basic building blocks for more complex financial instruments. Similarly, in Islamic finance, (*bay´u s-*)*salam* or/and forward contracts are the most basic financial derivatives.[16]

The essential difference between a *bay´u s-salam* and a forward contract is that, the agreed price has to be paid rather immediately close to time $t = 0$ at spot for a *bay´u s-salam*, while with a forward contract it is payable close to delivery at time $t = T$. This difference shows up in the valuation of the contracts.

For both contacts, we would like to know the fair price to be agreed at $t = 0$. In any case, the expected payoff of the contract should be zero at $t = 0$.

Relative to currency C, the expected payoff of the *bay´u s-salam* is

$$\mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)} - \frac{S_{0,agreed}}{C(0)}\right] = \mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right] - \frac{S_{0,agreed}}{C(0)} \overset{!}{=} 0 \ . \tag{15}$$

By (13), the fair price to be agreed is $S_{0,agreed} = S(0)$ for the *bay´u s-salam*. It is given directly by the actual price $S(0)$.

Let us consider now a forward contract. Relative to C its expected payoff is

$$\mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)} - \frac{S_{T,agreed}}{C(T)}\right] = \mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right] - \mathrm{E}_{\mathrm{Q}}\left[\frac{S_{T,agreed}}{C(T)}\right] \overset{!}{=} 0 \ . \tag{16}$$

Hence, the fair price to be agreed for a forward contract is

$$S_{T,agreed} = \frac{\mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right]}{\mathrm{E}_{\mathrm{Q}}\left[\frac{1}{C(T)}\right]} = \mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right] \cdot \mathrm{E}_{\mathrm{Q}}\left[C(T)^{-1}\right]^{-1} \tag{17}$$

[16] See e.g. Al-Bashir, Al-Amine (2005).

$$\overset{(13)}{=} \frac{S(0)}{C(0)} \cdot \left(\mathrm{E}_{\mathrm{Q}}\left[C(T)^{-1}\right]\right)^{-1} = \frac{S(0)}{\mathrm{E}_{\mathrm{Q}}\left[C(0)/C(T)\right]}$$

$$\neq \frac{S(0)}{C(0)} \cdot \mathrm{E}_{\mathrm{Q}}\left[C(T)\right] \neq \mathrm{E}_{\mathrm{Q}}\left[S(T)\right]$$

where we also wrote the last two inequalities as a warning against naïve simplifications. In particular, the difference between $\left(\mathrm{E}_{\mathrm{Q}}\left[C(T)^{-1}\right]\right)^{-1}$ and $\mathrm{E}_{\mathrm{Q}}\left[C(T)\right]$ is due to the non-linearity of the inverse.[17] Hence, fair estimation of the agreeable forward price in any auxiliary currency would require the estimation of the inverse performance $\mathrm{E}_{\mathrm{Q}}\left[C(0)/C(T)\right]$ of the currency.

In order to avoid a stochastic valuation, instead of the nominal contracting price $S_{T,agreed}$, we can specify a forwards contract in units of currency C only. In this case, relations (16) and (17) are replaced by a condition for the agreeable relative value

$$\left(\frac{S}{C}\right)_{T,agreed} = \mathrm{E}_{\mathrm{Q}}\left[\frac{S(T)}{C(T)}\right] = \frac{S(0)}{C(0)} \ . \tag{18}$$

At time T, the buyer will pay $(S/C)_{T,agreed} \cdot C(T)$. Note that such a contract may be used to hedge against a risk of a changing real value, rather than just a nominal risk w.r.t. nominal value.

From (16) we learn that from the exercise prices of *bay'u s-salam* and forward contract agree when expressed in terms of *C*. This means that the expectation of the future associated relative value is constant for all times. This is exactly meeting the intention of a *ribā*-free contract, with time-less value in terms of economically relevant real value as expressed in terms of *C*.

Let us consider a *salam* on a commodity or services between the buyer and the selling producer. Although there appears to exist no preference between *salam* and forward from the value point of view, in reality there may exist a preferences for the *salam* from the point of view of the seller, since only *bay'u s-salam* may provide to him some upfront extra liquidity, which could be necessary e.g. in order to run a production.

V. RISK PROFILES AND EQUIVALENT MARTINGALE MEASURES

Let us now consider the dynamics of the relative price, $\hat{S}(t) = S(t)/C(t)$. Since a future price $S(t)$, $t > 0$, is subject to the risk of change, its expected value $\mathrm{E}_{\mathrm{Q}}\hat{S}(t)$ is of particular importance, since it provides an objective value which is independent of actually traded prices. Nevertheless, the expected value E_Q depends on the measure Q related to the risk of change.

Measure Q may be chosen to compensate some particular risks. For example, the forward measure is conventionally chosen in order to compensate the risk of a changing price. Such a

[17] A similar difference appears due to the non-linear relation of zero bond price and forward rate in conventional finance. Practitioners often fix it by a convexity adjustment in second order.

choice of the valuation measure may be particularly suitable and legitimate for contracting counterparties, when the related risk is out of their responsibility. A realistic fair value is supposed to honour the real economic situation, in particular also the related risks. The computation accounts for a certain risk profile by a corresponding choice of the measure Q.

Under the assumptions of no arbitrage and a complete market, there exists a unique measure Q with a risk-neutral expectation, i.e.

$$\mathrm{E}_{\mathrm{Q}}\hat{S}(t) = \hat{S}(0) \tag{19}$$

which is called the martingale measure.

However, if the market is not complete, e.g. due to illiquidity, extreme events, or other reasons, the martingale measure Q and the derived arbitrage-free value are no longer unique. In case of such an incomplete market, additional input is required in order to fix the valuation measure. In this context, the individual risk profiles and preferences of the counterparties may determine the choice between different martingale measures.

Note that, any choice between alternative measures entails a choice between alternative numéraire assets. According to (6) and (20), different measures Q correspond also to different zero-curves $R(\cdot)$. A particular curve may be chosen according to a tailor-made dynamical risk-aversion profile, agreed by the counterparties of a contract. A particular risk aversion profile corresponds to a certain utility function, which in turn may be used to select a particular martingale measure among several equivalent ones. One might even consider time periods, within which $R(t)$ becomes negative.

Such a free and conscious choice of a zero curve $R(\cdot)$ may be an interesting alternative, particularly for direct contracts between investor and entrepreneur, where both of them have developed a common understanding of the risks involved in the project, and correspondingly have consciously both agreed on a particular non-standard form of the zero curve, because for the project under consideration this choice is more suitable than the plain reference to the value of the regional currency.

An important issue from the Islamic perspective of *gharar* is the mutual agreement about the risk profile implied by the choice of $R(\cdot)$. It puts the highest requirements on transparency of risks and their impact on contractual values. In cases where there is no detailed understanding and clarity about risks, or when the counterparty attitudes appear to be rather indifferent, then it might still be preferable to stick to a standard choice for the economic reference asset, e.g. the our proposed commodity-linked currency for the region.

With given commodity-linked numéraire C, the concept of zero-curves may be adapted from conventional financial engineering to the commodity-based valuation described above. In this case, a zero rate curve $T \mapsto R(T)$ can be derived from the reference asset, according to (17) by

$$1 + R(T) \cdot T := \frac{1}{\mathrm{E}_{\mathrm{Q}}\left[C(0)/C(T)\right]} \ . \tag{20}$$

VI. ASSET-RETURN RELATED TO *ṢUKŪK*

In conventional finance, government bonds are still the traditional instrument for providing extra liquidity to a government. The analogous Islamic instrument is the *ṣukūku s-salam*, briefly called *ṣukūk.*[18] It enables a country to sell its production of commodity (or service) in exchange for an expected rate of return. In this way, *ṣukūku s-salam* may be used to provide liquidity to a government by selling an asset it produces. With *ṣukūku s-salam* as alternative instruments, conventional government bonds may become dispensable.

Conventional zero curves may be constructed from government bonds. Similarly, expected return rates from Islamic finance instruments, e.g. *ṣukūk* or others, might be used to define a zero rate curve with respect to a suitable local currency. In particular, the return rate of a *ṣukūk* can be computed according to equation (19), using the relevant commodity.

In this context, a significant leap towards achieving a fully-fledged an Islamic yield curve is the launch of the Islamic Interbank Benchmark Rate (IIBR) in November 2011 by a number of industry leaders including Thomson Reuters, Islamic Development Bank, AAOIFI[19] and SESRIC together with a consortium of world's largest Islamic banks. Although the IIBR currently uses contributions of expected *murābaha* returns for *sharī´a*-compliant interbank funding denominated in US dollars, its instrument-neutral nature – as decided by the IIBR Islamic Benchmark Committee – no doubt paves the way for further expansion of its rate contribution base into other Islamic instruments, like *muḍāraba*, *wakala*, and in particular *ṣukūk,* as these instruments are becoming increasingly widespread. The latter is a frequently applied instrument for financing infrastructure and other development projects.

When returns are evaluated with respect to a foreign currency, artefacts from the foreign economy might be imported into the domestic economy. However, once each region defines its own currency, e.g. according to our outline above, the related returns may begin to reflect real economic value, expressed with reference to the region's development and needs.

Utilizing an economic rate of return for valuation still places some particular challenges to Islamic financial engineering. Firstly, volatilities currently cannot be calibrated like conventional rates to caps or swap markets, just because these markets do not exists in Islamic finance. Therefore the corresponding yield volatility has to be evaluated historically. Furthermore, in order to capture the risk of changing return (rates), here requires stochastic simulation and evaluation of partially dependent commodities. As it is known, stochastic distributions and processes of commodity prices may have features which are not as present with other asset classes.[20]

VII. CONCLUDING REMARKS

We have shown that the mathematical framework of risk-neutral valuation and arbitrage free contract prices may be applied for Islamic finance, similarly as in conventional finance, provided some modification on conventional inputs.

A rather mild modification for this goal consists in replacing them by performance rates of underlying assets. Above we considered in particular commodity, as assets which are particularly suited as numéraire. In particular, the conventional discount rates drawn from interbank markets forward interest rates have to be replaced by expected returns, drawn e.g.

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[18] See e.g. Janahi (2005)
[19] Accounting and Auditing Institution for Islamic Financial Institutions
[20] See e.g. Aydın, Rainer (2010), Aydın, Giebel, Rainer (2011), Giebel, Rainer (2011) and references therein

from *bay´u s-salam* or *ṣukūku s-salam.* In this way reference may be shifted away from fixed income interest to assets. Notably, we identified the recent instrument-flexible IIBR initiative as an important step for Islamic finance industry, in order to obtain the necessary, independent forward curves for *sharī´a*-compliant pricing of an increasing spectrum of Islamic instruments.

In fact, Islamic finance in general is genuinely based on asset return rather on conventional debt. Another important example not discussed in this paper is given by *murābaha* contracts. These most common instruments in Islamic finance are linked to the expected returns on investment projects. This strongly contrast with conventional cost-driven finance where the funding gap results in credits as the most common instrument, totally based on debt.

As a more precise, but also more challenging method, we propose not only to replace conventional zero rates by expected returns on assets, but even more to replace conventional risk-neutral numéraires related to conventional zero rates for nominal currencies. Our proposed risk-neutral reference asset is provided by a commodity-linked multi-dimensional reference currency system. Globally negotiated fraction β_w of the world part, and regionally agreed fraction β_r may provide easy political instruments to regulate the degree of required homogeneity of currencies globally or with in a defined region such as Europe respectively.

In contract valuation, admitting individual economic profiles and customized risk profiles is the most challenging approach. It is the one which leaves the most contractual freedom and responsibility at the same time to the contracting counterparties. Provided all counterparties have both, the necessary capability for risk analysis and commitment for risk transparency, then they might agree on a particular zero rate curve freely, according to their commonly agreed risk aversion profile. The zero rates in this case may be linked closely to their common judgement of risk for the project underlying to the contract. The zero rates in this case may be even negative, meaning that a later cash flow is strategically preferable to an earlier one.

As Islamic finance is maturing, it is becoming clear that, it implies a change of paradigm from debt-driven finance to asset and value-driven finance, from nominal money (back) to commodity-linked money, from nominal valuation to valuation relative to the real economy environment, which offers the potential of a real alternatives for a deep reform of the current global financial system, as it is overdue in the face of recent financial crises.

It must be noted that the principles of Islamic finance, most importantly the prohibition of *ribā* and *gharar* are in fact not proprietary to Islam, but find their counterpart in Christianity, also within the contemporary context of economic globalisation.[21] With social justice as a modern starting point in the last century[22], Islamic principles of economy and finance have made their way with their impact not restricted to the Muslim world.[23] The contribution Islamic finance might give to an alternative globalisation (*'aulama badīla*), integrating local particularities within a truly global culture (*ṯaqāfa 'alamīya*),[24] currently can hardly be overestimated. Islamic finance is currently bringing some of the most fundamental principles of economic ethics back on the global table of finance, and apparently, this is happening just in time.

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[21] AGAPE (2005)
[22] See e.g. Qutb (1949)
[23] E.g. in 2004 Sachsen-Anhalt was the first German country to issue a *ÒukÙk* .
[24] Hamzawy (2003) commenting on Samīr Amīn

In view of the recent crisis of conventional debt-based finance, it should not require to be an optimist to expect that, asset-based alternative finance, in particular Islamic finance, will gain increasing support. We believe that, financial instruments (particularly bonds) linked to asset performance, may not be restricted to the OIC region, but they might soon become more convenient also for other regions. Particularly within the EU, performance bonds linked to a basket of commodities, products, services or projects may soon provide a serious to the problematic debt-based eurobonds.

Corresponding financial engineering, and in particular its valuation methods, will be based on a new footing, tied more closely to real economy. More realistic measures of value and risk in terms of real economy have to be investigated. As we have seen in this paper, sustainable commodities may be a key input for such reference measures of sustainable value and productivity. Consequently, the methods of mathematical finance have to be refined further, in order to meet the detailed challenges posed by the valuation of multi-commodity-linked instruments.

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