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Deformation and consolidation around encased stone columns

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ABSTRACT

A new analytical solution is presented to study soft soil improvement by means of encased stone columns to reduce both settlement and consolidation time. The proposed solution aims to be a simple and useful tool for design. Only a unit cell, i.e. an end-bearing column and its surrounding soil, is modelled in axial symmetry under a rigid and uniform load. The soft soil is treated as an elastic material and the column as an elastic—plastic material using the Mohr—Coulomb yield criterion and a non-associated flow rule, with a constant dilatancy angle. An elasto-plastic behaviour is also considered for the encasement by means of a limit tensile strength. The solution is presented in a closed form and is directly usable in a spreadsheet. Parametric studies of the settlement reduction, stress concentration and consolidation time show the efficiency of column encasement, which is mainly ruled by the encasement stiffness compared to that of the soil. Column encasement is equally useful for common area replacement ratios but columns of smaller diameters are better confined. Furthermore, the applied load should be limited to prevent the encasement from reaching its tensile strength limit. A simplified formulation of the solution is developed assuming drained condition. The results are in agreement with numerical analyses.

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1. Introduction

Stone columns are one of the most common improvement techniques for foundation of embankments or structures constructed on soft soils. They are vertical columns in the ground, filled upwards with gravel compacted by means of a vibrator. Unlike other improvement techniques, stone columns are considered to not affect significantly the properties of the surrounding ground. The main effects usually considered with respect to the untreated ground conditions are: improvement of bearing capacity, reduction of total and differential settlements, acceleration of consolidation, improvement of the stability of embankments and natural slopes, and reduction of liquefaction potential.

The vertical capacity of stone columns is related to the lateral confinement provided by the surrounding soil. Very soft soils may not provide enough lateral support for a proper performance of a stone column treatment. The undrained shear strength of the surrounding soil is generally used as the criterion to decide the feasibility of the treatment, with lower bound in the range 5–15 kPa (Wehr, 2006). In recent years, geotextile encasement has been successfully used to extend the use of stone columns to extremely soft soils. Apart from the lateral support, geotextile

encasement acts as a filter between clay and sand; this ensures the effective drainage and avoids contamination of the sand with fines. Lately, other geosynthetics, such as geogrids, are also used for column encasement (Sharma et al., 2004; Gniel and Bouazza, 2009) because of their high tensile stiffness, yet they cannot avoid sand contamination.

Besides experimental work, most of the research on encased stone columns is done using numerical methods (e.g. Murugesan and Rajagopal, 2006; Malarvizhi and Ilamparuthi, 2007: Yoo, 2010) and very few analytical solutions are available (Raithel and Kempfert, 2000; Pulko et al., 2011). This paper presents a new analytical solution to study the settlement reduction and the acceleration of consolidation caused by encased stone columns. The proposed solution is an extension of another analytical solution recently developed by the authors for stone columns (Castro and Sagaseta, 2009). The solution assumes linear elastic behaviour of soil and linear elastic-perfectly plastic behaviour of encasement and column. Furthermore, the proper loading history is considered (undrained loading and consolidation analysis), and equilibrium and compatibility conditions, both in vertical and radial directions, are fulfilled. So, many of the limitations of the existing analytical solutions (Raithel and Kempfert, 2000) are overcome.

The analytical solution gives a quantitative assessment of the improvement introduced by the column encasement and the influence of its stiffness on the system performance. The axial



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Notation			
a _r	Area replacement ratio: $a_r = A_c/A_l$		
Cv	Coefficient of consolidation		
Cu	Undrained shear strength		
k	Coefficient of permeability		
p_{a}	Applied vertical unit load		
<i>r</i> _l , <i>r</i> _c	Radius of the unit cell, of the column		
S _r	Radial displacement		
Sz	Settlement		
s _{z0}	Settlement without columns		
Α	Cross-section		
Ε	Young's modulus		
Em	Oedometric (constrained) modulus: $E_m = [E(1 - \nu)]/$		
	$[(1+\nu)(1-2\nu)]$		
G	Shear modulus: $G = E/[2(1 + \nu)]$		
Jg	Tensile stiffness of the encasement		
$\bar{K_0}$	Coefficient of at rest lateral pressure		
Ka	Coefficient of active earth pressure		

tensile stress of the encasement is also evaluated. Finally, the analytical solution is compared with numerical analyses.

2. Proposed solution

2.1. Model

Column encasement may be very useful under concentrated loads (Murugesan and Rajagopal, 2010) but the presented analytical solution is based on a "unit cell" model (Fig. 1) and is, therefore, limited to distributed uniform loads. Because of the symmetry, only one column and its surrounding soil are studied in axial symmetry. Furthermore, the column is assumed to be fully penetrating through the soft soil and the applied load is considered as rigid, which results in uniform settlement. The area of soft soil, A_l , that is improved by each column, A_c , is generally expressed by the area replacement ratio, $a_r=A_c/A_l$, but sometimes is also defined in terms of the relation between diameters or radii, $N=r_l/r_c=\sqrt{1/a_r}$.

The solution is developed for a horizontal slice at a depth *z* of the unit cell, and consequently, shear stresses between slices at different depths are not considered. The overall behaviour of the whole unit cell is obtained by means of integration of the solution at the different depths.

2.2. Consolidation

Consolidation around encased stone columns is a fully coupled problem. However, a reasonably accurate simple solution can be obtained using the average value of the excess pore pressure along the radius, \bar{u} . The details of this type of approach can be found in Castro and Sagaseta (2009). Only one instantaneous load step is considered and remoulding effects during installation that may alter the soil permeability are neglected. A further assumption is the infinite permeability of the column (drain), which is doubtful for conventional stone columns but is reasonable if the columns are assumed to be encased in a geotextile. In this way, consolidation around encased stone columns can be studied using any conventional solution for radial consolidation (e.g. Barron, 1948) and a modified coefficient of consolidation that accounts for the influence of column and encasement.

L	Column length	
Ν	Column spacing ratio: $N = r_l/r_c$	
SCF	Stress concentration factor: SCF = σ_{zc}/σ_{zs}	
$T_{\rm g}$	Tensile stress of the encasement	
$T_{g,max}$	Tensile strength limit of the encasement	
β	Settlement reduction factor: $\beta = s_z/s_{z0}$	
φ	Friction angle	
γ	Unit weight	
λ	Lamé's constant: $\lambda = 2G\nu/(1-2\nu) = E_m - 2G$	
ν	Poisson's ratio	
ψ	Dilatancy angle	
Subscri	pts/superscripts:	
c,g,s,l	column, encasement, soil, elementary cell	
e,p	elastic, plastic	
i,u,f,y	initial (previous), undrained, final, at yielding	
r,z,θ	cylindrical coordinates	
⁻ (11006	er har) average value along the radius	

2.3. Encasement

Although stone columns are commonly encased using geosynthetics, or more precisely geotextiles or geogrids, the term encasement is preferred here for the sake of generality. It is modelled as a cylindrical shell of negligible thickness around the column. Encasement behaviour is supposed to be linear elasticperfectly plastic and characterized by a tensile stiffness, J_g , and a tensile strength, $T_{g,max}$ (Fig. 1). During column installation, the encasement is pre-stressed to an initial tensile stress, $T_{g,i}$. The encasement tensile stress obtained with the analytical solution is the increment from that value, ΔT_g .

In vertical direction, the unit cell is compressed and the encasement can only take tension because it is a flexible membrane. So, the encasement acts only in radial direction. Its equilibrium and compatibility conditions (Fig. 2) are those of a thin tube under internal, σ_{rc} , and external pressure, σ_{rs} .

$$\sigma_{\rm rc} = \frac{T_{\rm g}}{r_{\rm c}} + \sigma_{\rm rs} \tag{1}$$

$$T_{\rm g} = J_{\rm g} \frac{s_{\rm r}}{r_{\rm c}} \tag{2}$$

where s_r is the radial displacement of the interface.

Combining these two equations, the radial equilibrium between soil and column at their interface depends on the encasement properties (stiffness and radius) and its radial expansion.

$$\sigma_{\rm rc} = \frac{J_{\rm g} s_{\rm r}}{r_c^2} + \sigma_{\rm rs} \tag{3}$$

2.4. Elastic solution

As a first approximation, elastic behaviour is assumed for the soil, the encasement and the column. As previously mentioned, only a horizontal slice at a depth *z* of the unit cell is analysed (Fig. 3). The column is a vertical solid cylinder subjected to a vertical uniform pressure σ_{zc} and a radial pressure σ_{rc} at its lateral wall. The soil is a cylinder with a central cylindrical cavity, subjected to a vertical uniform effective pressure σ_{zs} ', a radial pressure σ_{rs} at the cavity wall (soil/encasement





interface) and an excess pore pressure \overline{u} . The encasement is a cylinder shell that relates the radial stresses of soil and column at their interface, as explained in the previous section. These five pressures determine the stresses and strains at any point of the soil and the column, and the tensile stress of the encasement.

The conditions of vertical equilibrium and compatibility and radial equilibrium (Eq. (3)) and compatibility of deformation at the encasement must be imposed. These four equations allow to express the above four vertical or radial pressures in terms of the pore pressure \bar{u} and the applied vertical pressure p_a only.

The development of the solution is analogous to that without encasement (Castro and Sagaseta, 2009) and most importantly, the solution is exactly the same but adjusting the constants H and F to account for the encasement.

$$H = \frac{1}{a_{\rm r}} (\lambda_{\rm c} + G_{\rm c} + G_{\rm s}) - (G_{\rm c} - G_{\rm s}) + \frac{1 - a_{\rm r}}{a_{\rm r}} \frac{J_{\rm g}}{2r_{\rm c}}$$
(4)

$$F = \frac{(1-a_{\rm r})(\lambda_{\rm c}-\lambda_{\rm s})}{2\left[a_{\rm r}(\lambda_{\rm s}-\lambda_{\rm c}+G_{\rm s}-G_{\rm c}) + \lambda_{\rm c}+G_{\rm c}+G_{\rm s}+(1-a_{\rm r})\frac{J_{\rm g}}{2r_{\rm c}}\right]}$$
(5)

H and *F* are constants of the solution for the undrained and final elastic states, respectively. For instance, they relate the applied load to the vertical strain as follows:



Fig. 2. Equilibrium and compatibility conditions of the encasement.

$$\varepsilon_{z,u} = \frac{p_a}{H - (1 - 3a_r)(G_c - G_s)}$$
(6)

$$\varepsilon_{z,f}^{e} = \frac{p_{a}}{(\lambda_{c} + 2G_{c})a_{r} + (\lambda_{s} + 2G_{s})(1 - a_{r}) - 2a_{r}(\lambda_{c} - \lambda_{s})F}$$
(7)

Both constants, H and F, have now a new term that accounts for the encasement (Eqs. (4) and (5)). As expected, encasing the column is more effective when the encasement stiffness is higher and the column diameter is lower. Eqs. (4) and (5) enable a quantitative assessment of the encasement influence and, for usual soil and column properties, the improvement provided by the encasement is really small. The encasement starts to be really useful only when the soil does not provide enough lateral support and the column yields. The small influence of the encasement in the elastic case is shown by the small increment of its tensile stress.

$$\Delta T_{g}^{e} = J_{g}F\left(\varepsilon_{z} - \frac{\overline{u}}{\lambda_{c} - \lambda_{s}}\right)$$
(8)



Fig. 3. Equilibrium and compatibility conditions between soil and column.

Table 1

Plastic increments of strains and stresses from the moment of column yielding for an increment of the applied load, p_a^p . Drained analysis.

	Column (c)	Soil (s)
Vertical strain, $\Delta \varepsilon_Z^p$	p_a^p/E_{ml}^p	p_a^p/E_{ml}^p
Radial displacement, Δs_r	$rrac{\Delta arepsilon_{p_{c}}^{2}}{2K_{arphi c}}$	$r rac{a_r}{1-a_r} rac{1-(r/r_l)^2}{(r/r_l)^2} rac{\Delta \varepsilon_z^p}{2K_{\psi c}}$
Radial effective stress, $\Delta \sigma'_r$	$J\Delta \varepsilon_Z^p$	$[\lambda_s + rac{G_s(r_c/r)^2 + a_r(G_s + \lambda_s)}{(1-a_r)K_{\psi_c}}]\Delta \varepsilon_z^p$
Hoop effective stress, $\Delta \sigma'_{\theta}$	$J\Delta arepsilon_Z^p$	$[\lambda_s - rac{G_s(r_c/r)^2 - a_r(G_s + \lambda_s)}{(1 - a_r)K_{\psi_c}}]\Delta \varepsilon_Z^p$
Vertical effective stress, $\Delta \sigma'_z$	$\frac{J\Delta \varepsilon_{r}^{p}}{K_{ac}}$	$[\lambda_s + 2G_s + rac{a_r\lambda_s}{(1-a_r)K_{\psi c}}]\Delta arepsilon_z^p$
Octahedral effective stress, $\Delta\sigma'_{oct}$	$\frac{(2+1/K_{ac})}{3}J\Delta\varepsilon_{z}^{p}$	$(\lambda_s + \frac{2}{3}G_s)[1 + \frac{a_r}{(1-a_r)K_{\psi c}}]\Delta \varepsilon_z^p$
Volumetric strain, $\Delta \varepsilon_{vol}$	$(1-rac{1}{K_{arepsilon c}})\Delta arepsilon_z^p$	$[1+rac{a_r}{(1-a_r)K_{\psi c}}]\Delta arepsilon_Z^p$

 $E_{ml}^{p} = (\lambda_{s} + 2G_{s}) (1 - a_{r}) + \frac{a_{r}\lambda_{s}}{K_{\psi c}} + \frac{a_{r}I}{K_{ac}}$

 $J = \lambda_s + \frac{G_s + a_r(G_s + \lambda_s)}{(1 - a_r)K_{\psi c}} + \frac{J_g}{2r_c K_{\psi c}}$

The modified coefficient of consolidation for the elastic case, c_{vr}^{Zre} , is the same as that without encasement but changing *H* consistently (Eq. (4)).

$$c_{\rm vr}^{\rm zre} = c_{\rm vr} \cdot \frac{E_{\rm mm}[H - (\lambda_{\rm c} - \lambda_{\rm s})] - (1 - a_{\rm r})(\lambda_{\rm c} - \lambda_{\rm s})^2}{(\lambda_{\rm s} + 2G_{\rm s})[H - (1 - 3a_{\rm r})(G_{\rm c} - G_{\rm s})]}$$
(9)

where $E_{mm} = a_r(\lambda_c + 2G_c) + (1 - a_r)(\lambda_s + 2G_s)$

The complete elastic solution is not detailed here to avoid repetition with Castro and Sagaseta (2009) and Balaam and Booker (1981).

2.5. Plastic deformation of the column

Since the encasement is useful only after column yields, the next step is to include an elastic-perfectly plastic behaviour of the column. Plastic strains in the column can be adequately modelled with the Mohr–Coulomb yield criterion and a non-associated flow rule, with a constant dilatancy angle ($\psi_c \neq \varphi_c$). The increments of elastic strains in the column during plastic deformation are neglected to keep the solution as simple as possible. Similar to the elastic case, the solution and its development are again analogous to those without encasement (Castro and Sagaseta, 2009). The only difference is in the constant *J*, whose value is now

$$J = \lambda_{\rm s} + \frac{G_{\rm s} + a_{\rm r}(G_{\rm s} + \lambda_{\rm s})}{(1 - a_{\rm r})K_{\psi \rm c}} + \frac{J_{\rm g}}{2r_{\rm c}K_{\psi \rm c}}$$
(10)

where $K_{\psi c} = (1 - \sin \psi_c)/(1 + \sin \psi_c)$



Fig. 4. Modified coefficient of consolidation. Plastic column. Influence of the encasement stiffness.

Similar to H and F for the undrained and final elastic states, J is a constant of the solution for the plastic increment and relates, for example, the excess pore pressure increment to the vertical strain increment:

$$\Delta \varepsilon_{z}^{p} = \frac{\left(1 - a_{r} + \frac{a_{r}}{K_{ac}}\right)\overline{\Delta u}}{(\lambda_{s} + 2G_{s})(1 - a_{r}) + \frac{a_{r}\lambda_{s}}{K_{\psi c}} + \frac{a_{r}J}{K_{ac}}}$$
(11)

The lateral confinement provided by the encasement is now considerable and its relevance depends on the ratio between the relative stiffness of the encasement and the soil. As a result, the increment of the tensile stress of the encasement is also significant:

$$\Delta T_{g}^{p} = \frac{J_{g}}{2K_{\psi c}} \Delta \varepsilon_{z}^{p}$$
(12)

In this elastic—plastic analysis, the initial stresses existing before load application must be included. As these stresses can vary with depth, the analysis depends on the depth *z*. The modified coefficient of consolidation (c_{VT}^{zrp}) is the same as that without encasement but changing the value of *J* (Eq. (10)) consistently.

$$c_{\rm vr}^{\rm zrp} = c_{\rm vr} \cdot \frac{(1-a_{\rm r}) + \frac{a_{\rm r}}{(\lambda_{\rm s}+2G_{\rm s})} \left(\frac{\lambda_{\rm s}}{K_{\psi \rm c}} + \frac{J}{K_{\rm ac}}\right)}{\left[1 + \frac{a_{\rm r}}{(1-a_{\rm r})K_{\psi \rm c}}\right] \left(1-a_{\rm r} + \frac{a_{\rm r}}{K_{\rm ac}}\right)}$$
(13)



Fig. 5. Time-settlement curves for common encasement stiffnesses.



Fig. 6. Stress concentration factor through time. Influence of the encasement stiffness.

The time of yielding, the solution of the plastic increment and its integration for the whole column are not repeated here because they are the same as those without encasement (Castro and Sagaseta, 2009) but changing the value of J (Eq. (10)) consistently.







Fig. 8. Stress concentration factor through time. Plastic column. Influence of the tensile strength of the encasement.

2.6. Tensile strength of the encasement

The considerable increment of the tensile stress of the encasement when the column yields (Eq. (12)) may cause the encasement to reach its tensile strength, $T_{g,max}$. Therefore, the condition of encasement yielding must be imposed.

$$T_{g,max} - T_{g,i} - \Delta T_g^y = \Delta T_{g,max}^p = \frac{J_g}{2K_{\psi c}} \Delta \varepsilon_z^{py}$$
(14)

 ΔT_g^y is the tensile stress increment at column yielding (Eq. (8)). The superscript "py" refers to plastic column and encasement yielding. The pore pressure at the moment of encasement yielding, \overline{u}_{py} , is related to $\Delta \varepsilon_{zy}^{py}$ (Eq. (11)).

$$\Delta \varepsilon_{z}^{\text{py}} = \frac{\left(1 - a_{\text{r}} + \frac{a_{\text{r}}}{K_{\text{ac}}}\right) \left(\overline{u}_{\text{py}} - \overline{u}_{\text{y}}\right)}{(\lambda_{\text{s}} + 2G_{\text{s}})(1 - a_{\text{r}}) + \frac{a_{\text{r}}\lambda_{\text{s}}}{K_{\psi\text{c}}} + \frac{a_{\text{r}}J}{K_{\text{ac}}}}$$
(15)

where J is defined by Eq. (10).

After encasement yielding, the solution is the same as that without encasement and it can be found in Castro and Sagaseta (2009). Note: *J* does not include the influence of the encasement, because it expands at constant hoop stress.



Fig. 9. Tensile stress of the encasement. Plastic column. Influence of the tensile strength of the encasement.



 $\ensuremath{\textit{Fig. 10.}}$ Comparison of the tensile stress of the encasement with numerical calculations.





b Influence of the encasement tensile strength

Fig. 11. Settlement reduction factor, β , with the applied load.



Fig. 12. Settlement reduction factor, β . Influence of the encasement stiffness for different area replacement ratios.

2.7. Drained solution

The presented solution considers an undrained loading followed by a consolidation process. However, consolidation around stone columns, especially if the columns are encased with a geotextile, may be nearly as fast as the loading pace, which means that for those cases drained condition is a more reasonable assumption. In any case, depending on the soil permeability and the loading pace, the real behaviour is between drained condition and an undrained loading followed by consolidation. These two different assumptions or limit cases have already been used for conventional stone columns. Pulko and Majes (2005) developed a drained solution while the authors considered the consolidation process (Castro and Sagaseta, 2009). Both solutions are slightly different when the column yields, because it follows different stress paths.

In this section, the solution is developed for drained condition, obtaining a simplified formulation for the settlement reduction factor and the tensile stress of the encasement.

Balaam and Booker (1981) proposed the drained solution for conventional stone columns when soil and column are elastic. The drained solution for encased stone columns is the same but changing *F* accordingly (Eq. (5)). So, for example, the settlement reduction factor for this elastic case, β^{e} , is the ratio of vertical strain with columns ($\varepsilon_{z} = p_{a}/E_{ml}^{e}$) and without columns ($\varepsilon_{z0} = p_{a}/E_{ms}^{e}$):



Fig. 13. Settlement reduction factor, β . Comparison with field measurements.



Fig. 14. Comparison of drained and consolidation analyses.

$$\beta^{\rm e} = \frac{E_{\rm ms}}{E_{\rm ml}^{\rm e}} \tag{16}$$

where E_{ml}^{e} is a kind of average oedometric modulus of the unit cell.

$$E_{\rm ml}^{\rm e} = (1 - a_{\rm r})E_{\rm ms} + a_{\rm r}E_{\rm mc} - 2a_{\rm r}(\lambda_{\rm c} - \lambda_{\rm s})F \tag{17}$$

The first and second terms are the weighted values of the soil and column moduli respectively and the third term is the influence of the radial displacement of the soil/column interface where the encasement is placed.

Assuming an initial geostatic stress state and neglecting the influence of column installation, the initial stresses are

$$\sigma_{zs,i} = \gamma'_{s} z$$

$$\sigma_{zc,i} = \gamma'_{c} z$$

$$\sigma'_{rs,i} = \sigma_{rc,i} = K_{0s} \sigma'_{zs,i}$$

$$u_{i} = 0$$
(18)

Now, column yielding at a specific depth, z, does not depend on the time and is controlled only by the applied load, p_{a}^{y} .

$$P_{a}^{y} = Yz$$

$$Y = \frac{E_{ml}^{e}(K_{0s}\dot{\gamma_{s}} - K_{ac}\dot{\gamma_{c}})}{2G_{c}(K_{ac} + F) - \lambda_{c}(1 - 2F)(1 - K_{ac})}$$
(19)

The drained solution when the column is at its active limit state for a plastic increment $p_a^p = p_a - p_a^y$, is detailed in Table 1, and in particular, the settlement reduction factor for a plastic column is the ratio of vertical strain with columns ($\varepsilon_z^p = p_a^p/E_{ml}^p$) and without columns ($\varepsilon_{z0}^{p} = p_{a}^{p}/E_{ms}$):

$$\beta^p = \frac{E_{ms}}{E_{ml}^p} \tag{20}$$

where E_{ml}^{p} is again a kind of average modulus of the unit cell as (17) but for a plastic column.

$$E_{\rm ml}^{\rm p} = (1 - a_{\rm r})E_{\rm ms} + a_{\rm r}\frac{\lambda_{\rm s}}{K_{\rm \psi c}} + \frac{Ja_{\rm r}}{K_{\rm ac}}$$
(21)

Now, the first term is the weighted value of the oedometric modulus of the soil, the second one accounts for the plastic strains of the column (dilatancy) and the third one is the influence of the radial interaction between soil, encasement and column.

If for the sake of simplicity the encasement is assumed elastic, the average settlement reduction factor depends on the applied load that cause the yielding of the column, p_a^y .

$$\beta = \beta^{e} \frac{p_{a}^{y}}{p_{a}} + \beta^{p} \left(1 - \frac{p_{a}^{y}}{p_{a}} \right)$$
(22)

 p_{λ}^{y} depends on the depth, z, and so does the reduction factor, β . As encased stone columns are useful in very soft soils, whose properties generally vary with depth, it is common to split the problem into different slices. However, if the soil properties are supposed constant with depth, Eq. (22) may be integrated for the whole column of height, L. Column yielding starts at the surface, where the initial stresses are null, and progresses downwards as the applied load increases. Then, for an applied load, the column yields until a depth $z^{y} = p_{a}/Y$. The integration is different if only the upper part of the column is at its active limit state $(z^y \le L)$ or if the whole column has yielded ($z^{y} \ge L$). The solution for the later case is equivalent to applying Eq. (22) to the mean depth of the column (z = L/2).

$$\beta = \beta^{e} \left(1 - \frac{p_{a}}{2LY} \right) + \beta^{p} \frac{p_{a}}{2LY} \quad ; \quad z^{y} \leq L$$

$$\beta = \beta^{e} \frac{LY}{2p_{a}} + \beta^{p} \left(1 - \frac{LY}{2p_{a}} \right) \quad ; \quad z^{y} \geq L$$
(23)

If no encasement is considered in the presented drained solution $(J_g = 0)$, it perfectly agrees with that of Pulko and Majes (2005) for conventional stone columns.

Another interesting result of the drained solution is a simplified expression for the tensile stress of the encasement at a depth, z.

$$T_{\rm g} = J_{\rm g} \left[\frac{F}{E_{\rm ml}^{\rm e}} p_{\rm a}^{\rm y} + \frac{1}{2K_{\psi c} E_{\rm ml}^{\rm p}} p_{\rm a}^{\rm p} \right]$$
(24)

3. Parametric study and numerical analyses

3.1. Numerical model

Coupled numerical analyses of the unit cell (Fig. 1) were performed using the finite element code Plaxis v8.6 (Brinkgreve, 2007). For comparison purposes, the same boundary conditions and material properties of the analytical solution were chosen for the numerical models. Therefore, a rigid plate was set on top of the unit cell, the soil was modelled as elastic and the encasement and the column as elastic-perfectly plastic. The encasement is modelled as a flexible membrane fully tied to the continuum elements of the soil and the column.

A common range of geometries and material properties were modelled for parametric studies and comparisons with the presented analytical solution. The results of the numerical simulations reveal the accuracy of the analytical solution and the influence of its hypotheses, such as neglecting the shear stresses and using an average pore water pressure along the radius.

3.2. Consolidation

The consolidation process around an encased stone column can be studied using any conventional solution for radial consolidation (e.g. Barron, 1948) and a modified consolidation coefficient that accounts for the influence of column and encasement. When the column is elastic, the encasement influence is small and the modified coefficient of consolidation, c_{vr}^{zre} , is nearly the same as that without encasement (2-3 times the basic one for normal geometries and soil and column properties). By contrast, the modified coefficient of consolidation for a plastic column, c_{Vr}^{zrp} , is clearly influenced by the tensile stiffness of the encasement (Fig. 4). If there is no encasement (stone column, $J_g = 0$) or the encasement has reached its tensile strength, c_{Vr}^{zrp} is lower (0.8–0.9 times) than the basic one, while for usual tensile stiffnesses of the encasement, c_{Vr}^{zrp} is slightly higher (1–1.2 times) than the basic one.

The accuracy of the proposed solution in modelling the consolidation process is verified by numerical results in Fig. 5, which shows settlement development with time. However, as it happens for the stone column solution (Castro and Sagaseta, 2009), the agreement for low degrees of consolidation (<30%) is not very good due to inherent assumptions of Barron's solution. The small differences in the final values of the settlement will be analysed in its own section.

3.3. Stress concentration

The ratio between the vertical stress on the column and the soil is usually called stress concentration factor (SCF = σ_{zc}/σ_{zs}) and gives an idea of the part of the applied load that soil transfers to the column. Fig. 6 shows its variation with time. The vertical stresses on the soil and the column may vary with the radius, and therefore, their averaged values are used to calculate the SCF. Again, the influence of the encasement stiffness is only noticeable for a plastic column. In that case, the lateral confinement provided by the encasement allows the column to support a higher load. The distribution of the applied load between soil and column and how it is influenced by the tensile stress of the encasement are plotted in Fig. 7. The stiffer the encasement is, the higher tensile stress it supports and the more lateral confinement it provides to the column. The tensile stress of the encasement increases notably only after column yielding.

That increase of the tensile stress may cause the encasement to reach its tensile strength and then, it no longer helps the column to support any more vertical load (Fig. 8). The numerical analyses show up how well the analytical solution captures the influence of the encasement stiffness and its tensile strength. The tensile stress of the encasement increases with time until it reaches its maximum value (Fig. 9). Now, a direct comparison with the numerical analyses at a specific depth is not possible because in the numerical model, as column yields, shear bands develop in the column causing the tensile stress of the encasement to fluctuate (Fig. 10). However, the agreement between the numerical and the analytical results is very good but for those fluctuations of the numerical values.

3.4. Settlement reduction

The settlement reduction factor, defined as the ratio between the final settlement with and without improvement, $\beta = s_z/s_{z0}$, is used in practice to evaluate the efficiency of the improvement method. The settlement reduction decreases with the applied load, p_a , from an elastic value, β^e , and approaches a plastic one, β^p , at the same rate as plastic strains develop in the column (Fig. 11). The applied load is normalised by the initial vertical stress because column yielding depends on that factor, $p_a/(L\gamma_s')$. Fig. 11(a) illustrates the effectiveness of encasing the columns if the tensile stiffness of the encasement is high enough compared to that of the soil $(J_g/(r_c E_s) > 1)$. So, encasing stone columns is recommended in very soft soils for moderate loads and using a stiff material for the encasement. If the applied load is high, the tensile stress of the encasement reaches its maximum value, $T_{g,max}$, and its effectiveness is severely reduced (Fig. 11(b)).

On the other hand, the settlement reduction introduced by the encasement is nearly the same for different area replacement ratios (Fig. 12), which means that column encasement is equally useful for different area replacement ratios, yet columns of smaller diameters are better confined. In Figs. 11 and 12, the numerical results validate the accuracy of the analytical solution, but the agreement gets slightly worse as the tensile stiffness of the encasement increases. Hence, the only assumption that has a slightly noticeable effect in the results is neglecting the elastic strains in the column during its plastic deformation.

The influence of the geotextile stiffness on the settlement reduction factor has been measured in the field (Kempfert, 2003). Although it is not possible to make a detailed comparison because of the lack of information of the different field sites, the values measured in the field of the settlement reduction factor and their variation with the geotextile stiffness are in good agreement with the proposed analytical solution (Fig. 13). For a typical soft soil stiffness ($E_s = 1$ MPa) and a column radius of $r_c = 0.5$ m, factors of $J_g/(r_c E_s) = 2-5$ imply a geotextile stiffness in the range 1–2.5 MN/m.

3.5. Drained solution

Along with the consolidation analysis, a drained solution has been also developed. Since the stress paths followed by the column in both analyses are not the same, different results are expected. To evaluate those differences, the settlement reduction factor is compared in Fig. 14.

For further comparison, the settlement reduction factor was numerically calculated assuming drained condition. The differences between a drained and a consolidation analysis using finite elements are very small and are only noticeable for low loads when the whole column has not yielded yet. By contrast, the simplified drained formulation of the proposed solution differs visibly from the consolidation approach. The differences are greater for higher encasement stiffnesses. As it was mentioned in the previous section, the analytical solution predicts slightly lower settlements than the numerical analyses because it disregards the elastic strains in the column during plastic deformation. That phenomenon is clearer in a drained approach because the column yields earlier than it does if undrained loading and consolidation are modelled. Nevertheless, the drained solution is still very useful for a quick rough estimate of the settlement reduction factor (Eq. (23)). Pulko et al. (2011) developed a new analytical solution for drained condition that includes the elastic strains in the column during its plastic deformation. The formulation is slightly more complex but the results are very accurate.

4. Conclusions

A new analytical solution has been developed to study the deformation and consolidation around encased stone columns. The solution considers column and encasement yielding. The governing parameters are identified and their influence on the settlement, stress concentration and consolidation process is evaluated. The solution is presented in a closed form and is directly usable in a spreadsheet.

Column encasement has negligible effect for an elastic column and starts to be useful only after column yielding. The effectiveness of the encasement is directly related to its stiffness through the factor $J_g/(r_c E_s)$. Therefore, encasing stone columns is recommended in soft soils using stiff encasements and under moderate loads because for high applied loads, the encasement reaches its tensile strength and does not provide any further improvement. The settlement reduction provided by the encasement does not depend on the area replacement ratio.

A simplified formulation of the proposed solution is obtained assuming drained condition.

Comparisons of the proposed solution with numerical analyses show a good agreement, which confirms the validity of the solution and its hypotheses. So, the proposed solution is a simple and accurate tool for the design of encased stone columns.

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