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An inventory control model with consideration of remanufacturing and product life cycle

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ABSTRACT

This paper investigates inventory control policies in a manufacturing/remanufacturing system during the product life cycle, which consists of four phases: introduction, growth, maturity, and decline. Both demand rate and return rate of products are random variables with normal distribution; the mean of the distribution varies according to the time in the product life cycle. Closed-form formulas of optimal production lot size, reorder point, and safety stock in each phase of the product life cycle are derived. A numerical example is presented with sensitivity analysis. The result shows that different inventory control policies should be adopted in different phases of the product life cycle. It is also found that the optimal production lot size and reorder point are not sensitive to the phase length and the demand changing rate.

1. Introduction

For the past few decades, electronic companies have faced two major pressures: short product life cycle and environmental sustainability. First, technology advances have shortened the life cycles for many products. Product demand may increase rapidly at first and then decrease a few months later due to the emergence of new products. Inventory management under a short product life cycle is not easy. Many problems such as large safety stock, high obsolescence costs, and high forecasting errors will arise. It is necessary to consider the constantly varying demand and its uncertainty when making inventory control policy. In addition, due to the short product life cycle and the emergence of new products, an outdated product may be returned even if it is still in good condition. For example, a customer may buy a new mobile phone to replace his/her old mobile phone just because he/she likes the new one, although the old mobile phone is still in good condition.

Second, due to environmental and ecological responsibility, enterprises are trying to reuse, remanufacture, and recycle the used products in order to reduce the negative impact on environment. Companies in many countries are required to conform to the Waste of Electric and Electronic Equipment (WEEE) directives (Rahimifard and Clegg, 2007). Environmental sustainability and green supply chain management have received increasing attention since the 1990s (Seuring and Müller, 2008). Several international journals have published special issues about sustainable/green supply chain management in recent years (Piplani et al., 2007; Rahimifard and Clegg, 2007; Jayaram et al., 2007; Seuring et al., 2008; see also Srivastava, 2007; Seuring and Müller, 2008).

The pressures of a short product life cycle and environmental sustainability make remanufacturing a reasonable choice. Remanufacturing is an industrial process, whereby used/broken products are restored to useful life. Remanufacturing is also an important part of sustainable supply chain and reverse logistics. The motives for product remanufacturing include legislation, increased profitability, ethical responsibility, secured spare part supply, and brand protection. Reasons for returning used products include end-of-life returns, end-of-use returns, commercial returns, and reusable components (Östlin et al., 2008). After remanufacturing, the returned products, along with the new products, comprise the serviceable inventory and satisfy customer demand. Inventory control in such remanufacturing systems becomes complicated. In many cases, used products are assumed to be collected and remanufactured to a good-as-new state, such as car batteries, printer cartridges, one-time use cameras, and some electronic components. Customers cannot distinguish 'new' (i.e. manufactured) products from repaired products (i.e. remanufactured), or they consider these two products as interchangeable. For example, about 90% of Kodak one-time use cameras (OTUCs) are produced from recycled camera bodies, and about 90% (by weight) of a used Kodak (2005) OTUC body is directly reused in the manufacture of new cameras (Mukhopadhyay and Ma, 2009).

The purpose of this paper is to investigate the effects of the product life cycle on inventory control in a manufacturing/remanufacturing system and to determine the optimal production

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lot size, reorder point, and safety stock during each phase of the product life cycle. The product life cycle is divided into following phases: introduction (phase 1), growth (phase 2), maturity (phase 3), and decline (phases 4 and 5). Both demand rate and return rate of products are random variables with normal distribution; the mean of the distribution varies according to the time in the product life cycle. Before introducing our model, we present a brief literature review.

van der Laan et al. (1996a, 1996b) consider several inventory control strategies with remanufacturing and disposal. Product demands and returns are assumed to be independent Poisson processes; push and pull strategies are considered in the inventory model (van der Laan and Salomon, 1997; van der Laan et al., 1999a; van der Laan and Teunter, 2006) to coordinate production, remanufacturing, and disposal operations. Lead time effects are further investigated in a similar remanufacturing system to improve system performance (van der Laan et al., 1999b; Kiesmüller, 2003a, 2003b). Recently, Mukhopadhyay and Ma (2009) review joint procurement and production decisions in remanufacturing under quality and demand uncertainty. Three different cases are presented, and the optimal procurement and the production quantity for the firm are determined.

All the above articles have an assumption that the demand rate and return rate are independent. In contrast, Kiesmüller and van der Laan (2001) develop an inventory model in which the random returns depend explicitly on the demand stream. They assume a constant probability that an item is returned. Dobos (2003) considers inventory strategies for a reverse logistics system in which demand is a known continuous function in a given planning horizon and the return rate of used items is also a given function of time; there is a constant delay between these two functions. To take stochastic demand rate and return rate into consideration, most relevant articles assume that demand rate and return rate follow specific distributions with fixed parameters, which are consistent through the product life cycle. However, Östlin et al. (2009) have developed strategies to balance supply and demand for remanufactured products during the product life cycle, but they do not present a clear inventory control policy.

As previously mentioned, the product life cycle is shorter than before, especially in the electronics industry. Product demand may increase rapidly at first and then decrease a few months later. In addition, the product may be returned even if it is still in good condition. Therefore, the product life cycle influences not only long-term strategies but also operational activities. If the product life cycle is not considered in inventory control, then product shortage or overstocking is more likely to occur. Reiner et al. (2009) point out that when the life cycle structure is not considered in the demand model, forecasting errors may become uncomfortably high, leading to large safety inventories and a substantial risk of high obsolescence costs.

To our knowledge, very few articles consider product life cycle, inventory control, and remanufacturing simultaneously. Ahiska and King (2010) use a discrete-time Markov decision process to find the optimal inventory policy (i.e. the manufacturing and the remanufacturing strategy that have the smallest cost) in each life cycle stage. Unlike in our paper, the same inventory policy is adopted within a stage, because the mean demand and the mean return are both assumed to be constant within each stage. Also, the length of a stage is considered to be long enough so that the problem can be treated as a set of infinite-horizon problems. Chung and Wee (2011) also develop an integrated production inventory model with short life cycles to consider green product design and remanufacturing with re-usage concept. An optimal replenishment policy is derived. The result of the analysis shows that the re-manufacturability and the component life cycle of product design are interrelated. They have shown that new technology evolution, remanufacturing ratios, and system's holding costs are critical factors affecting decision making in a green supply chain inventory control system.

The rest of this paper is organized as follows. Section 2 presents the assumptions and notations. Section 3 explains the mathematical modeling. Section 4 provides numerical examples and sensitivity analysis. The paper concludes in Section 5.

2. Assumptions and notations

2.1. Notations

Decision variables:

y_i	number of production activities in phase i of the product life cycle
s_i	safety stock in phase i of the product life cycle

Dependent variables:

$Q_{i,j}$	production lot size in the jth production activity in phase i
$ROP_{i,j}$	inventory level of reorder point for the jth production activity in phase i
$D_{i,j}$	mean of the total demand during the lead time of the jth production activity in phase i
TC_i	sum of the fixed cost of manufacturing orders and the holding cost in phase i
$I(t)$	expected inventory level at time t

Parameters:

$\lambda(t)$	mean of the demand rate at time t
σ_λ^2	variance of the demand rate
$\gamma(t)$	mean of the return rate at time t
σ_γ^2	variance of the return rate
$Cov_{\lambda\gamma}$	covariance between the demand and return rates
$\tilde{\lambda}(t)$	mean of net demand rate at time t; $\tilde{\lambda}(t)=\lambda(t)-\gamma(t)$
T_i	length of phase i
τ	lead time for manufacturing
K	fixed cost per manufacturing order
h	holding cost of a product per unit time
a_i, b_i	constants
β	preset fill rate, referring to the fraction of product demand that is met from products in inventory, i.e., the probability of no stockout

2.2. Assumptions

The scheme of the manufacturing/remanufacturing system in this paper is illustrated in Fig. 1. The serviceable inventory stocks the manufactured and remanufactured products to satisfy the demand. There are two ways to replenish the serviceable inventory: by manufacturing new products or by remanufacturing returned products. The remanufactured products are assumed to be as good as the new ones. We also assume that both the demand and the return rate of products are random variables with normal distribution and that the mean of the distribution varies according to the time spent in the product life cycle (Fig. 2). We can see that the return rate is not independent of the demand rate. There is a time lag between the two functions, and the peak of the return rate function decreases.

The product life cycle has four phases (i.e., introduction, growth, maturity, and decline) that can be demarcated according to several factors (e.g. sales, demand, profits, and competitors).

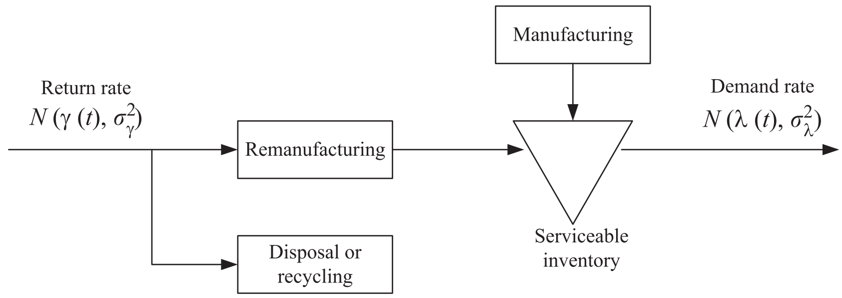

Fig. 1. Scheme of the manufacturing/remanufacturing system.

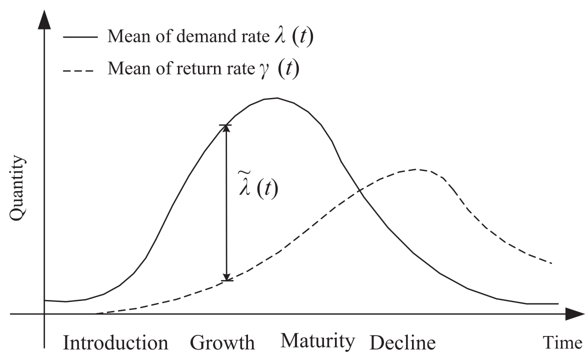

Fig. 2. Product life cycle and the relation between demand rate and return rate.

A firm can also use its experience or the historical data of similar products to predict the phase lengths of a product's life cycle. In this paper, without loss of generality, different phases of a product's life cycle are demarcated according to the demand. Each phase is described as follows:

(1) *Introduction*: demand remains at a low level, i.e., $\lambda(t)=a_1$, where $a_1>0$.
(2) *Growth*: demand begins to increase rapidly, i.e., $\lambda(t)=at+b$, where $a>0$.
(3) *Maturity*: demand remains in a steady state, without significant increase or decrease, i.e., $\lambda(t)=a'$, where $a'>a_1$.
(4) *Decline*: demand starts to decrease, i.e., $\lambda(t)=at+b$, where $a<0$.

We also make the following assumptions: (1) the lead time for manufacturing is a constant; (2) a returned product is either remanufactured or recycled/disposed immediately; the remanufacturing time is ignored; (3) the unit cost for remanufacturing a returned product is less than the unit cost for manufacturing a new product; and (4) no salvage value or disposal costs are applied to a recycled/disposed product.

3. Mathematical modeling

In this section, we discuss how to manage the inventory during different phases of the product life cycle, including how to determine the optimal production lot size, reorder point, and safety stock.

3.1. Introduction (phase 1)

In this phase, demand remains at a low level; the returned products are rarely seen and can be ignored. Suppose $\lambda(t)=a_1$ and $\lambda(t)$ is a constant, then we can use the Economic Order Quantity (EOQ) model to determine the optimal production lot size. The sum of the fixed cost of manufacturing orders and the holding cost is expressed as follows:

$$TC_1=(a_1T_1/Q_1)K+(Q_1/2)T_1h \tag{1}$$

According to the EOQ model, we can derive the optimal production lot size Q_1 for each production activity in phase 1 as follows:

$$Q_1=\sqrt{\frac{2a_1K}{h}} \tag{2}$$

Since the total demand during the lead time is also a random variable that follows normal distribution $N(\tau a_1,\tau^2\sigma_\lambda^2)$, the safety stock is necessary to prevent stockout and to maintain a high fill rate β. The higher the fill rate, the greater the amount of safety stock needed.

Suppose that F^{-1} is the reverse cumulated probability function of the standard normal distribution. The safety stock can then be derived as follows:

$$s_1=F^{-1}(\beta)\tau\sigma_\lambda \tag{3}$$

After the safety stock is determined, the reorder point for each production activity in this phase can be calculated as

$$ROP_1=\tau a_1+s_1 \tag{4}$$

Once the inventory level drops down to ROP_1, an order for Q_1 products is placed.

3.2. Growth (phase 2)

In this phase, demand begins to rise rapidly, and some returned products emerge due to end-of-use or end-of-life. For simplification, we suppose that the means of both the demand and the return rates increase linearly over time in this phase. Since the demand rate is greater than the return rate and the unit cost for remanufacturing a returned product is less than the unit cost for manufacturing a new product, all returned products will be remanufactured. The mean of the net demand rate for new manufactured products is equal to $\lambda(t)-\gamma(t)$, which is a linear function $\tilde{\lambda}(t)=a_2t+b_2$ (Fig. 3). Since $\tilde{\lambda}(t)$ increases over time, the production lot size may not be the same each time. A production lot size must be equal to the sum of the net demands during the time before next production activity. In other words, the production lot size $Q_{2,j}$ is the integral of the net demand rate function $\tilde{\lambda}(t)$ between jth and $j+1$th production activities, as illustrated in Fig. 3. We can see that the production lot size increases with production activities. Suppose that there are y_2 production activities in this phase and that the length of time between production activities is the same. Once y_2 is determined, the production lot sizes $Q_{2,j}$ can be calculated as

$$Q_{2,j}=\int_{jT_2/y_2}^{(j+1)T_2/y_2}\tilde{\lambda}(t)dt=\frac{a_2}{2}(2j+1)\left(\frac{T_2}{y_2}\right)^2+b_2\left(\frac{T_2}{y_2}\right),\quad j=0,\ldots,y_2-1 \tag{5}$$

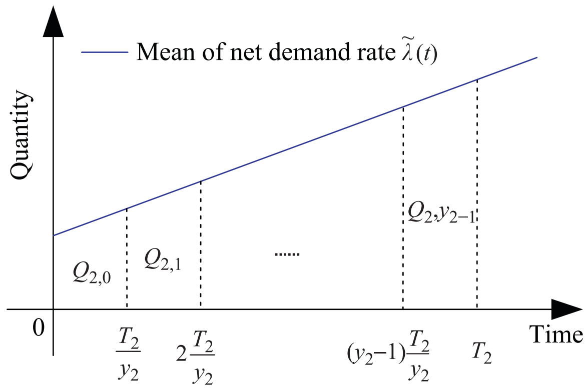

Fig. 3. Illustration of production lot size $Q_{2,j}$ in phase 2.

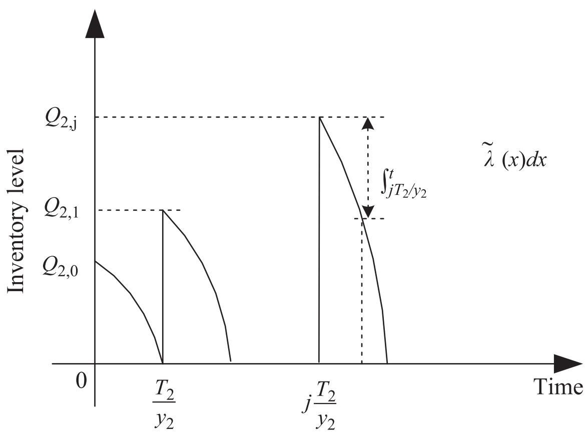

Fig. 4. Expected inventory level (safety stock not considered yet) in phase 2.

To choose an optimal y_2, we establish the following mathematical model, in which TC_2 is the sum of the fixed costs of manufacturing orders and the holding cost:

$$\min_{y_2} TC_2 = Ky_2 + \sum_{j=0}^{y_2-1} h \int_{jT_2/y_2}^{(j+1)T_2/y_2} \left(Q_{2,j} - \int_{jT_2/y_2}^{t} \tilde{\lambda}(x)dx \right) dt \tag{6}$$

The second term in Eq. (6) refers to the holding cost, which is the integral of the expected inventory level multiplied by the unit holding cost h. As illustrated in Fig. 4, the expected inventory level function $I(t)$ (safety stock not yet considered) is zigzag, which is a common characteristic in inventory management. However, different features are that (t) is not linearly decreasing and that the production lot size is increasing due to the increasing $\tilde{\lambda}(t)$. The expected inventory level function can be derived as follows:

$$I(t) = Q_{2,j} - \int_{jT_2/y_2}^{t} \tilde{\lambda}(x)dx = Q_{2,j} - \left(\frac{a_2}{2}\left(t^2 - \left(j\frac{T_2}{y_2} \right)^2 \right) + b_2\left(t - j\frac{T_2}{y_2} \right) \right)$$

$$j = 0,\ldots,y_2-1; \quad jT_2/y_2 \le t \le (j+1)T_2/y_2 \tag{7}$$

where $\int_{jT_2/y_2}^{t} \tilde{\lambda}(x)dx$ refers to the cumulative net demand from the beginning of that production activity to time t.

Applying Eqs. (5) and (7) to Eq. (6), we obtain the following model:

$$\min_{y_2} TC_2 = Ky_2 + h\left(\frac{a_2}{12} T_2^3 y_2^{-2} + \left(\frac{a_2}{4} T_2^3 + \frac{b_2}{2} T_2^2 \right) y_2^{-1} \right) \tag{8}$$

Because TC_2 is a convex function (see Appendix A for proof), we let $dTC_2/dy_2 = 0$ to calculate the optimal y_2 to minimize TC_2:

$$Ky_2^3 - h\left(\frac{a_2}{4} T_2^3 + \frac{b_2}{2} T_2^2 \right) y_2 - \frac{a_2}{6} h T_2^3 = 0 \tag{9}$$

Eq. (9) is a cubic equation in y_2 that can be solved using the Cardano formula or other suitable methods (Witula and Slota, 2010). The solution obtained from the Cardano formula may not be an integer solution. Since TC_2 is a convex function, however, we can calculate TC_2 values of the nearest two integers to the solution obtained from the Cardano formula and choose the one with the smaller TC_2 value as the optimal solution for y_2.

The net demand for new manufactured products at time t is assumed to be normal distribution with mean $\tilde{\lambda}(t) = \lambda(t) - \gamma(t) = a_2 t + b_2$ and variance $\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}$. Therefore, the total demand for new manufactured products during the lead time of the jth production activity in this phase also follows normal distribution with mean

$$D_{2,j} = \int_{jT_2/y_2-\tau}^{jT_2/y_2} \tilde{\lambda}(t)dt = a_2 j\tau \frac{T_2}{y_2} - \frac{a_2}{2}\tau^2 + b_2\tau, \quad j = 1,\ldots,y_2 \tag{10}$$

and variance $\tau^2(\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma})$. To prevent stockout, the following safety stock is necessary:

$$s_2 = F^{-1}(\beta)\tau\sqrt{\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}} \tag{11}$$

The reorder point for jth production activity in this phase is then derived as

$$ROP_{2,j} = D_{2,j} + s_2 = D_{2,j} + F^{-1}(\beta)\tau\sqrt{\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}}, \quad j = 1,\ldots,y_2 \tag{12}$$

Once the inventory level drops down to $ROP_{2,j}$, an order for $Q_{2,j}$ products is placed. Note that both the reorder point and the production lot size will increase due to the increasing net demand in this phase.

3.3. Maturity (phase 3)

In this phase, demand remains in a steady state without significant increase or decrease, while more and more end-of-use and end-of-life products are returned. For simplification, we suppose that the mean of the demand rate $\lambda(t)$ is a constant and the mean of the return rate $\gamma(t)$ increases linearly over time in this phase. Fig. 5 illustrates the production lot size $Q_{3,j}$ in this phase. If $\tilde{\lambda}(t) = a_3 t + b_3$, then we can derive the production lot sizes $Q_{3,j}$ and the reorder point $ROP_{3,j}$ following a procedure similar to the one used in phase 2. First, $Q_{3,j}$ can be derived as

$$Q_{3,j} = \int_{jT_3/y_3}^{(j+1)T_3/y_3} \tilde{\lambda}(t)dt = \frac{a_3}{2}(2j+1)\left(\frac{T_3}{y_3} \right)^2 + b_3\left(\frac{T_3}{y_3} \right), \quad j = 0,\ldots,y_3-1 \tag{13}$$

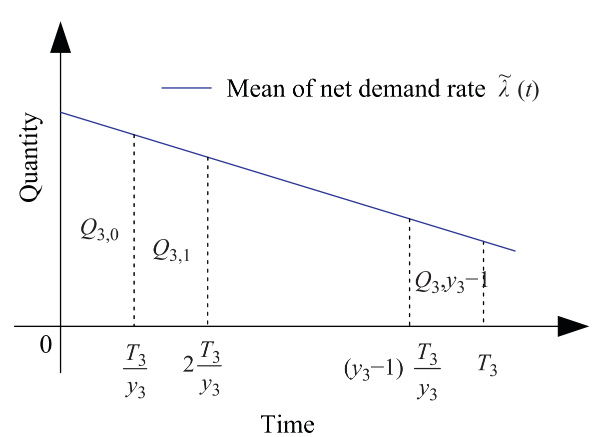

Fig. 5. Illustration of production lot size $Q_{3,j}$ in phase 3.

where y_3 is obtained by solving the following mathematical model:

$$\min_{y_3} T\,C_3 = Ky_3 + \sum_{j=0}^{y_3-1} h \int_{jT_3/y_3}^{(j+1)T_3/y_3} \left(Q_{3,j} - \int_{jT_3/y_3}^{t} \tilde{\lambda}(x)dx \right) dt \quad (14)$$

Eq. (14) can be simplified as

$$\min_{y_3} TC_3 = Ky_3 + h\left(\frac{a_3}{12} T_3^3 y_3^{-2} + \left(\frac{a_3}{4} T_3^3 + \frac{b_3}{2} T_3^2 \right) y_3^{-1} \right) \quad (15)$$

Because TC_3 is a convex function (see Appendix B for proof), we let $dTC_3/dy_3 = 0$ to derive the optimal y_3 to minimize TC_3:

$$Ky_3^3 - h\left(\frac{a_3}{4} T_3^3 + \frac{b_3}{2} T_3^2 \right) y_3 - \frac{a_3}{6} hT_3^3 = 0 \quad (16)$$

Eq. (16) is a cubic equation in y_3 and can be solved using the Cardano formula. We then calculate the TC_3 values of the nearest two integers to the solution obtained from the Cardano formula and choose the one with the smaller TC_3 value as the optimal solution for y_3.

The net demand for new manufactured products at time t is assumed to be normal distribution with mean $\tilde{\lambda}(t) = \lambda(t) - \gamma(t) = a_3 t + b_3$ and variance $\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}$. Therefore, the total demand for new manufactured products during the lead time of the jth production activity in this phase also follows normal distribution with mean

$$D_{3,j} = \int_{jT_3/y_3-\tau}^{jT_3/y_3} \tilde{\lambda}(t)dt = a_3 j\tau \frac{T_3}{y_3} - \frac{a_3}{2}\tau^2 + b_3\tau, \quad j = 1,\ldots,y_3 \quad (17)$$

and variance $\tau^2(\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma})$. To prevent stockout, the following safety stock is necessary:

$$s_3 = F^{-1}(\beta)\tau\sqrt{\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}} \quad (18)$$

The reorder point for jth production activity in this phase is then derived as

$$ROP_{3,j} = D_{3,j} + s_3 = D_{3,j} + F^{-1}(\beta)\tau\sqrt{\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}}, \quad j = 1,\ldots,y_3 \quad (19)$$

Once the inventory level drops down to $ROP_{3,j}$, an order for $Q_{3,j}$ products is placed. Note that both the reorder point and the production lot size will decrease each time due to the decreasing net demand in this phase.

From Eqs. (11) and (18), we can see that the safety stocks required in phases 2 and 3 are the same. This result shows that the safety stock is only relevant to the length of lead time, the variance of net demand, and the required fill rate. If all of these items are the same, then the safety stocks will be the same too.

3.4. Decline (phases 4 and 5)

In this phase, the demand rate starts to decline. If the demand rate is still greater than the return rate, then all returned products will be remanufactured; otherwise, there will be no production of new products, and some of the returned products will be disposed or recycled. We further discuss these two scenarios in the following sections.

3.4.1. Decline phase I (phase 4)

In this phase, we suppose $\lambda(t)$ is decreasing linearly but still greater than $\gamma(t)$, which is also assumed to be a linearly increasing function over time. The net demand rate for new manufactured products is defined as $\tilde{\lambda}(t) = a_4 t + b_4$, which is also a decreasing linear function. Therefore, we can repeat the analysis procedure in Section 3.3 to derive the production lot size:

$$Q_{4,j} = \int_{jT_4/y_4}^{(j+1)T_4/y_4} \tilde{\lambda}(t)dt = \frac{a_4}{2}(2j+1)\left(\frac{T_4}{y_4}\right)^2 + b_4\left(\frac{T_4}{y_4}\right), \quad j = 0,\ldots,y_4-1 \quad (20)$$

where y_4 is obtained by solving the following integer cubic equation:

$$Ky_4^3 - h\left(\frac{a_4}{4} T_4^3 + \frac{b_4}{2} T_4^2 \right) y_4 - \frac{a_4}{6} hT_4^3 = 0, \quad y_4 \in \text{integer}, \quad y_4 \geq 1 \quad (21)$$

The reorder point is determined as

$$ROP_{4,j} = D_{4,j} + F^{-1}(\beta)\tau\sqrt{\sigma_\lambda^2 + \sigma_\gamma^2 + 2Cov_{\lambda\gamma}}, \quad j = 1,\ldots,y_4 \quad (22)$$

where

$$D_{4,j} = a_4 j\tau \frac{T_4}{y_4} - \frac{a_4}{2}\tau^2 + b_4\tau, \quad j = 1,\ldots,y_4$$

3.4.2. Decline phase II (phase 5)

In this phase, the demand rate keeps declining and is lower than the return rate. There is an excess of returned products, some of which will be disposed or recycled to reduce unnecessary remanufacturing and holding costs. All products come from remanufacturing, and no new products are produced. As a result, there is no need to calculate the production lot size or reorder point. However, to ensure the required fill rate, we must control the inventory $I(t)$ at the following level:

$$I(t) = \lambda(t) + F^{-1}(\beta)\sigma_\lambda \quad (23)$$

When a product is returned at time t, it will be remanufactured if the current inventory level is less than $\lambda(t) + F^{-1}(\beta)\sigma_\lambda$; otherwise, it will be disposed or recycled.

3.5. Summary

The production lot size Q_1 and the reorder point ROP_1 in phase 1 are irrelevant to the phase length according to Eqs. (2)–(4); prediction of the demand rate is more important in this phase. On the contrary, the phase lengths in phase 2 through phase 4 affect inventory policies. In addition, the production lot size and reorder points in phase 2 through phase 4 can be calculated using the same formula, as long as the net demand rate function $\tilde{\lambda}(t)$ is derived first.

4. Numerical examples and sensitivity analysis

4.1. A basic numerical example

Fig. 6 shows the mean demand rate and return rate functions for the numerical example. Other parameters are as follows: $\tau = 3$, $K = 500$, $h = 0.1$, $\sigma_\lambda^2 = 100$, $\sigma_\lambda^2 = 200$, $Cov_{\lambda\gamma} = 50$, and $\beta = 0.97$. The inventory control policy in each phase is discussed below.

Phase 1:

Let $a_1 = 30$. The optimal production lot size, reorder point, and safety stock are derived as

$$Q_1 = 547.72, \quad ROP_1 = 146.42, \quad s_1 = 56.42$$

Phase 2:

From Fig. 6, we have $a_2 = 3$, $b_2 = 30$, $T_2 = 60$. First, we must calculate the optimal number of production activities y_2. From Eq. (9), we have

$$y_2^3 - 43.2y_2 - 21.6 = 0 \quad (24)$$

Solving Eq. (24) using the Cardano formula, we find that $y_2 = 6.809685$. The integer solution is $y_2 = 7$, because $TC_2(6) >$

$TC_2(7)$. The optimal production lot size and reorder point are then derived as follows:

$$Q_{2,j}=220.41j+367.35,\quad j=0,\ldots,6$$

$$ROP_{2,j}=77.14j+189.35,\quad j=1,\ldots,7$$

The safety stock s_2=112.85.

Phase 3:

Let $a_3=-2$, b_3=210, T_3=45. From Eq. (16), we have

$$500y_3^3-16706.25y_3+3037.5=0 \tag{25}$$

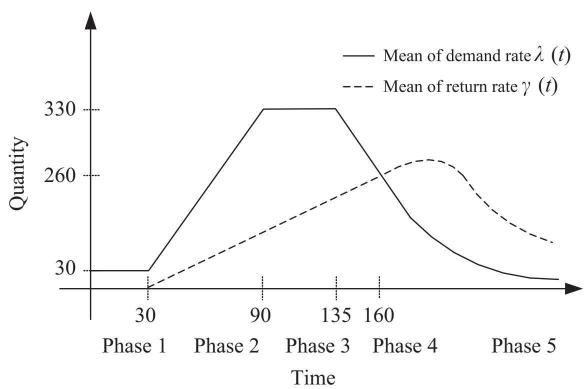

Fig. 6. Mean demand rate and return rate functions for the numerical example.

Table 1
Results of numerical examples.

	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
Production lot size	547.72	367.35 587.76 808.16 1028.57 1248.98 1469.39 1689.80	1518.75 1406.25 1293.75 1181.25 1068.75 956.25	1500	260
Reorder point	146.42	266.50 343.63 420.78 497.92 575.06 652.20 729.35	706.85 661.85 616.85 571.85 526.85 481.85	134.45	–
Safety stock	56.42	112.85	112.85	112.85	18.81

Solving Eq. (25), we find that y_3=5.687206. The integer solution is y_3=6, because $TC_3(5)>TC_3(6)$. The optimal production lot size and reorder point are then derived as

$$Q_{3,j}=-112.5j+1518.75,\quad j=0,\ldots,5$$

$$ROP_{3,j}=-45j+751.85,\quad j=1,\ldots,6$$

The safety stock s_3=112.85.

Phase 4:

Let $a_4=-4.8$, b_4=120, T_4=25. From Eq. (21), we have

$$y_4^3-3.75y_4+2.5=0 \tag{26}$$

Solving Eq. (26), we find that y_4=1.402801. The integer solution is y_4=1, because $TC_4(2)>TC_4(1)$. The optimal production lot size and reorder point are $Q_{4,0}$=1500 and $ROP_{4,1}$=134.45, respectively. The safety stock s_3=112.85.

Phase 5:

In this phase, we must control the inventory level at

$$I(t)=\lambda(t)+18.81 \tag{27}$$

A returned product will be disposed or recycled if the current inventory level is greater than $\lambda(t)+18.81$.

Under the above inventory control policy, we summarize the optimal production lot size, reorder point, and safety stock during different phases of the product life cycle in Table 1. The expected inventory level over time is illustrated in Fig. 7.

4.2. Sensitivity analysis

As mentioned in Sections 3.4.2 and 3.5, the length of phases 1 and 5 is irrelevant to the production lot size and the reorder point, but the phase lengths in phase 2 through phase 4 affect the inventory policies. Unfortunately, the prediction of the phase lengths is not very accurate. Therefore, we perform sensitivity analysis on the phase lengths. Note that the length of phase 4 is not an independent parameter; it is defined as $-b_4/a_4$ when $\tilde{\lambda}(t)=0$.

We take $Q_{2,3}$ and $ROP_{2,3}$ for testing phase lengths varying from 51 to 70. Fig. 8 shows that $Q_{2,3}$ will increase as the phase lengths increase and decrease when y_2 increases. It fluctuates from 936.91 to 1232.63. Similar results are found for $ROP_{2,3}$, although the influence is not as significant.

Another parameter that needs estimation is the slope of $\tilde{\lambda}(t)$. Again, we take $Q_{2,3}$ and $ROP_{2,3}$ for testing slopes of $\tilde{\lambda}(t)$, i.e., a_2 varying from 2 to 4. Similar to Fig. 8, the zigzag lines appear in Fig. 9. By performing additional sensitivity analyses, we discover that the zigzags in Figs. 8 and 9 are due to the integralization of y_2. If we ignore the integer property of y_2 and adopt the original

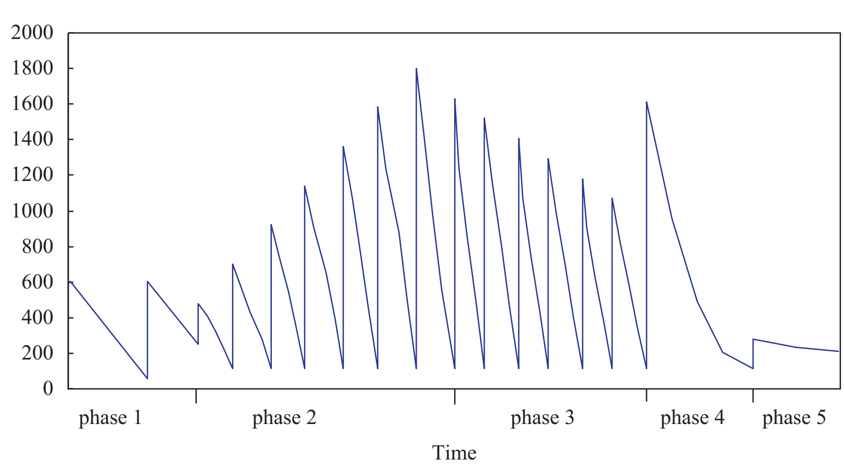

Fig. 7. Inventory level over time.

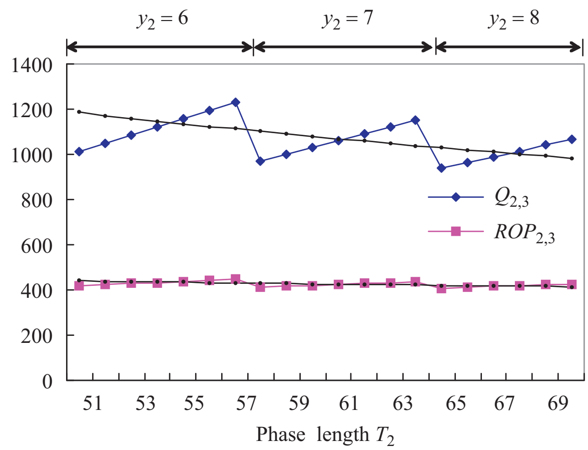

Fig. 8. Sensitivity analysis on phase lengths.

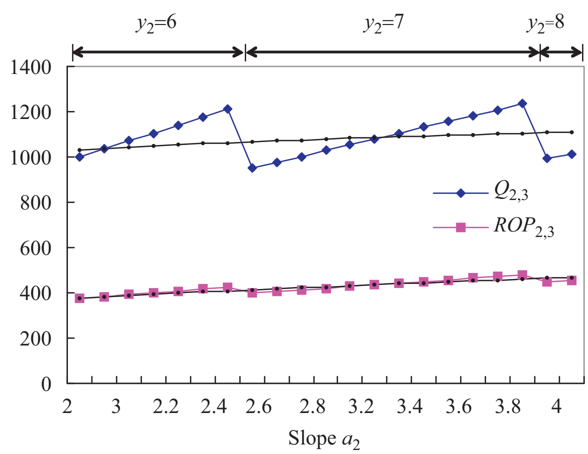

Fig. 9. Sensitivity analysis on a_2.

real number solution, the resulting $Q_{2,3}$ and $ROP_{2,3}$ become more stable, as indicated by the dotted lines in Figs. 8 and 9.

The above analysis indicates that the impact of inaccurate estimation for the phase length and the demand changing rate on the inventory control is not very significant. This implies that our inventory control model is robust in an uncertain environment. However, in other aspects of management, especially strategic or long-term decisions, the prediction of these two parameters remains an important issue.

5. Conclusion and future research

This paper analyzes the relationship between the demand rate and the return rate in a manufacturing/remanufacturing system during each phase of the product life cycle. The major contribution of the paper is that the closed-form formulas of optimal production lot size, reorder point, and safety stock in each phase of the product life cycle are successfully derived. The numerical example shows the practicability of our model and indicates that different inventory control policies should be adopted in different phases of the product life cycle. In phase 1, the EOQ model with safety stock is enough. The production lot size should increase with production activities in phase 2 and decrease in phases 3 and 4. There is no need to manufacture new products in phase 5, during which some returned products are discarded to reduce unnecessary remanufacturing and holding costs. Phase 5 only requires maintaining inventory at a decreasing level to ensure the necessary fill rate. In addition, the results of sensitivity analysis show that the inventory control policy is not sensitive to the phase length and the demand changing rate.

When applying the proposed inventory control policy to a real case, it is particularly suitable for the products with very short life cycle, such as in the mobile phone industry. The demand for these products changes quickly, but traditional inventory models typically assume that demand is stationary. Therefore, traditional inventory models cannot consider the change in demand even in a certain stage of the product life cycle. Our model, however, can provide a detailed inventory control policy within a stage, which may have a dramatically changing demand.

Several assumptions in this paper can be relaxed for future research. Customers may consider remanufactured and manufactured products as two different ones. Other distribution can be adopted for demand and return rate. Nonlinear mean demand and return rate functions also deserve investigation.

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Appendix A

To prove that TC_2 is a convex function, we calculate the first and second order derivatives of TC_2:

$$\frac{dTC_2}{dy_2} = K - h\left(\frac{a_2}{6}T_2^3y_2^{-3} + \left(\frac{a_2}{4}T_2^3 + \frac{b_2}{2}T_2^2\right)y_2^{-2}\right)$$

$$\frac{d^2TC_2}{dy_2^2} = h\left(\frac{a_2}{2}T_2^3y_2^{-4} + \left(\frac{a_2}{2}T_2^3 + b_2T_2^2\right)y_2^{-3}\right)$$

$$\because\ a_2,\ b_2,\ h,\ T_2 > 0 \text{ and } y_2 \geq 1$$

$$\therefore\ \frac{d^2TC_2}{dy_2^2} > 0,\ TC_2 \text{ is convex}$$

Appendix B

To prove that TC_3 is a convex function, we calculate the first and second order derivatives of TC_3:

$$\frac{dTC_3}{dy_3} = K - h\left(\frac{a_3}{6}T_3^3y_3^{-3} + \left(\frac{a_3}{4}T_3^3 + \frac{b_3}{2}T_3^2\right)y_3^{-2}\right)$$

$$\frac{d^2TC_3}{dy_3^2} = h\left(\frac{a_3}{2}T_3^3y_3^{-4} + \left(\frac{a_3}{2}T_3^3 + b_3T_3^2\right)y_3^{-3}\right)$$

$$= hT_3^2y_3^{-4}\left(\frac{a_3}{2}T_3(1-y_3) + (a_3T_3 + b_3)y_3\right)$$

$$\because\ a_3T_3 + b_3 \geq 0,\ a_3 < 0 \text{ and } y_3 \geq 1$$

$$\therefore\ \frac{d^2TC_3}{dy_3^2} \geq 0,\ TC_3 \text{ is convex}$$

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