On the Down-Link Performance of Multi-Carrier CDMA Systems With Partial Equalization

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Abstract— **This paper addresses the performance evaluation of multi-carrier code division multiple access systems. A partial equalization technique depending on a parameter adapted to different conditions such as the number of sub-carriers, the number of active users and the mean signal-to-noise ratio is evaluated for the down-link. We analytically derive the performance of this system and the value of the equalization parameter that maximizes the performance. The partial equalization technique is shown to have the same complexity of the well known maximal ratio combining, equal gain combining and orthogonality restoring combining techniques, with significative performance improvement. Analytical results are compared with simulations showing a perfect agreement.**

Index Terms— **Fading channel, MC-CDMA, partial equalization, performance evaluation.**

I. Introduction

MULTI-CARRIER code division multiple access (MC-CDMA) techniques are considered for fourth generation mobile radio systems due to the efficiency in counteracting both multi-user interference and frequency selective fading for high data-rate communications (see, e.g., [1]–[5]). Different schemes have been proposed in the Literature, for which an overview can be found in [6].

We investigate the down-link performance of MC-CDMA in a frequency selective fading channel following the scheme presented in [1], [7] modified for what concerns the equalization and combining technique of signals coming from different sub-carriers. In this scheme the spreading is done in the frequency-domain, with spreading factor equal to the number of sub-carriers, and Walsh-Hadamard (W-H) codes are used (see Figs. 1 and 2 for the transmitter and receiver block schemes, respectively).[1]

Even if it can be assumed that at the receiver side of the down-link the information associated to all users experiences the same channel and the system remains always synchronous, the orthogonality between the sequences of different users is lost, in spite of the use of W-H code, due to the different fading in each sub-channel. Therefore, the choice of the combining technique becomes critical.

Within the family of linear combining techniques, different schemes based on the channel state information are known in the literature (see, e.g., [10]), where signals coming from different sub-carriers are weighted by suitable coefficients G_m (m being the sub-carrier index). The equal gain combining (EGC) consists in weighting equally each sub-channel contribution and compensating only the phases as in (1):

$$G_m = \frac{H_m^*}{|H_m|}\,, \tag{1}$$

where by G_m we indicate the $m^{\underline{\text{th}}}$ complex channel gain and H_m is the $m^{\underline{\text{th}}}$ channel coefficient.[2] As investigated in [1], if the number of active users is negligible with respect to the number of sub-carriers, that is the system is noise-limited, the best choice is represented by a combination in which the sub-channel with higher signal-to-noise ratio (SNR) has the higher weight, as in the maximal ratio combining (MRC):

$$G_m = H_m^*\,. \tag{2}$$

On the other hand, this choice totally destroys the orthogonality between the codes. For this reason, if the number of active user is high (the system is interference-limited), a good choice is given by restoring at the receiver the orthogonality between the sequences. This means to cancel the effects of the channel on the sequences as in the orthogonality restoring combining (ORC), where:

$$G_m = \frac{1}{H_m}\,. \tag{3}$$

This implies a total cancellation of the multiuser interference, but, on the other hand, this method enhances the noise, because the sub-channels with low SNR have higher weights. It is evaluated in [1] that for a Rayleigh fading channel it raises to infinity the noise contribution (i.e., the $\mathbb{E}\left\{|H_m|^{-2}\right\}$ approaches infinity). Consequently, a correction on G_m is introduced in [11], as follows:

$$G_m = u(|H_m| - \rho_{TH})\frac{1}{H_m}, \tag{4}$$

where $u(\cdot)$ is the unitary-step function and the threshold ρ_{TH} is introduced to cancel the contributions of sub-channels highly corrupted by the noise. This method is the so-called controlled equalization (CE) or threshold orthogonality restoring combining (TORC) technique.

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[1]Note that other solutions (see, e.g., [8], [9]) consist in performing the spreading in the time domain.

[2]Operation * is the complex conjugate, whereas $\mathbb{E}\{\cdot\}$ represents the statistical expectation.

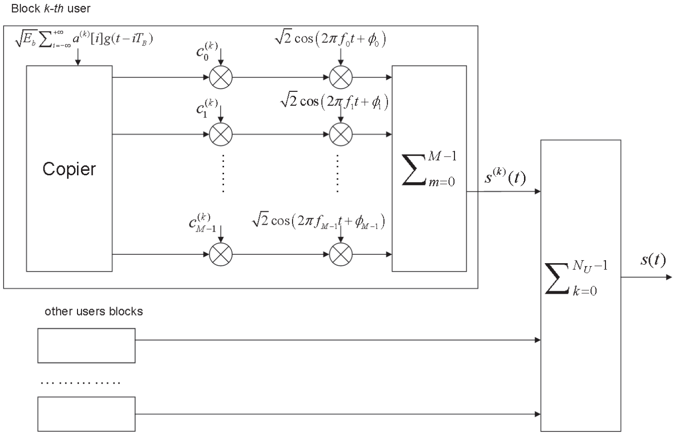

Fig. 1. Transmitter block-scheme.

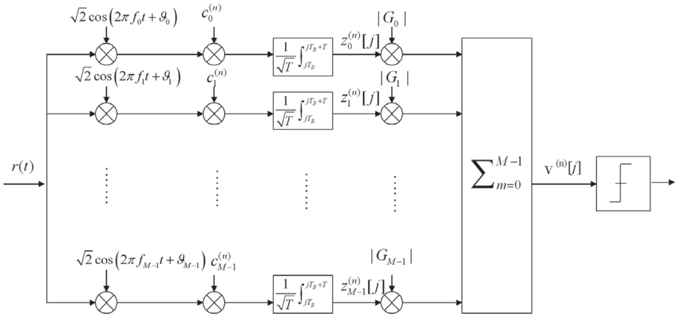

Fig. 2. Receiver block-scheme for the n^{th} user.

However, exception made for the two opposite cases of one active user (giving MRC) and negligible noise (giving ORC) the presented methods do not represent the optimum solution for real cases of interest. The optimum choice for linear equalization is the minimum mean square error (MMSE) technique. For the particular case of fully-loaded system (i.e., the number of users is equal to the number of sub-carriers), in [12] it has been proved that it results in:

$$G_m = \frac{H_m^*}{|H_m|^2 + \frac{1}{N_u \overline{\gamma}}}, \tag{5}$$

where N_u is the number of users and $\overline{\gamma}$ is the mean SNR averaged over fast fading. When the system is not full-loaded it is cumbersome to obtain the MMSE coefficients [12].

More complex non-linear equalizers, such as the maximum likelihood detection (MLD), and iterative detection presented attain better performance [4]. However, since we are analyzing the down-link of a mobile radio system, the computation is done in the mobile unit and it is fundamental to have a detection scheme capable to attain good performance with low complexity.

Then, we will focus on linear equalization techniques with the same complexity of EGC, MRC and ORC, but more robust to both multiuser interference and thermal noise.

In particular, we will analyze a partial equalization technique where coefficients G_m depend on a parameter β as follows:

$$G_m = \frac{H_m^*}{|H_m|^{1+\beta}}\,. \tag{6}$$

Note that (1), (2), (3) can be viewed as particular cases of (6) for which the parameter β assumes the values 0 (EGC), -1 (MRC) and 1 (ORC), respectively. The key idea is that, since MRC and ORC are optimum in the extreme cases of noise-limited and interference-limited systems, respectively, for each intermediate situation there should exist an optimum value of the parameter β which minimizes the mean bit error probability (BEP) averaged over fast fading.

In this paper we analytically evaluate the optimum β for all possible number of sub-carriers, active users, and for all possible values of the SNR. After that, it will be shown that a partial equalization technique with parameter β properly chosen (e.g., to be optimum for a fully-loaded condition) improves the system performance still maintaining the same complexity of MRC, EGC and ORC. The accuracy of analytical results will be verified by comparison with simulations. In addition, as a bench-mark, our results will be compared with those in [13] where a similar technique was studied by simulations (in the particular case of full-load system, $M = 64$ sub-carriers and some given values of the SNR) and it was pointed out that it can attain BEP not far from the one of MMSE. The great advantage of the proposed solution is that (once β is fixed) it has the same complexity of MRC and ORC schemes.

The paper is organized as follows: methodology, assumptions and system model are discussed in Section II and Section III. The test statistic and the BEP are analytically derived in Section IV and Section V, respectively. Numerical results are reported in Section VI, and our conclusions are given in Section VII.

II. Methodology and Assumptions

We are dealing with the down-link of a MC-CDMA system for which we make the following commonly accepted assumptions (see, e.g., [1], [7], [13]):

- the system remains always synchronous, and possible different delays affecting each sub-carriers are assumed to be perfectly compensated;
- the channel is time invariant for several symbol-time and each sub-carrier experiments flat fading uncorrelated on different sub-carriers;
- we assume perfect channel state information for each sub-carrier;
- we adopt the central limit theorem (CLT) and the law of large numbers (LLN) to assess the BEP analysis.

From one hand, the assumption of uncorrelated fading among sub-carriers represents a realistic case when the sub-carriers are sufficiently spaced in frequency (i.e., more than the coherence bandwidth) or when only a subset of the amount of sub-carriers is used for a symbol transmission. On the other hand, it allows a completely analytical evaluation of the performance with careful investigation of dependencies between system parameters, the comparison with previous results appeared in the literature, and a lower-bound of the performance in realistic scenarios.

With regard to the adoption of CLT and LLN, it is reasonable for sufficiently high number of sub-carriers, as for practical systems, such as the terrestrial digital video broadcasting (DVB-T), where the number of sub-carriers is

2K or 8K. The suitability of the approximation on the BEP obtained through these assumptions will also be checked by simulation.

On the other hand, it is important to underline that the main objective of this work is not to assess an exact expression for the BEP itself, but to derive the value of β which provides the lower BEP. We will show through both the analytical methodology and simulations, that the optimum value of β which minimizes the exact BEP is in perfect agreement with the value that results in the lower approximated BEP obtained through the analytical methodology. The main advantage of this approach is to derive the analytical dependence between the optimum β and other parameters (such as the number of active users, the signal-to-noise ratio, the number of sub-carriers, etc.) representing slow processes with respect to the performance perceived by users. Then two interesting cases can be investigated: one in which β is adaptively changed tracking the variation of slow processes and the other in which β is assumed to be fixed at a particular value suggested by analytical results.

For a discussion on the performance evaluation in the presence of processes varying with different rapidity see, as example, [14]. The BEP averaged over fast fading is a common performance figure for applications in which the user's perceived quality of service is related to the error rate observed in a time interval of few seconds. Thus, the variability of processes affecting the useful signal during this interval has to be carefully taken into account (e.g., due to the shadowing, $\overline{\gamma}$ slowly varies with respect to the BEP).

III. SYSTEM MODEL

A. Transmitter

A block scheme of the transmitter is depicted in Fig. 1. Following, as an example, the MC-CDMA architecture in [6], the number of sub-carriers, M, is equal to the spreading factor, N. Each data-symbol is copied over all sub-carriers, and multiplied by the chip assigned to each particular sub-carrier. Consequently, the spreading is performed in the frequency-domain.

We consider W-H orthogonal code sequences for the multiple access and binary phase shift keying (BPSK) modulation; the transmitted signal referred to the $k^{\underline{\text{th}}}$ user, can be written as follows:

$$s^{(k)}(t) = \sqrt{\frac{2E_b}{M}} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} c_m^{(k)} a^{(k)}[i] g(t - iT_b) \times \cos(2\pi f_m t + \phi_m), \tag{7}$$

where E_b is the energy per bit, i denotes the data index, m is the sub-carrier index, c_m is the $m^{\underline{\text{th}}}$ chip (taking value ± 1)[3], $a^{(k)}[i]$ is the data-symbol transmitted during the $i^{\underline{\text{th}}}$ time-symbol, $g(t)$ is a rectangular pulse waveform, with duration $[0, T]$ and unitary energy, T_b is the bit-time, $f_m = f_0 + m \cdot \Delta f$ is the sub-carrier-frequency (with $\Delta f \cdot T$ and $f_0 T$ integers that implies orthogonal frequencies) and ϕ_m is the random phase uniformly distributed within $[-\pi, \pi]$.

[3]We assume orthogonal sequences $\overline{c^{(k)}}$ for different users, such that:

$$< \overline{c^{(k)}}, \overline{c^{(k')}} >= \sum_{m=0}^{M-1} c_m^{(k)} c_m^{(k')} = \begin{cases} M & k = k', \\ 0 & k \neq k'. \end{cases}$$

In particular, $T_b = T + T_g$ is the total OFDM symbol duration, increased with respect to T of a time-guard T_g (inserted between consecutive multi-carrier symbols to eliminate the residual inter symbol interference, ISI, due to the channel delay spread). Considering that, exploiting the orthogonality of the code, all the different users use the same carriers, the total transmitted signal results in:

$$s(t) = \sum_{k=0}^{N_u-1} s^{(k)}(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_u-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} c_m^{(k)} a^{(k)}[i] \times g(t - iT_b) \cos(2\pi f_m t + \phi_m), \tag{8}$$

where N_u is the number of active users and, because of the use of orthogonal codes, $N_u \leq M$.

B. Channel Model

Since we are considering the down-link, we assume that, focusing on the $n^{\underline{\text{th}}}$ receiver, the information associated to different users experiments the same fading. Due to the CDMA structure of the system, each user receives the information of all the users and select only its own data through the spreading sequence. We assume the impulse response of the channel $h(t)$ as time-invariant during many symbol intervals. We employ a frequency-domain channel model in which the transfer function, $H(f)$, is given by:

$$H(f) \simeq H(f_m) = \alpha_m e^{j\psi_m} \text{ for } |f - f_m| < \frac{W_s}{2}, \forall\, m, \tag{9}$$

where α_m and ψ_m are the $m^{\underline{\text{th}}}$ amplitude and phase coefficients, respectively, and W_s is the the transmission bandwidth of each sub-carrier. The assumption in (9) means that the pulse shaping still remains rectangular even if the non-distortion conditions are not perfectly verified. Hence, the response $g'(t)$ to $g(t)$ is a rectangular pulse with unitary energy and duration $T' \triangleq T + T_d$, being $T_d \leq T_g$ the time delay.

We assume that each $H(f_m)$ is independent identically distributed (i.i.d.) complex zero-mean Gaussian random variable (r.v.) with variance, σ_H^2, related to the path-loss L_p as $1/L_P = \mathbb{E}\left\{\alpha^2\right\} = 2\sigma_H^2$.

C. Receiver

The received signal can be written as

$$r(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_u-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)} a^{(k)}[i] g'(t - iT_b) \times \cos(2\pi f_m t + \overbrace{\phi_m + \psi_m}^{\vartheta_m}) + n(t), \tag{10}$$

where $n(t)$ is the additive white Gaussian noise with two-side power spectral density (PSD) $N_0/2$ and $\vartheta_m \triangleq \phi_m + \psi_m$. Note that, since ϑ_m can be considered uniformly distributed in $[-\pi, \pi]$, we can consider $\angle H(f_m) \sim \vartheta_m$ in the following.

The receiver structure is depicted in Fig. 2. Focusing, without loss of generality, to the $l^{\underline{\text{th}}}$ sub-carrier of user n, the receiver performs the correlation at the $j^{\underline{\text{th}}}$ instant (perfect synchronization and phase tracking are assumed) of the received signal with the signal $c_l^{(n)}\sqrt{2}\cos(2\pi f_l t+\vartheta_l)$, as:

$$z_l^{(n)}[j]=\frac{1}{\sqrt{T}}\int_{jT_b}^{jT_b+T} r(t)\ c_l^{(n)}\sqrt{2}\cos(2\pi f_l t+\vartheta_l)dt\,. \quad (11)$$

Substituting (10) in (11), the term $z_l^{(n)}[j]$ results in (12), which is found at the bottom of the page. It is shown in Appendix I that $u_{m,l}[j]$ is independent on j and its value is 1 for $m=l$, 0 for $m\neq l$. Then the second term in (12) (i.e., the ISI) is zero. In addition, it can be proved that terms $n_l[j]$ are zero mean Gaussian r.v.'s with variance $N_0/2$. After some algebra, (12) results in

$$\begin{aligned} z_l^{(n)}[j] &= \sqrt{\frac{E_b\delta_d}{M}}\alpha_l a^{(n)}[j] \\ &+ \sqrt{\frac{E_b\delta_d}{M}}c_l^{(n)}\alpha_l\sum_{k=0,k\neq n}^{N_u-1}c_l^{(k)}a^{(k)}[j]+n_l[j]\,, \end{aligned} \quad (13)$$

where $\delta_d\triangleq 1/(1+T_d/T)$ represents the loss of energy caused by the time-spreading of the impulse.

IV. Test Statistic

The decision variable (i.e., the test statistic), $v^{(n)}[j]$, is obtained by linearly combining the weighted signals from each sub-carrier as follows:[4]

$$v^{(n)}=\sum_{l=0}^{M-1}|G_l|z_l^{(n)}\,, \quad (14)$$

where $|G_l|$ is a suitable amplitude of the $l^{\underline{\text{th}}}$ equalization coefficient. By considering MC-CDMA with partial equalization, the weight for the $l^{\underline{\text{th}}}$ sub-carrier is given by:

$$G_l=\frac{H^*(f_l)}{|H^*(f_l)|^{1+\beta}}\,,\quad -1\leq\beta\leq 1\,. \quad (15)$$

Note that, the correlation of the signal on each sub-carrier with $\sqrt{2}\cos(2\pi f_l t+\vartheta_l)$ is equivalent to multiply by $H^*(f_l)/|H(f_l)|$. Hence, each sub-carrier is weighted by a gain

$$|G_l|=|H(f_l)|^{-\beta}=\alpha_l^{-\beta}\,,\quad -1\leq\beta\leq 1\,. \quad (16)$$

Therefore, from (13) and (14) we can write:

$$\begin{aligned} v^{(n)} &= \overbrace{\sqrt{\frac{E_b\delta_d}{M}}\sum_{l=0}^{M-1}\alpha_l^{1-\beta}a^{(n)}}^{U}+\overbrace{\sum_{l=0}^{M-1}\alpha_l^{-\beta}n_l}^{N} \\ &+ \overbrace{\sqrt{\frac{E_b\delta_d}{M}}\sum_{l=0}^{M-1}\sum_{k=0,k\neq n}^{N_u-1}\alpha_l^{1-\beta}c_l^{(n)}c_l^{(k)}a^{(k)}}^{I}\,. \end{aligned} \quad (17)$$

At this point, the distribution of the test statistic can be obtained by studying the statistics U, I and N in (17).

[4]For the sake of conciseness in our notation, since ISI is avoided, we will neglect the time-index j in the following.

1) Interference Term: Exploiting the properties of orthogonal codes as in [7], after some algebra the interference term can be rewritten as:

$$I=\sqrt{\frac{E_b\delta_d}{M}}\sum_{k=0,k\neq n}^{N_u-1}a^{(k)}\left(\overbrace{\sum_{h=1}^{\frac{M}{2}}\alpha_{x_h}^{1-\beta}}^{A_1}-\overbrace{\sum_{h=1}^{\frac{M}{2}}\alpha_{y_h}^{1-\beta}}^{A_2}\right)\,, \quad (18)$$

where indexes x_h and y_h, thus A_1 and A_2, depend on user k and define the following partition

$$c^{(n)}[x_h]c^{(k)}[x_h]=1 \quad (19)$$

$$c^{(n)}[y_h]c^{(k)}[y_h]=-1 \quad (20)$$

$$\{x_h\}\cup\{y_h\}=0,1,2,....,M-1\,. \quad (21)$$

For large M, we can apply the CLT to each one of the internal sums in (18) obtaining:

$$A_1\sim\mathcal{N}\left(\frac{M}{2}\mathbb{E}\left\{\alpha^{1-\beta}\right\},\frac{M}{2}\zeta_\beta(\alpha)\right) \quad (22)$$

$$A_2\sim\mathcal{N}\left(\frac{M}{2}\mathbb{E}\left\{\alpha^{1-\beta}\right\},\frac{M}{2}\zeta_\beta(\alpha)\right)\,, \quad (23)$$

where $\zeta_\beta(\alpha)$ indicates the variance of $\alpha^{1-\beta}$ that is given by

$$\zeta_\beta(\alpha)\triangleq\mathbb{E}\left\{(\alpha^{1-\beta})^2\right\}-(\mathbb{E}\left\{\alpha^{1-\beta}\right\})^2\,. \quad (24)$$

Therefore, $A\triangleq A_1-A_2$ is distributed as:

$$A\sim\mathcal{N}\left(0,M\ \zeta_\beta(\alpha)\right)\,. \quad (25)$$

By exploiting the symmetry of the Gaussian p.d.f. and the property of the sum of uncorrelated (and thus independent) Gaussian r.v.'s i.i.d. ($a^{(k)}A\sim\mathcal{N}(0,M\ \zeta_\beta(\alpha))$), the interference term is distributed as:

$$I\sim\mathcal{N}\left(0,\sigma_I^2\triangleq E_b\delta_d(N_u-1)\zeta_\beta(\alpha)\right)\,. \quad (26)$$

2) Noise Term: The thermal noise at the combiner output is given by

$$N=\sum_{l=0}^{M-1}\alpha_l^{-\beta}n_l\,, \quad (27)$$

where terms α_l and n_l are independent and n_l is zero mean. Thus, N consists on a sum of i.i.d zero mean r.v.'s with variance $N_0/2\ \mathbb{E}\left\{\alpha^{-2\beta}\right\}$. Following the approximation done in [1] and in [7], involving the CLT, we approximate the unconditioned noise term N as:

$$N\sim\mathcal{N}\left(0,\sigma_N^2\triangleq M\frac{N_0}{2}\mathbb{E}\left\{\alpha^{-2\beta}\right\}\right)\,. \quad (28)$$

3) Useful Term: By applying the CLT, the gain U on the useful term in (17) results distributed as

$$U\sim\mathcal{N}\left(\sqrt{E_b\delta_d M}\mathbb{E}\left\{\alpha_l^{1-\beta}\right\},E_b\delta_d\zeta_\beta(\alpha)\right)\,. \quad (29)$$

4) Independence Between Each Term: By noting that $a^{(k)}$ is zero mean and statistically independent on α_l, A, and n_l, it follows that $\mathbb{E}\{I\ N\}=\mathbb{E}\{I\ U\}=0$. Since n_l and α_l are statistically independent, the $\mathbb{E}\{N\ U\}=0$. The fact that I, N and U are uncorrelated Gaussian r.v.'s implies they are also independent.

V. Bit Error Probability Evaluation

From (26) and (28) we obtain

$$I + N \sim \mathcal{N}\left(0, E_b\delta_d(N_u-1)\zeta_\beta(\alpha) + M\mathbb{E}\left\{\alpha^{-2\beta}\right\}\frac{N_0}{2}\right), \quad (30)$$

that can be applied to the test statistic in (17) to derive the BEP conditioned to the r.v. U as

$$P_{\mathrm{b}}|_U = \frac{1}{2}\mathrm{erfc}\left\{\frac{U}{\sqrt{2(\sigma_I^2+\sigma_N^2)}}\right\}. \quad (31)$$

By applying the LLN as in [1], that is approximating $\sum_{l=0}^{M-1}\alpha_l^{1-\beta}$ with $M\mathbb{E}\left\{\alpha^{1-\beta}\right\}$, we can derive the unconditioned BEP as follows:

$$P_{\mathrm{b}} \simeq \frac{1}{2}\mathrm{erfc}\left\{\sqrt{\frac{E_b\delta_d(\mathbb{E}\left\{\alpha^{1-\beta}\right\})^2}{2E_b\delta_d\frac{N_u-1}{M}\zeta_\beta(\alpha)+\mathbb{E}\left\{\alpha^{-2\beta}\right\}N_0}}\right\}. \quad (32)$$

In Appendix II we show that:

$$\mathbb{E}\left\{\alpha^{1-\beta}\right\} = (2\sigma_H^2)^{\frac{1-\beta}{2}}\Gamma\left(\frac{3-\beta}{2}\right), \quad (33)$$

$$\mathbb{E}\left\{\alpha^{-2\beta}\right\} = \left(2\sigma_H^2\right)^{-\beta}\Gamma(1-\beta), \quad (34)$$

$$\zeta_\beta(\alpha) = (2\sigma_H^2)^{1-\beta}\left[\Gamma(2-\beta) - \Gamma^2\left(\frac{3-\beta}{2}\right)\right], \quad (35)$$

where $\Gamma(z)$ is the well known Euler Gamma function. By substituting (33), (34) and (35) in (32) we obtain

$$P_{\mathrm{b}} \simeq \frac{1}{2}\mathrm{erfc}\{\sqrt{\mathrm{SNIR}}\}, \quad (36)$$

where

$$\mathrm{SNIR} = \frac{\Gamma^2\left(\frac{3-\beta}{2}\right)\frac{2\sigma_H^2E_b\delta_d}{N_0}}{2\frac{N_u-1}{M}\left[\Gamma(2-\beta)-\Gamma^2\left(\frac{3-\beta}{2}\right)\right]\frac{2\sigma_H^2E_b\delta_d}{N_0}+\Gamma(1-\beta)}. \quad (37)$$

By defining the mean SNR at the receiver as $\overline{\gamma} \triangleq 2\sigma_H^2\delta_dE_b/N_0$, the BEP results in a function of the mean SNR and the combining parameter β as in (36) with

$$\mathrm{SNIR} = \frac{\Gamma^2\left(\frac{3-\beta}{2}\right)\overline{\gamma}}{2\frac{N_u-1}{M}\left[\Gamma(2-\beta)-\Gamma^2\left(\frac{3-\beta}{2}\right)\right]\overline{\gamma}+\Gamma(1-\beta)}. \quad (38)$$

Note that this BEP expression is general in β and it is immediate to verify that results in the expressions for EGC ($\beta = 0$) and MRC ($\beta = -1$) found in [1] as:

$$\textbf{(EGC)}\ P_{\mathrm{b}}(0,\overline{\gamma}) \simeq \frac{1}{2}\mathrm{erfc}\left\{\sqrt{\frac{\frac{\pi}{4}\overline{\gamma}}{2\frac{N_u-1}{M}\left(1-\frac{\pi}{4}\right)\overline{\gamma}+1}}\right\}, \quad (39)$$

$$\textbf{(MRC)}\ P_{\mathrm{b}}(-1,\overline{\gamma}) \simeq \frac{1}{2}\mathrm{erfc}\left\{\sqrt{\frac{\overline{\gamma}}{2\frac{N_u-1}{M}\overline{\gamma}+1}}\right\}. \quad (40)$$

As a benchmark, note that for MRC with one active user (i.e., $N_u = 1$) the (40) becomes:

$$P_{\mathrm{b}}(-1,\overline{\gamma})|_{N_u=1} \simeq \frac{1}{2}erfc(\sqrt{\overline{\gamma}}), \quad (41)$$

that is independent on the number of sub-carrier M and represents the well known limit of the antipodal waveforms in AWGN channel. This means that the approximation due to LLN is equivalent to assume that we have a number of sub-carriers (M) sufficiently high to saturate the frequency-diversity, then the transmission performs as in the absence of fading.

A. Optimum Choice of the Combining Parameter β

Now we will analyze the proposed partial equalization technique with the aim of finding the optimum value of β, defined as the value within the range $[-1, 1]$ that minimizes the BEP as expressed in (38):

$$\beta^{(opt)} = \arg\min_\beta\{P_{\mathrm{b}}(\beta,\overline{\gamma})\} \simeq \quad (42)$$
$$\arg\max_\beta\left\{\frac{\Gamma^2\left(\frac{3-\beta}{2}\right)\overline{\gamma}}{2\frac{N_u-1}{M}\left[\Gamma(2-\beta)-\Gamma^2\left(\frac{3-\beta}{2}\right)\right]\overline{\gamma}+\Gamma(1-\beta)}\right\}.$$

It will be shown in the numerical results that the approximation on the BEP does not significantly affect the optimized value of the parameter β. By forcing to zero the derivative of the argument in (42), calling $\Gamma'(x) \triangleq d\Gamma(x)/dx$, and considering $\overline{\gamma} > 0$ together with $\Gamma\left(\frac{3-\beta}{2}\right) \neq 0$ for $-1 \leq \beta \leq 1$, after some algebra we obtain the following equation:

$$\begin{aligned}&-\Gamma'\Big(\frac{3-\beta}{2}\Big)\Big\{2\overline{\gamma}\frac{N_u-1}{M}\Big[\Gamma\Big(2-\beta\Big)-\Gamma^2\Big(\frac{3-\beta}{2}\Big)\Big]\\&+\Gamma\Big(1-\beta\Big)\Big\}=\Gamma\Big(\frac{3-\beta}{2}\Big)\Big\{2\overline{\gamma}\frac{N_u-1}{M}\Big[-\Gamma'(2-\beta)\\&+\Gamma\Big(\frac{3-\beta}{2}\Big)\Gamma'\Big(\frac{3-\beta}{2}\Big)\Big]-\Gamma'(1-\beta)\Big\}. \quad (43)\end{aligned}$$

$$\begin{aligned}z_l^{(n)}[j] &= 2\sqrt{\frac{E_b}{MT}}\sum_{i=-\infty}^{+\infty}\int_{jT_b}^{jT_b+T}\sum_{k=0}^{N_u-1}\sum_{m=0}^{M-1}\alpha_m c_m^{(k)}c_l^{(n)}a^{(k)}[i]\ g'(t-iT_b)\cos(2\pi f_mt+\vartheta_m)\cos(2\pi f_lt+\vartheta_l)dt \quad (12)\\&+\overbrace{\int_{jT_b}^{jT_b+T}\sqrt{2}\frac{c_l^{(n)}}{\sqrt{T}}n(t)\cos(2\pi f_lt+\vartheta_l)dt}^{n_l[j]} = \sqrt{\frac{E_b}{M}\frac{T}{T'}}\sum_{k=0}^{N_u-1}\sum_{m=0}^{M-1}\overbrace{\frac{1}{T}\int_{jT_b}^{jT_b+T}2\cos(2\pi f_mt+\vartheta_m)\cos(2\pi f_lt+\vartheta_l)dt}^{u_{m,l}[j]}\\&\times\alpha_mc_m^{(k)}c_l^{(n)}a^{(k)}[j]+n_l[j] = \sqrt{\frac{E_b}{M}\frac{T}{T'}}\left\{\sum_{k=0}^{N_u-1}\alpha_lc_l^{(k)}c_l^{(n)}a^{(k)}[j]u_{l,l}[j]+\sum_{m=0,\ m\neq l}^{M-1}\sum_{k=0}^{N_u-1}\alpha_mc_m^{(k)}c_l^{(n)}a^{(k)}[j]u_{m,l}[j]\right\}+n_l[j].\end{aligned}$$

It is known that $\Gamma'(x) = \Psi(x)\Gamma(x)$, where $\Psi(x)$ is the logarithmic derivative of the Gamma Function, the so-called Digamma-function defined as (see, e.g., [17]) $\Psi(x) \triangleq d\ln\Gamma(x)/dx$. Then, we are able to rewrite (43) as

$$\Gamma\ (1-\beta)\left[\Psi\left(\frac{3-\beta}{2}\right)-\Psi(1-\beta)\right] = 2\overline{\gamma}\frac{N_u-1}{M}\Gamma(2-\beta)\left[\Psi(2-\beta)-\Psi\left(\frac{3-\beta}{2}\right)\right]. \quad (44)$$

Since $\Gamma(x+1) = x\Gamma(x)$ and $\Gamma(1-\beta) \neq 0$ for $-1 \leq \beta \leq 1$, we obtain:

$$\Psi\left(\frac{3-\beta}{2}\right)-\Psi(1-\beta) = 2\overline{\gamma}\frac{N_u-1}{M}(1-\beta)\left[\Psi(2-\beta)-\Psi\left(\frac{3-\beta}{2}\right)\right]. \quad (45)$$

Considering also that (see, e.g., [17]) $\Psi(x+1) = \Psi(x)+1/x$, after some manipulations and defining the parameter

$$\xi \triangleq 2\overline{\gamma}\frac{N_u-1}{M}, \quad (46)$$

we obtain the following expression:

$$\left[\Psi\left(\frac{3-\beta}{2}\right)-\Psi(1-\beta)\right]\left[\frac{1}{\xi}+(1-\beta)\right]-1=0. \quad (47)$$

Note that the parameter ξ quantifies how much the system is noise-limited (low values) or interference-limited (high values), and (47) represents the implicit solution, for the problem of finding the optimum value of β for all possible values of SNR, number of sub-carriers and number of users. Indeed, (47) open the way to an important consideration. In fact, the optimum β only depends, through ξ, on slowly varying processes such as the SNR (averaged over fast fading then randomly varying according to shadowing), the number of users and the number of sub-carriers. This means that it could be reliable an adaptive partial equalization technique in which β is slowly adapted to the optimum value for the current set of $\overline{\gamma}$, N_u and M.

B. Bit Error Probability for CE Detection

For a wider comparison of results, we also evaluate the performance when a CE detector is implemented. The test statistic for this case is evaluated in Appendix III resulting in (69), (71) and (72). Hence, we obtain that $I+N$ are Gaussian distributed as following:

$$\mathcal{N}\left(0, E_b\delta_d(N_u-1)\zeta_\beta(\alpha)+M\mathbb{E}\left\{(u(\alpha-\rho_{TH}))^2\right\}\frac{N_0}{2}\right), \quad (48)$$

and by applying the same methodology as for the partial equalization technique, we derive the BEP conditioned to the r.v. U as in (36) with SNIR equal to

$$\frac{\frac{E_b\delta_d}{M}\left[\sum_{l=0}^{M-1}u(\alpha_l-\rho_{TH})\right]^2}{2E_b\delta_d(N_u-1)\zeta(\alpha)+M\mathbb{E}\left\{\left[\alpha_l^{-1}u(\alpha_l-\rho_{TH})\right]^2\right\}N_0}.$$

Through the LLN as in [1], that is approximating $\sum_{l=0}^{M-1}u(\alpha_l-\rho_{TH})$ with $M\mathbb{E}\{u(\alpha_l-\rho_{TH})\}$, and since

$$\mathbb{E}\{u(\alpha-\rho_{TH})\} = e^{-\frac{\rho_{TH}^2}{2\sigma_H^2}}, \quad (49)$$

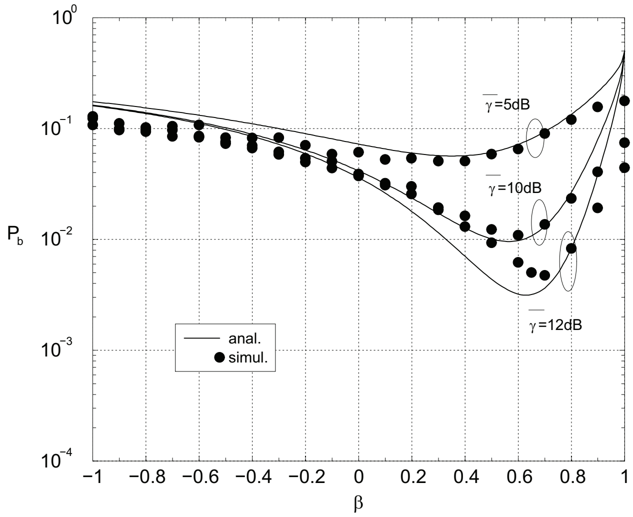

Fig. 3. Analytical and simulated BEP as a function of the parameter β for $\overline{\gamma} = 5$, 10dB and 12dB with $M = N_u = 1024$.

$$\mathbb{E}\left\{(\alpha^{-1}u(\alpha-\rho_{TH}))^2\right\} = \frac{1}{2\sigma_H^2}\Gamma\left[0, \frac{\rho_{TH}^2}{2\sigma_H^2}\right], \quad (50)$$

$$\zeta(\alpha) = \left[e^{\left(-\frac{\rho_{TH}^2}{2\sigma_H^2}\right)}-e^{\left(-\frac{\rho_{TH}^2}{\sigma_H^2}\right)}\right], \quad (51)$$

we obtain the BEP expression in the case of CE detector as given by (36) with SNIR equal to

$$\text{SNIR} = \frac{\overline{\gamma}e^{-\frac{\rho_{TH}^2}{\sigma_H^2}}}{2\frac{N_u-1}{M}\left[e^{\left(-\frac{\rho_{TH}^2}{2\sigma_H^2}\right)}-e^{\left(-\frac{\rho_{TH}^2}{\sigma_H^2}\right)}\right]\overline{\gamma}+\Gamma\left[0, \frac{\rho_{TH}^2}{2\sigma_H^2}\right]}. \quad (52)$$

VI. Numerical Results

In this section, numerical results on the BEP and the optimum β in different system conditions will be shown. Firstly, the goodness of the presented approach is proved by comparison with simulations. In particular, Fig. 3 shows the BEP as a function of β for different values of $\overline{\gamma}$ (5dB, 10dB and 12dB) and $N_u = M = 1024$. Analytical results coming from (38) and simulative results appear to be in a good agreement, in particular for what concerns the value of β providing the minimum for the BEP. Moreover, it can be noted that the choice of the optimum value of β guarantees a significant improvement in the performance with respect to the cases of MRC ($\beta = -1$), EGC ($\beta = 0$) and ORC ($\beta = 1$); this improvement appears more relevant as the SNR increases.

Fig. 4 shows the optimum value of β for different combinations of $\overline{\gamma}$, M, N_u, that is for different values of ξ. The analytical curve is obtained by simply noting that the β satisfying the (47) can be written as a function of ξ by inverting

$$\xi = \left(\frac{1}{\Psi\left(\frac{3-\beta}{2}\right)-\Psi(1-\beta)}+\beta-1\right)^{-1}. \quad (53)$$

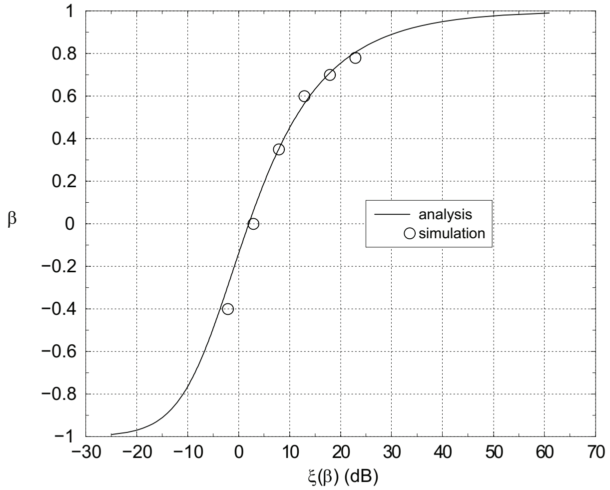

Fig. 4. Analytical and simulated optimum β as a function of the parameter ξ.

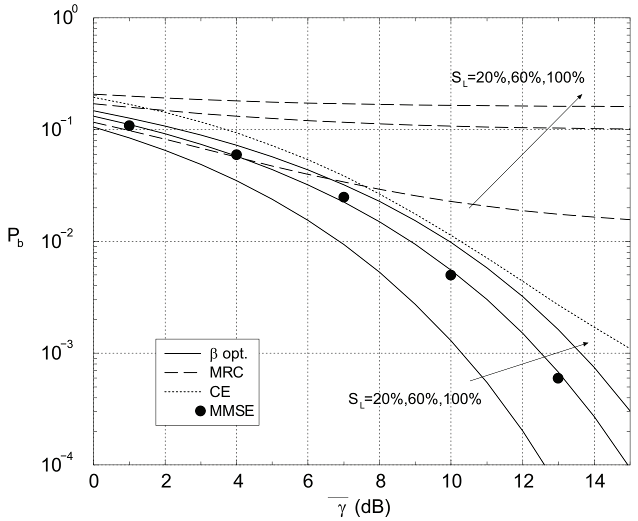

Fig. 5. BEP as a function of the mean SNR for system load $S_L = (N_u - 1)/M$ equal to 20%, 60% and fully-loaded when MRC or partial equalization with optimum β are adopted. For the fully-loaded case, the comparison includes also MMSE (from [13]) and CE detectors.

An immediate comment is necessary at this point. In fact, this means that the optimum β does not depend on $\overline{\gamma}$, M and N_u separately, but it is a function of the ensemble parameter ξ. In addition, a perfect agreement with simulations can be appreciated. In particular, this figure confirms that if the system is noise-limited (low ξ), MRC solution is optimum, while, if the system is interference-limited (high ξ), a choice close to ORC is required.

The performance improvement of partial equalization technique with optimum β with respect to classical MRC can be evaluated, for different system load $S_L = (N_u - 1)/M$ and SNRs, by observing Fig. 5. As an example, at $\overline{\gamma} = 8$dB with $S_L = 20\%$ the BEP is about 0.005 with optimum β against 0.03 with MRC, whereas for $S_L = 60\%$ is about 0.015 and 0.11, for optimum β and MRC, respectively. When the system is fully-loaded, Fig. 5 also shows a comparison with MMSE (from [13]) and CE detectors. For CE we checked that $\rho_{TH} = 0.25$ is a good value for the SNR range considered. As can be observed, MMSE always provides the better performance and it is about $1 \div 1.5$ dB away from that obtained with partial equalization technique with optimum β. Note also that the system with optimum β and system load 60% performs as fully-loaded MMSE.

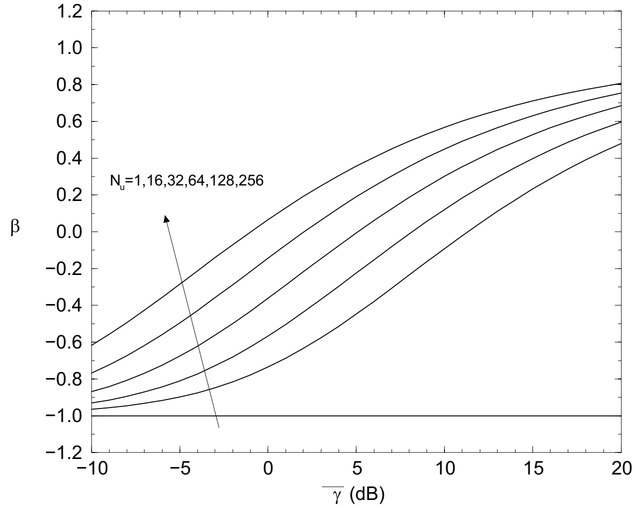

Fig. 6. Optimum β as a function of the SNR for different numbers of active users and $M = 256$.

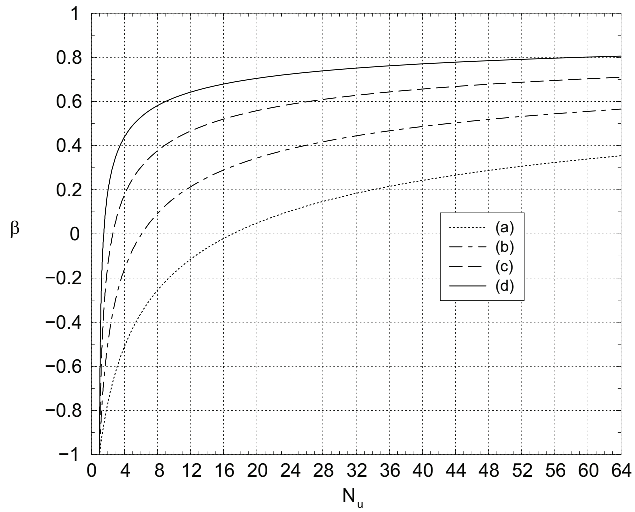

Fig. 7. Optimum β as a function of the number of active users for $M = 64$, $\overline{\gamma}$=5dB (a), $\overline{\gamma}$=10dB (b), $\overline{\gamma}$=15dB (c) and $\overline{\gamma}$=20dB (d).

For a fixed value of sub-carriers $M = 256$, the Fig. 6 shows the optimum value of β as a function of the SNR for different numbers of active users, that is for different system loads (interference free, $1/16$, $1/8$, $1/4$, $1/2$ and fully-loaded). For low SNRs, the MRC represents the best choice, while for high SNRs, the optimum tends to the ORC technique. For a fixed SNR, the optimum β increases with the number of users.

To compare our results also with those obtained in [13], we report in Fig. 7 the optimum value of β for $M = 64$ as a function of the number of active users (i.e., the system load) for four different values of the SNR $(5, 10, 15, 20\text{dB})$. In [13] the fully-loaded case was examined providing values of β in agreement with our analytical results for $N_u = 64$.

In Fig. 8 the impact of different equalization strategies on

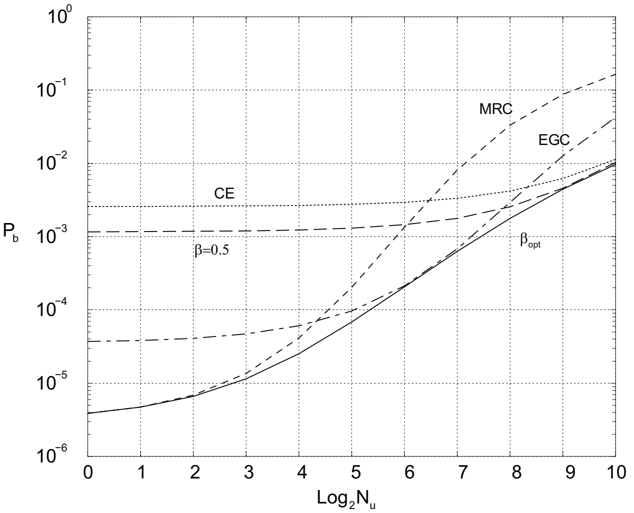

Fig. 8. The impact of the parameter β on the BEP as a function of the number of users for $M = 1024$ and $\overline{\gamma} = 10$dB for different detection techniques.

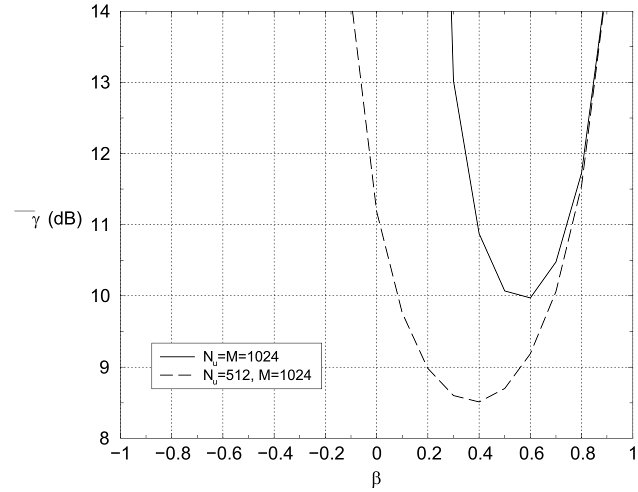

Fig. 9. SNR required for a target $P_b = 0.01$, as a function of β, for $N_u = M = 1024$ (fully loaded) and $N_u = M/2 = 512$ (half load).

the BEP as a function of the number of active users, N_u, is reported for $\overline{\gamma} = 10$dB and $M = 1024$. First of all it can be noticed that the optimum β always provides the better performance; then, it can be observed that when few users are active MRC represents a good solution, approaching the optimum, crossing the performance of EGC for a system load about $1/64 \div 1/32$ (i.e., $N_u = 16 \div 32$) and the performance of a CE detector with $\rho_{TH} = 0.25$ for a system load about $1/16 \div 1/8$. Note that a fixed value of β equal to 0.5 represents a solution close to the optimum for system loads ranging in $1/4 \div 1$ (i.e., $N_u = 256 \div 1024$) and the performance still remain in the same order for all system loads.

Since the point of view of a system designer is to evaluate the SNR required by different solutions to obtain a target BEP, in Fig. 9 the required SNR for $P_b = 0.01$ (a typical value of interest for uncoded systems) is plotted as a function of β for $N_u = M = 1024$ (fully loaded) and $N_u = M/2 = 512$ (half load). These results show a strong impact of the choice of β on the required SNR: as an example, to obtain the target BEP with a SNR up to 1dB above the value required with the optimum β, the admitted ranges for β are $0.4 \div 0.75$ for the fully loaded and $0.1 \div 0.7$ for the half load.

VII. Conclusions

In this paper we analytically evaluated the performance of MC-CDMA systems adopting a partial equalization technique. This technique depends on a parameter β for which the value optimizing the performance has been analytically derived depending on the SNR, the number of sub-carriers and the number of active users. The proposed technique is shown to have the same complexity of the well known maximal ratio combining, equal gain combining and orthogonality restoring combining techniques, with significative performance improvement. Different situations for which β is varied adaptively to slowly varying processes or maintained constant to a value about 0.5 have been compared with MRC, EGC, ORC and CE techniques. The gain in terms of required SNR to achieve a target BEP has been reported in different system conditions. In addition, comparisons with simulations have been made and the perfect agreement validates the proposed methodology.

Appendix I

In this appendix we evaluate the term $u_{m,l}[j]$ in (12). We distinguish two cases ($m = l$ and $m \neq l$) and we use the well known property $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ and the fact that, for the assumptions, f_0T and $\Delta f \cdot T$ are integers. If $m = l$ then

$$u_{l,l}[j] = \frac{\sin(Q_0 + 4\pi n_0) - \sin Q_0}{4\pi n_0} + 1 = 1\,, \qquad (54)$$

where $n_0 \triangleq f_0T + l\Delta fT$ integer and $Q_0 \triangleq 4\pi f_l jT_b + 2\vartheta_l$. If $m \neq l$ then

$$u_{m,l}[j] = \frac{\sin(Q_1 + 2\pi n_1) - \sin Q_1}{2\pi n_1} + \frac{\sin(Q_2 + 2\pi n_2) - \sin Q_2}{2\pi n_2} = 0\,, \qquad (55)$$

where $Q_1 \triangleq 2\pi(f_m + f_l)jT_b + \vartheta_m + \vartheta_l$, $Q_2 \triangleq 2\pi(f_m - f_l)jT_b + \vartheta_m - \vartheta_l$, $n_1 \triangleq 2f_0T + (m+l)\Delta f \cdot T$ integer and $n_2 \triangleq (m-l)\Delta f \cdot T$ integer. Summarizing, we have:

$$u_{m,l}[j] = \begin{cases} 1 & m = l, \\ 0 & m \neq l. \end{cases}, \qquad (56)$$

independently on the index j.

Appendix II

In this appendix we evaluate $\mathbb{E}\left\{\alpha^{1-\beta}\right\}$, $\mathbb{E}\left\{\alpha^{-2\beta}\right\}$ and $\zeta_\beta(\alpha) = \mathbb{E}\left\{(\alpha^{1-\beta})^2\right\} - (\mathbb{E}\left\{\alpha^{1-\beta}\right\})^2$ where α is a Rayleigh distributed r.v. Thus, the p.d.f. of α is

$$\alpha \sim \frac{\alpha}{\sigma_H^2} e^{-\frac{\alpha^2}{2\sigma_H^2}}\,, \qquad (57)$$

for $\alpha \geq 0$ and 0 otherwise. It is known from [17] that

$$\int_0^{+\infty} x^{a-1} e^{-px^2} dx = \frac{1}{2} p^{-\frac{a}{2}} \Gamma\left(\frac{a}{2}\right)\,,\ a > 0\,, \qquad (58)$$

where $\Gamma(z)$ represents the Euler Gamma function. Hence:

$$\mathbb{E}\left\{\alpha^{1-\beta}\right\} = \int_0^{+\infty} \alpha^{1-\beta}\frac{\alpha}{\sigma_H^2}e^{-\frac{\alpha^2}{2\sigma_H^2}}d\alpha = (2\sigma_H^2)^{\frac{1-\beta}{2}}\Gamma\left(\frac{3-\beta}{2}\right), \quad (59)$$

$$\mathbb{E}\left\{\alpha^{-2\beta}\right\} = \int_0^{+\infty} \alpha^{-2\beta}\frac{\alpha}{\sigma_H^2}e^{-\frac{\alpha^2}{2\sigma_H^2}}d\alpha = (2\sigma_H^2)^{-\beta}\Gamma(1-\beta), \quad (60)$$

$$\mathbb{E}\left\{(\alpha^{1-\beta})^2\right\} = \int_0^{+\infty} \alpha^{2-2\beta}\frac{\alpha}{\sigma_H^2}e^{-\frac{\alpha^2}{2\sigma_H^2}}d\alpha = \left(2\sigma_H^2\right)^{1-\beta}\Gamma(2-\beta). \quad (61)$$

>From (59) and (61) results

$$\zeta_\beta(\alpha) = \left(2\sigma_H^2\right)^{1-\beta}\Gamma(2-\beta) - \left[(2\sigma_H^2)^{\frac{1-\beta}{2}}\Gamma\left(\frac{3-\beta}{2}\right)\right]^2 = (2\sigma_H^2)^{1-\beta}\left[\Gamma(2-\beta) - \Gamma^2\left(\frac{3-\beta}{2}\right)\right]. \quad (62)$$

Appendix III

In this appendix we evaluate the test statistic for a CE (or TORC) detector. Starting from (4),(13) and (14), the decision variable $v^{(n)}$ is given by:

$$v^{(n)} = \overbrace{\sqrt{\frac{E_b\delta_d}{M}}\Sigma_{l=0}^{M-1}u(\alpha_l-\rho_{TH})\,a^{(n)}}^{U} + \overbrace{\sqrt{\frac{E_b\delta_d}{M}}\Sigma_{l=0}^{M-1}\Sigma_{k=0,k\neq n}^{N_u-1}u(\alpha_l-\rho_{TH})c_l^{(n)}c_l^{(k)}a^{(k)}}^{I} + \overbrace{\Sigma_{l=0}^{M-1}\alpha_l^{-1}u(\alpha_l-\rho_{TH})n_l}^{N}. \quad (63)$$

The distribution of the test statistic can be obtained by studying the statistics U, I and N in (63) as in the following.

1) Interference Term: Exploiting the properties of orthogonal codes as in [7], after some algebra the interference term can be rewritten as:

$$I = \sqrt{\frac{E_b\delta_d}{M}}\sum_{k=0,k\neq n}^{N_u-1}a^{(k)} \times \left(\overbrace{\sum_{h=1}^{\frac{M}{2}}u(\alpha_{x_h}-\rho_{TH})}^{B_1} - \overbrace{\sum_{h=1}^{\frac{M}{2}}u(\alpha_{y_h}-\rho_{TH})}^{B_2}\right), \quad (64)$$

where indexes x_h and y_h, thus B_1 and B_2, depend on user k and define the partition as in (19), (20) and (21).

For large M, we can apply the CLT to each one of the internal sums in (64) obtaining

$$B_1 \sim \mathcal{N}\left(\frac{M}{2}\mathbb{E}\left\{u(\alpha-\rho_{TH})\right\}, \frac{M}{2}\zeta(\alpha)\right), \quad (65)$$

$$B_2 \sim \mathcal{N}\left(\frac{M}{2}\mathbb{E}\left\{u(\alpha-\rho_{TH})\right\}, \frac{M}{2}\zeta(\alpha)\right), \quad (66)$$

where $\zeta(\alpha)$ indicates the variance of $u(\alpha-\rho_{TH})$ that is given by

$$\zeta(\alpha) \triangleq \mathbb{E}\left\{(u(\alpha-\rho_{TH}))^2\right\} - \left(\mathbb{E}\left\{u(\alpha-\rho_{TH})\right\}\right)^2 = e^{-\frac{\rho_{TH}^2}{2\sigma_H^2}} - e^{-\frac{\rho_{TH}^2}{\sigma_H^2}}. \quad (67)$$

Therefore, $B \triangleq B_1 - B_2$ is distributed as:

$$B \sim \mathcal{N}(0, M\zeta(\alpha)). \quad (68)$$

By exploiting the symmetry of the Gaussian p.d.f. and the property of the sum of uncorrelated (and thus independent) Gaussian r.v.'s i.i.d. ($a^{(k)}B \sim \mathcal{N}(0, M\zeta(\alpha))$), the interference term is distributed as:

$$I \sim \mathcal{N}\left(0, \sigma_I^2 \triangleq E_b\delta_d(N_u-1)\zeta(\alpha)\right). \quad (69)$$

2) Noise Term: The thermal noise at the combiner output is given by

$$N = \sum_{l=0}^{M-1}\alpha_l^{-1}u(\alpha_l-\rho_{TH})n_l, \quad (70)$$

where terms α_l and n_l are independent and n_l is zero mean. Thus, N consists on a sum of i.i.d zero mean r.v.'s with variance $N_0/2\ \mathbb{E}\left\{(\alpha_l^{-1}u(\alpha_l-\rho_{TH}))^2\right\}$. Following the approximation done in [1], [7], involving the CLT, the unconditioned noise term N results in:

$$N \sim \mathcal{N}\left(0, \sigma_N^2 \triangleq M\frac{N_0}{2}\mathbb{E}\left\{(\alpha_l^{-1}u(\alpha_l-\rho_{TH}))^2\right\}\right), \quad (71)$$

with $\mathbb{E}\left\{(\alpha_l^{-1}u(\alpha_l-\rho_{TH}))^2\right\} = \frac{1}{2\sigma_H^2}\Gamma\left[0, \frac{\rho_{TH}^2}{2\sigma_H^2}\right]$; $\Gamma[0,x]$ being the incomplete Euler Gamma function.

3) Useful Term: By applying the CLT, the term U results in

$$U = \sqrt{\frac{E_b\delta_d}{M}}\sum_{l=0}^{M-1}u(\alpha-\rho_{TH}) \sim \mathcal{N}\left(\sqrt{E_b\delta_d M}\mathbb{E}\left\{u(\alpha-\rho_{TH})\right\}, E_b\delta_d\zeta(\alpha)\right), \quad (72)$$

where $\mathbb{E}\left\{u(\alpha-\rho_{TH})\right\} = e^{-\frac{\rho_{TH}^2}{2\sigma_H^2}}$.

4) Independence Between Each Term: By noting that $a^{(k)}$ is zero mean and statistically independent on α_l, B, and n_l, it follows that $\mathbb{E}\{I\ N\} = \mathbb{E}\{I\ U\} = 0$. Since n_l and α_l are statistically independent, the $\mathbb{E}\{N\ U\} = 0$. Since I, N and U are uncorrelated Gaussian r.v.'s, they are also independent.

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