ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2012) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A production-repairing inventory model with fuzzy rough coefficients under inflation and time value of money

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ARTICLE INFO

Article history:
Received 9 November 2011
Received in revised form 8 June 2012
Accepted 6 July 2012
Available online xxxx

Keywords:
Dynamic production rate
Trust measure
Fuzzy rough variable
Fuzzy rough expectation

ABSTRACT

In this paper, a production-repairing inventory model in fuzzy rough environment is proposed incorporating inflationary effects where a part of the produced defective units are repaired and sold as fresh units. Here, production and repairing rates are assumed as dynamic control variables. Due to complexity of environment, different costs and coefficients are considered as fuzzy rough type and these are reduced to crisp ones using fuzzy rough expectation. Here production cost is production rate dependent, repairing cost is repairing rate dependent and demand of the item is stock-dependent. Goal of the research work is to find decisions for the decision maker (DM) who likes to maximize the total profit from the above system for a finite time horizon. The model is formulated as an optimal control problem and solved using a gradient based non-linear optimization method. Some particular cases of the general model are derived. The results of the models are illustrated with some numerical examples.

1. Introduction

Different types of uncertainty such as randomness [1], fuzziness [2,3] and roughness [4] are common factors in any real life problem including inventory control. Well established mathematical tools are available to deal with problems involving these uncertainties [2,3,5,1]. But in real life, some problems occur where both fuzziness and roughness exist simultaneously. To overcome these situations normally fuzzy rough variables are used to model the problem. Dubois and Prade [6] introduced the concept of fuzzy rough sets. After that, some researchers [7,8] defined fuzzy rough set as a more general case. Using this approach some researchers modelled different problems where fuzziness and roughness occur simultaneously [9–14]. Liu [15] proposed some definitions and discussed some valuable properties of fuzzy rough variable.

In many cases, it is found that some inventory parameters involve both the fuzzy and rough uncertainties. For example, the inventory related costs – holding cost, set-up cost, production cost, repairing cost, disposal cost, etc. depend on several factors such as bank interest, inflation, labour wages, wear and tear cost, etc. which are uncertain in fuzzy rough sense. To be more specific, inventory holding cost is sometimes represented by a fuzzy number and it depends on the storage amount which may be imprecise and range within an interval due to several factors such as scarcity of storage space, market fluctuation, human estimation/ thought process i.e. it may be represented by a rough set. In the literature, some researchers

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http://dx.doi.org/10.1016/j.apm.2012.07.024

[16,1,17,18] developed and solved inventory models in fuzzy random environment. Very few investigations [9,10] are available for inventory or production-inventory problems with fuzzy rough impreciseness.

Production-repairing system is, now-a-day an important area of inventory studies due to growing environmental concern, environmental regulations in industry and gradually decreasing resources in the world. Beside fabricating the finished product from raw materials, it may be possible to repair the defective units produced from the production. In these cases, recovery of the defective units is economically more attractive than disposal. Furthermore, in the recent past, the growth of environment movement has given the reuse system increasing attention [19]. Initial attempts to address the inventory of repairing items or products dates back to the 1960s, with Schrady [20] being the first to investigate a repair-inventory system. After that, extensive research works have been made to develop real-life recycling models during last two decades[3,21–24]. Some scholars also studied inventory problem in repairing/reworked processes under mixed uncertain environment [10,25].

Defective units may be reworked in the same cycle along with the normal production after some time from the initial commencement of production engaging some additional labour forces and machinery for repair. But all of the defective units can not be considered for reworking. Some of the defective units may be of very poor quality so that it will be expensive to repair those units, which should be avoided for rework. Therefore, a certain percentage of the defective units may be considered for reworking. The demand of the units will be met not only from the produced perfect units but also from the reworked units.

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. Most of the classical inventory models did not take into account the effects of inflation and time value of money. This has happened mostly because of the belief that inflation and time value of money do not influence the cost and price components (i.e., the inventory policy) to any significant degree. But, during last few decades, due to high inflation and consequent sharp decline in the purchasing power of money in the developing countries like India, Bangladesh etc., the financial situation has been changed and so it is not possible to ignore the effect of inflation and time value of money any further. Misra [26], Chang [27], Sarkar and Moon [28] and Sarkar et al. [29,30] have extended their approaches to different inventory models by considering the time value of money, different inflation rates for the costs, finite replenishment, shortages etc.

After all these studies, some lacunas in the formation of the models and the shortcomings may be summarized as below:

- Most of the earlier models considered constant production/repairing rates and constant production/repairing costs. But in many production systems, production cost is function of production rate [31,32] with imprecise parameters.
- One of the weaknesses of major production-inventory models is the unrealistic assumption that all units produced are of good quality. But production of defective units [33,34] is a natural phenomenon in any production process.
- Normally production inventory models are formulated for infinite time horizon which is not realistic [5]. Due to rapid change of world economy, specially for fashionable/luxarious items, manufacturers very frequently change their product specification with new features, names and packets to attract the customers. As a result lifetime of these products (with respect to demand) in the market is finite[2,32]. Not much attention has been paid on production inventory models in finite time horizon specially for the production repairing system.
- A little study has been made on production repairing model considering selling price, production cost, holding cost, set-up cost, etc. as fuzzy rough parameters [9].
- No investigation on production-repairing inventory system has been made considering dynamic production and repairing rates.

In order to overcome the above mentioned limitations of reworked inventory models, in this paper, a production-repairing inventory model incorporating inflation and time value of money with fuzzy rough coefficients is considered with displayed inventory level dependent demand, dynamic production and repairing rates, production rate dependent production cost and repairing rate dependent repairing cost. Some particular cases of the general model such as models with constant demand having constant/linear production and repairing rates and costs are presented. Numerical examples are provided to illustrate the model.

Rest of the paper is organized as follows. In Section 2, some preliminaries and deductions are presented. In Section 3, assumptions and notations of the proposed inventory model are listed. In Section 4, mathematical formulation of the inventory model is presented. Numerical examples to illustrate the models are provided in Section 5. Some particular cases are presented in Section 6, also. Finally, a brief conclusion is drawn in Section 7. Some mathematical calculations are presented in Appendix.

2. Preliminaries and deductions

2.1. Possibility (Pos), necessity (Nes) measure

Any fuzzy subset $\tilde{a}$ of $\mathcal{R}$ with membership function $\mu_{\tilde{a}}(x) : \mathcal{R} \to [0,1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy quantities with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Dubois and Prade [35,36] and Liu and Iwamura [37]

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$$Pos(\tilde{a} * \tilde{b}) = \{sup(min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)),\ x, y \in \mathcal{R}, x * y\}, \tag{1}$$

$$Nes(\tilde{a} * \tilde{b}) = \{inf(max(1 - \mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)),\ x, y \in \mathcal{R}, x * y\}, \tag{2}$$

where $*$ is any arithmetic relational operator and $\mathcal{R}$ is set of real numbers.

2.1.1. Fuzzy extension principle [38]

If $\tilde{a}, \tilde{b} \in \mathcal{R}$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be a binary operation then membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as

$$\mu_{\tilde{c}}(z) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)),\ x, y \in \mathcal{R} \text{ and } z = f(x, y),\ \forall z \in \mathcal{R}\}. \tag{3}$$

2.1.2. Credibility measure [18]

If A be a fuzzy event then credibility measure of A is denoted by $Cr(A)$ and defined as

$$Cr(A) = \frac{1}{2}[Pos(A) + Nes(A)].$$

More generally, according to Maity [2] the above form can be consider as

$$Cr(A) = [\rho Pos(A) + (1 - \rho)Nes(A)] \quad \text{where } 0 \leqslant \rho \leqslant 1.$$

2.2. Fuzzy expectation [39]

Let $\widetilde{X}$ be a normalized fuzzy variable. The expected value of the fuzzy variable $\widetilde{X}$ is denoted by $E[\widetilde{X}]$ and defined by

$$E[\widetilde{X}] = \int_0^\infty Cr(\widetilde{X} \geqslant r)dr - \int_{-\infty}^0 Cr(\widetilde{X} \leqslant r)dr. \tag{4}$$

When the right hand side of (4) is of form ∞ to $-\infty$ the expected value is not defined.

Lemma 1 ([2]). *If $\tilde{a} = (a_1, a_2, a_3)$ is a TFN and r is a crisp number, then expected value of $\tilde{a}$, $E[\tilde{a}]$, is given by*

$$E[\tilde{a}] = \frac{1}{2}[(1 - \rho)a_1 + a_2 + \rho a_3] \quad \text{where } 0 \leqslant \rho \leqslant 1.$$

2.3. Rough space [15]

Let Λ be a non empty set, κ a σ algebra of subsets of Λ, and Δ an element in κ and π a trust measure. Then $(\Lambda, \Delta, \kappa, \pi)$ is called a rough space.

2.4. Rough variable [15]

Let $(\Lambda, \Delta, \kappa, \pi)$ be a rough space. A rough variable ξ is a measurable function from the rough space $(\Lambda, \Delta, \kappa, \pi)$ to the set of real numbers. i.e. for every Borel set B of $\mathcal{R}$, $\{\lambda \in \Lambda | \xi(\lambda) \in B\} \in \kappa$.

The lower $(\underline{\xi})$ and upper $(\overline{\xi})$ approximations of the rough variable ξ are given by $\underline{\xi} = \{\xi(\lambda) | \lambda \in \Delta\}$ and $\overline{\xi} = \{\xi(\lambda) | \lambda \in \Lambda\}$.

2.5. Trust measure [15]

Let $(\Lambda, \Delta, \kappa, \pi)$ be a rough space. The trust measure of event A is denoted by $Tr\{A\}$ and defined by $Tr\{A\} = \frac{1}{2}(\underline{Tr}\{A\} + \overline{Tr}\{A\})$, where $\underline{Tr}\{A\}$ denotes the lower trust measure of event A, defined by $\underline{Tr}\{A\} = \frac{\pi\{A \bigcap \Delta\}}{\pi\{\Delta\}}$, and $\overline{Tr}\{A\}$ denotes the upper trust measure of event A, defined by $\overline{Tr}\{A\} = \frac{\pi\{A\}}{\pi\{\Lambda\}}$. When the enough information about the measure π is not given, it may be treated as the Lebesgue measure. Then we can get the trust measure of the rough event $\hat{\xi} \geqslant t$, $Tr\{\hat{\xi} \geqslant t\}$ and its function curve (cf. Fig. 1) as presented below where t is a crisp number, $\hat{\xi}$ is a rough variable given by $\hat{\xi} = ([a, b][c, d]),\ 0 \leqslant c \leqslant a \leqslant b \leqslant d$.

$$Tr\{\hat{\xi} \geqslant t\} = \begin{cases} 0 & \text{for } d \leqslant t, \\ \frac{(d-t)}{2(d-c)} & \text{for } b \leqslant t \leqslant d, \\ \frac{1}{2}\left(\frac{(d-t)}{(d-c)} + \frac{(b-t)}{(b-a)}\right) & \text{for } a \leqslant t \leqslant b, \\ \frac{1}{2}\left(\frac{(d-t)}{(d-c)} + 1\right) & \text{for } c \leqslant t \leqslant a, \\ 1 & \text{for } t \leqslant c. \end{cases}$$

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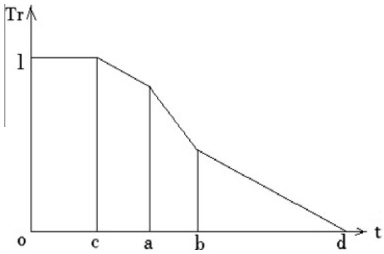

Fig. 1. Tr $\{\hat{\xi} \geqslant t\}$ function curve.

2.6. Rough expectation [15]

Let $\widehat{X}$ be a rough variable. The expected value of the rough variable $\widehat{X}$ is denoted by $E[\widehat{X}]$ and defined by

$$E[\widehat{X}] = \int_0^{\infty} Tr(\widehat{X} \geqslant r)dr - \int_{-\infty}^{0} Tr(\widehat{X} \leqslant r)dr \tag{5}$$

provided that at least one of the two integrals is finite. When the right hand side of (5) is of form ∞ to $-\infty$ the expected value is not defined.

Lemma 2. *If $\hat{\xi} = ([a,b][c,d])$ is a rough variable and r is a crisp number, then expected value of $\hat{\xi}$, $E[\hat{\xi}]$, is given by*

$$E[\hat{\xi}] = \frac{1}{4}[a+b+c+d].$$

Proof. Since $\hat{\xi} = ([a,b][c,d])$ is a rough variable and r is a crisp number, then from definition of trust measure we have

$$Tr\{\hat{\xi} \geqslant r\} = \begin{cases} 0 & \text{for } d \leqslant r, \\ \frac{(d-r)}{2(d-c)} & \text{for } b \leqslant r \leqslant d, \\ \frac{1}{2}\left(\frac{(d-r)}{(d-c)} + \frac{(b-r)}{(b-a)}\right) & \text{for } a \leqslant r \leqslant b, \\ \frac{1}{2}\left(\frac{(d-r)}{(d-c)} + 1\right) & \text{for } c \leqslant r \leqslant a, \\ 1 & \text{for } r \leqslant c, \end{cases}$$

$$Tr\{\hat{\xi} \leqslant r\} = \begin{cases} 0 & \text{for } r \leqslant c, \\ \frac{(r-c)}{2(d-c)} & \text{for } c \leqslant r \leqslant a, \\ \frac{1}{2}\left(\frac{(r-c)}{(d-c)} + \frac{(r-a)}{(b-a)}\right) & \text{for } a \leqslant r \leqslant b, \\ \frac{1}{2}\left(\frac{(r-c)}{(d-c)} + 1\right) & \text{for } b \leqslant r \leqslant d, \\ 1 & \text{for } d \leqslant r. \end{cases}$$

So the expected value of $\hat{\xi}$ is calculated using (5) as follows:

$$\begin{aligned} E[\hat{\xi}] &= \int_0^{\infty} Tr(\hat{\xi} \geqslant r)dr - \int_{-\infty}^{0} Tr(\hat{\xi} \leqslant r)dr \\ &= \int_0^c 1dr + \int_c^a \frac{1}{2}\left(\frac{(d-r)}{(d-c)} + 1\right)dr + \int_a^b \frac{1}{2}\left(\frac{(d-r)}{(d-c)} + \frac{(b-r)}{(b-a)}\right)dr + \int_b^d \frac{(d-r)}{2(d-c)}dr = \frac{1}{4}[a+b+c+d]. \end{aligned}$$

2.7. Fuzzy rough variable [15]

A fuzzy rough variable is a measurable function from a rough space $(\Lambda, \Delta, \kappa, \pi)$ to the set of fuzzy variables. More generally, a fuzzy rough variable is a rough variable taking fuzzy values.

2.8. Fuzzy-rough expectation [15]

Let $\hat{\tilde{X}}$ be a fuzzy rough variable. The expected value of the fuzzy rough variable $\hat{\tilde{X}}$ is denoted by $E[\hat{\tilde{X}}]$ and defined by

$$E[\hat{\tilde{X}}] = \int_0^\infty Tr(\lambda \in \Lambda | E[\tilde{X}(\lambda)] \geqslant r)dr - \int_{-\infty}^0 Tr(\lambda \in \Lambda | E[\tilde{X}(\lambda)] \leqslant r)dr \tag{6}$$

provided that at least one of the two integrals is finite.

Lemma 3. *Let $\hat{\tilde{\xi}} = (\hat{\xi} - L, \hat{\xi}, \hat{\xi} + R)$ be a fuzzy rough variable, where $\hat{\xi} = ([a,b][c,d])$ is a rough variable. Then expected value of $\hat{\tilde{\xi}}$ is*

$$E[\hat{\tilde{\xi}}] = \frac{1}{4}[a+b+c+d] + \frac{\rho R - (1-\rho)L}{2} \quad \text{where } 0 \leqslant \rho \leqslant 1.$$

Proof. Since $\hat{\tilde{\xi}} = (\hat{\xi} - L, \hat{\xi}, \hat{\xi} + R)$ where $\hat{\xi} = ([a,b][c,d])$ is a fuzzy rough variable, then using Lemma 1 we get,

$$E[\hat{\tilde{\xi}}] = E[\frac{1}{2}[(1-\rho)(\hat{\xi} - L) + \hat{\xi} + \rho(\hat{\xi} + R)]] \quad \text{where } 0 \leqslant \rho \leqslant = E[\hat{\xi} + \theta] \quad \text{where } \theta = \frac{\rho R - (1-\rho)L}{2}.$$

Again using Lemma 2 we get,

$$E[\hat{\xi} + \theta] = \frac{1}{4}[a+b+c+d] + \theta = \frac{1}{4}[a+b+c+d] + \frac{\rho R - (1-\rho)L}{2}.$$

Therefore

$$E[\hat{\tilde{\xi}}] = \frac{1}{4}[a+b+c+d] + \frac{\rho R - (1-\rho)L}{2} \quad \text{where } 0 \leqslant \rho \leqslant 1.$$

2.9. Single objective fuzzy rough expected value model

In order to solve the uncertain model with fuzzy rough parameters, we must convert it into a deterministic one. The technique of computing the expected value is an efficient method and is easily realized.

2.9.1. Basic model

Consider the following problem with fuzzy rough coefficients

$$Max f(x, \hat{\tilde{\xi}}), \tag{7}$$

where x is decision vector, $\hat{\tilde{\xi}}$ is a fuzzy rough vector, $f(x, \hat{\tilde{\xi}})$ is objective function. As optimization of a fuzzy rough objective is not well defined the above problem can not be solved directly. According to Liu [15], above problem is equivalent to

$$Max\, E[f(x, \hat{\tilde{\xi}})]. \tag{8}$$

3. Assumptions and notations for the proposed models

3.1. Assumptions

Following assumptions are made for developing the proposed production-repairing inventory model

(i) Inventory system involves only one item.
(ii) Production and repairing rates are time dependent.
(iii) Defective rate is constant and known.
(iv) Certain percentage of defective units is repairable and known.
(v) The cycle lengths are all equal.
(vi) After some production cycles repairing process starts. After that both production and repairing processes are continued.
(vii) The effects of inflation and time value of money are considered. Let k and r be the inflation rate and discount rate respectively. Therefore, the net discount rate of inflation, R is defined as: $R = r - k$.
(viii) Demand is displayed inventory level dependent.
(ix) Shortages are not allowed.
(x) Production and repairing costs are known function of production and repairing rates respectively.
(xi) This is a multi period production-inventory model with finite time horizon.
(xii) Repaired units are treated as serviceable units.

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3.2. Notations

Following notations are used for developing the model

$u(t)$	$(u_0 + u_1 t + u_2 t^2)$ production rate at any time t where u_0, u_1 and u_2 are decision variables.
δ	constant defective rate of production.
t_u	the cycle length for both production and repairing.
m	number of production cycles with out repairing.
n	total number of cycles.
$x(t)$	the serviceable stock level at time t.
$x_R(t)$	the repairable stock level at time t.
$p(t)$	$(p_0 + p_1 t)$ repairing rate at any time t where p_0 and p_1 are decision variables.
$D(t)$	$(d_0 + d_1 x(t))$ demand rate, where d_0 and d_1 are constant and known.
$\hat{\tilde{c}}_u$	$(\hat{\tilde{c}}_0 + \hat{\tilde{c}}_1/u(t) + \hat{\tilde{c}}_2 u(t))$ the per unit production cost where $\hat{\tilde{c}}_0$ is raw material cost, $\hat{\tilde{c}}_1$ is labour charges and $\hat{\tilde{c}}_2$ is tool wear and tear cost in \$.
t'_{ui}	the duration of production of the i-th cycle.
$\hat{\tilde{c}}_r$	$(\hat{\tilde{c}}_{r1}/p(t) + \hat{\tilde{c}}_{r2} p(t))$ the per unit repairing cost where $\hat{\tilde{c}}_{r1}$ is labour charges, and $\hat{\tilde{c}}_{r2}$ is tool wear and tear cost in \$.
$\hat{\tilde{s}}_u$	the set up cost of each cycle in \$.
$\hat{\tilde{c}}_{h1}$	the holding cost of per serviceable unit per unit time in \$.
$\hat{\tilde{c}}_{h2}$	the holding cost of per repairable unit per unit time in \$.
T	the finite time horizon in week.
$\hat{\tilde{c}}_s$	the per unit selling price in \$.
μ	the constant percentage of defective units for repair.
$\hat{\tilde{c}}_z$	the per unit disposal cost in \$.
ρ	the degree of optimism of expected profit.

Symbols $\tilde{}$, $\hat{}$ and $\hat{\tilde{}}$ that are used on the top of the above notations, indicate fuzzy parameters, rough parameters and fuzzy rough parameters respectively.

4. Model development and analysis

4.1. Proposed production-repairing model in fuzzy rough environment

In the development of the model it is assumed that demand of the item in the market exists for a finite time T, which is the planning horizon of the model. Total planning horizon is divided into n cycles. At the beginning of i-th (i = 1,2,...,n) cycle production starts and continues for a time t'_{ui}. Among produced units $\delta\%$ units are defective units and remaining units are serviceable units. Among produced defective units $\mu\%$ units are repairable and remaining units are disposed. Repairable units are stored for first m cycles so that its amount becomes sufficient to start repairing process. Along with production, repairing starts simultaneously from $m+1$th cycle and repaired units are as good as non-defective (serviceable) units. From this cycle repairing process continues in each cycle for a duration in which production process runs and at the end of last production cum repairing process excess repairable units are disposed. Total process is depicted in Fig. 2. Demand of the item is met using serviceable units. Production rate is quadratic function in time t and repairing rate is a linear function in time t whose coefficients are so determined that produced serviceable (including repaired) units are just sufficient to meet the demand of the cycle. Here production cost depends on production rate, repairing cost depends on repairing rate and some other inventory costs are assumed as fuzzy rough in nature. Goal is to find decision for a DM who likes to maximize the total expected profit under inflationary effect from the planning horizon. According to these assumptions instantaneous stock $x(t)$ of serviceable units at any time $t \in [0, mt_u]$ is given by

$$\frac{dx(t)}{dt} = \begin{cases} (1-\delta)u(t) - D(t) & \text{for } (i-1)t_u \leqslant t \leqslant (i-1)t_u + t'_{ui}, \\ -D(t) & \text{for } (i-1)t_u + t'_{ui} \leqslant t \leqslant it_u, \end{cases} \tag{9}$$

with boundary conditions $x((i-1)t_u) = 0,\ x(it_u) = 0$ for $i = 1, 2, \ldots, m$.

Solving the above differential equations we get,

$$x(t) = \begin{cases} \frac{((1-\delta)u_0 - d_0)(1-e^{-d_1(t-(i-1)t_u)})}{d_1} + (1-\delta)\Big[u_1\Big\{\frac{(t-(i-1)t_u e^{-d_1(t-(i-1)t_u)})}{d_1} \\ -\frac{(1-e^{-d_1(t-(i-1)t_u)})}{d_1^2}\Big\} + u_2\Big\{\frac{(t^2-(i-1)^2 t_u^2 e^{-d_1(t-(i-1)t_u)})}{d_1} \\ -\frac{2(t-(i-1)t_u e^{-d_1(t-(i-1)t_u)})}{d_1^2} + \frac{2(1-e^{-d_1(t-(i-1)t_u)})}{d_1^3}\Big\}\Big] & \text{for } (i-1)t_u \leqslant t \leqslant (i-1)t_u + t'_{ui}, \\ \frac{d_0(e^{d_1(it_u - t)} - 1)}{d_1} & \text{for } (i-1)t_u + t'_{ui} \leqslant t \leqslant it_u, \end{cases} \tag{10}$$

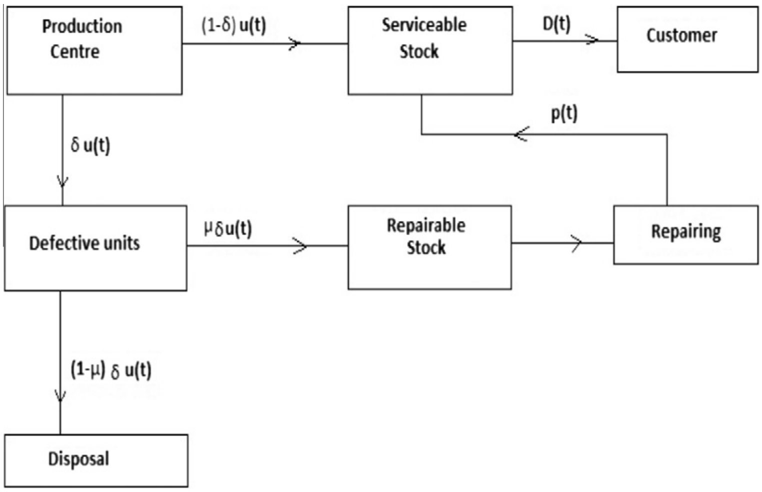

Fig. 2. The material flow in the production-repairing model.

Similarly instantaneous stock $x(t)$ of serviceable units at any time $t \in [mt_u, T]$ is given by

$$\frac{dx(t)}{dt} = \begin{cases} (1-\delta)u(t) - D(t) + p(t) & \text{for } (j-1)t_u \leqslant t \leqslant (j-1)t_u + t'_{uj}, \\ -D(t) & \text{for } (j-1)t_u + t'_{uj} \leqslant t \leqslant jt_u, \end{cases} \tag{11}$$

with boundary conditions $x(jt_u) = 0$ where $j = m+1, m+2, \ldots, n$.

Solving the above differential equations we get,

$$x(t) = \begin{cases} \frac{((1-\delta)u_0 - d_0 + p_0)(1-e^{-d_1(t-(j-1)t_u)})}{d_1} + (1-\delta)\Big[\Big(u_1 + \frac{p_1}{1-\delta}\Big)\Big\{\frac{(t-(j-1)t_u e^{-d_1(t-(j-1)t_u)})}{d_1} & \\ -\frac{(1-e^{-d_1(t-(j-1)t_u)})}{d_1^2}\Big\} + u_2\Big\{\frac{(t^2-(j-1)^2 t_u^2 e^{-d_1(t-(j-1)t_u)})}{d_1} & \\ -\frac{2(t-(j-1)t_u e^{-d_1(t-(j-1)t_u)})}{d_1^2} + \frac{2(1-e^{-d_1(t-(j-1)t_u)})}{d_1^3}\Big\}\Big] & \text{for } (j-1)t_u \leqslant t \leqslant (j-1)t_u + t'_{uj}, \\ \frac{d_0(e^{d_1(jt_u-t)}-1)}{d_1} & \text{for } (j-1)t_u + t'_{uj} \leqslant t \leqslant jt_u, \end{cases} \tag{12}$$

Instantaneous stock for repairable units at any time $t \in [0, T]$ is given by

$$\frac{dx_R(t)}{dt} = \begin{cases} \mu\delta u(t) & \text{for } (i-1)t_u \leqslant t \leqslant (i-1)t_u + t'_{ui}, \\ 0 & \text{for } (i-1)t_u + t'_{ui} \leqslant t \leqslant it_u, \\ \mu\delta u(t) - p(t) & \text{for } (j-1)t_u \leqslant t \leqslant (j-1)t_u + t'_{uj}, \\ 0 & \text{for } (j-1)t_u + t'_{uj} \leqslant t \leqslant jt_u, \end{cases} \tag{13}$$

with boundary conditions $x_R(0) = 0$, $x_R(nt_u) \geqslant 0$, $x_R((i-1)t_u + t'_{ui}) = x_R(it_u)$, $x_R((j-1)t_u + t'_{uj}) = x_R(jt_u)$ for $i = 1, 2, \ldots, m$ and $j = m+1, m+2, \ldots, n$.

Solving the above differential equations we get,

$$x_R(t) = \begin{cases} x_R((i-1)t_u) + \mu\delta[u_0(t-(i-1)t_u) & \\ +\frac{u_1(t^2-(i-1)^2 t_u^2)}{2} + \frac{u_2(t^3-(i-1)^3 t_u^3)}{3}] & \text{for } (i-1)t_u \leqslant t \leqslant (i-1)t_u + t'_{ui}, \\ x_R((i-1)t_u + t'_{ui}) & \text{for } (i-1)t_u + t'_{ui} \leqslant t \leqslant it_u \\ x_R((j-1)t_u) - p_0(t-(j-1)t_u) & \\ -\frac{p_1(t^2-(j-1)^2 t_u^2)}{2} + \mu\delta[u_0(t-(j-1)t_u) & \\ +\frac{u_1(t^2-(j-1)^2 t_u^2)}{2} + \frac{u_2(t^3-(j-1)^3 t_u^3)}{3}] & \text{for } (j-1)t_u \leqslant t \leqslant (j-1)t_u + t'_{uj}, \\ x_R((j-1)t_u + t'_{uj}) & \text{for } (j-1)t_u + t'_{uj} \leqslant t \leqslant jt_u. \end{cases} \tag{14}$$

Therefore the total profit from the planning horizon is given by

$$\begin{aligned}\hat{\tilde{J}}(u_0,u_1,u_2,p_0,p_1) &= \hat{\tilde{c}}_s\int_0^T e^{-Rt}D(t)dt - \hat{\tilde{c}}_{h1}\int_0^T e^{-Rt}x(t)dt - \hat{\tilde{c}}_{h2}\int_0^T e^{-Rt}x_R(t)dt - \hat{\tilde{c}}_0\int_0^T e^{-Rt}u(t)dt - \hat{\tilde{c}}_1\int_0^T e^{-Rt}dt \\ &\quad - \hat{\tilde{c}}_2\int_0^T e^{-Rt}\{u(t)\}^2dt - \hat{\tilde{c}}_{r1}\int_0^T e^{-Rt}dt - \hat{\tilde{c}}_{r2}\int_0^T e^{-Rt}\{p(t)\}^2dt - \hat{\tilde{c}}_z\delta(1-\mu)\int_0^T e^{-Rt}u(t)dt \\ &\quad - \hat{\tilde{c}}_z x_R(nt_u)\int_0^T e^{-Rt}dt - n\hat{\tilde{s}}_u\int_0^T e^{-Rt}dt \\ &= \hat{\tilde{c}}_sA - \hat{\tilde{c}}_{h1}B_1 - \hat{\tilde{c}}_{h2}B_2 - \hat{\tilde{c}}_0P_1 - \hat{\tilde{c}}_1P_2 - \hat{\tilde{c}}_2P_3 - \hat{\tilde{c}}_{r1}R_1 - \hat{\tilde{c}}_{r2}R_2 - \hat{\tilde{c}}_zD_1 - \hat{\tilde{s}}_uS = (\widehat{J}-j_l,\widehat{J},\widehat{J}+j_r),\end{aligned} \tag{15}$$

where $\widehat{J} = ([j_1,j_2][j_3,j_4])$ and $A,\ B_1,\ B_2,\ P_1,P_2,\ P_3,\ R_1,\ R_2,\ D_1,\ S$ are given in Appendix (cf. Eq. (30) to (39)).

Let $\hat{\tilde{c}}_s = (\hat{c}_s - c_{sl},\ \hat{c}_s, \hat{c}_s + c_{sr}),\ \hat{c}_s = ([c_{s1},c_{s2}][c_{s3},c_{s4}]),\ \hat{\tilde{c}}_{h1} = (\hat{c}_{h1} - c_{h1l}, \hat{c}_{h1}, \hat{c}_{h1} + c_{h1r}),\ \hat{c}_{h1} = ([c_{h11},c_{h12}][c_{h13},c_{h14}]),\ \hat{\tilde{c}}_{h2} = (\hat{c}_{h2} - c_{h2l}, \hat{c}_{h2}, \hat{c}_{h2} + c_{h2r}),\ \hat{c}_{h2} = ([c_{h21},c_{h22}][c_{h23},c_{h24}]),\ \hat{\tilde{c}}_0 = (\hat{c}_0 - c_{0l}, \hat{c}_0, \hat{c}_0 + c_{0r}),\ \hat{c}_0 = ([c_{01},c_{02}][c_{03},c_{04}]),\ \hat{\tilde{c}}_1 = (\hat{c}_1 - c_{1l}, \hat{c}_1, \hat{c}_1 + c_{1r}),\ \hat{c}_1 = ([c_{11},c_{12}][c_{13},c_{14}]),\ \hat{\tilde{c}}_2 = (\hat{c}_2 - c_{2l}, \hat{c}_2, \hat{c}_2 + c_{2r}),\ \hat{c}_2 = ([c_{21},c_{22}][c_{23},c_{24}]),\ \hat{\tilde{c}}_{r1} = (\hat{c}_{r1} - c_{r1l}, \hat{c}_{r1}, \hat{c}_{r1} + c_{r1r}),\ \hat{c}_{r1} = ([c_{r11},c_{r12}][c_{r13},c_{r14}]),\ \hat{\tilde{c}}_{r2} = (\hat{c}_{r2} - c_{r2l}, \hat{c}_{r2}, \hat{c}_{r2} + c_{r2r}),\ \hat{c}_{r2} = ([c_{r21},c_{r22}][c_{r23},c_{r24}]),\ \hat{\tilde{c}}_z = (\hat{c}_z - c_{zl}, \hat{c}_z, \hat{c}_z + c_{zr}), \hat{c}_z = ([c_{z1},c_{z2}][c_{z3},c_{z4}])$ and $\hat{\tilde{s}}_u = (\hat{s}_u - s_{ul}, \hat{s}_u, \hat{s}_u + s_{ur}),\ \hat{s}_u = ([s_{u1},s_{u2}][s_{u3},s_{u4}])$. therefore

$$j_1 = c_{s1}A - c_{h12}B_1 - c_{h22}B_2 - c_{02}P_1 - c_{12}P_2 - c_{22}P_3 - c_{r12}R_1 - c_{r22}R_2 - c_{z2}D_1 - s_{u2}S, \tag{16}$$

$$j_2 = c_{s2}A - c_{h11}B_1 - c_{h21}B_2 - c_{01}P_1 - c_{11}P_2 - c_{21}P_3 - c_{r11}R_1 - c_{r21}R_2 - c_{z1}D_1 - s_{u1}S, \tag{17}$$

$$j_3 = c_{s3}A - c_{h14}B_1 - c_{h24}B_2 - c_{04}P_1 - c_{14}P_2 - c_{24}P_3 - c_{r14}R_1 - c_{r24}R_2 - c_{z4}D_1 - s_{u4}S, \tag{18}$$

$$j_4 = c_{s4}A - c_{h13}B_1 - c_{h23}B_2 - c_{03}P_1 - c_{13}P_2 - c_{23}P_3 - c_{r13}R_1 - c_{r23}R_2 - c_{z3}D_1 - s_{u3}S, \tag{19}$$

$$j_l = c_{sl}A - c_{h1r}B_1 - c_{h2r}B_2 - c_{0r}P_1 - c_{1r}P_2 - c_{2r}P_3 - c_{r1r}R_1 - c_{r2r}R_2 - c_{zr}D_1 - s_{ur}S \tag{20}$$

$$\text{and } j_r = c_{sr}A - c_{h1l}B_1 - c_{h2l}B_2 - c_{0l}P_1 - c_{1l}P_2 - c_{2l}P_3 - c_{r1l}R_1 - c_{r2l}R_2 - c_{zl}D_1 - s_{ul}S. \tag{21}$$

So the problem reduces to

$$\text{Maximize}\,\hat{\tilde{J}}(u_0,u_1,u_2,p_0,p_1). \tag{22}$$

4.2. Equivalent deterministic representation of the proposed model

According to the Section 2.8. the expected value of $\hat{\tilde{J}}(u_0,u_1,u_2,p_0,p_1)$ is given by

$$E[\hat{\tilde{J}}(u_0,u_1,u_2,p_0,p_1)] = E_j = \frac{1}{4}[j_1+j_2+j_3+j_4] + \frac{\rho j_r - (1-\rho)j_l}{2}. \tag{23}$$

Then according to Section 2.9. the above problem (22) reduces to following single objective optimization problem:

$$\text{Maximize}\,E[\hat{\tilde{J}}(u_0,u_1,u_2,p_0,p_1)]. \tag{24}$$

Now the problem stated in (24) is solved using a gradient based optimization technique-Generalized Reduced Gradient method (LINGO 9.0 software).

5. Numerical illustration

5.1. Input data

To illustrate the above production-repairing model numerically, following input data are considered:

$d_0 = 18,\ d_1 = 0.2,\ T = 25,\ \delta = 0.1,\ R = 0.0757,\ \mu = 0.85,\ \hat{\tilde{c}}_s = (\sigma - 0.8, \sigma, \sigma + 0.5),\ \sigma = ([7,7.3][6.8,7.5]),\ \hat{\tilde{c}}_{h1} = (\zeta_1 - 0.02, \zeta_1, \zeta_1 + 0.03),\ \zeta_1 = ([0.18,0.2][0.15,0.22]),\ \hat{\tilde{c}}_{h2} = (\zeta_2 - 0.01, \zeta_2, \zeta_2 + 0.02),\ \zeta_2 = ([0.13,0.16][0.11,0.18]),\ \hat{\tilde{c}}_0 = (\alpha_0 - 0.2, \alpha_0, \alpha_0 + 0.3),\ \alpha_0 = ([1.8,2.1][1.5,2.2]),\ \hat{\tilde{c}}_1 = (\alpha_1 - 0.05, \alpha_1, \alpha_1 + 0.05),\ \alpha_1 = ([0.8,1.2][0.5,1.3]),\ \hat{\tilde{c}}_2 = (\alpha_2 - 0.003, \alpha_2, \alpha_2 + 0.005),\ \alpha_2 = ([0.04,0.08][0.03,0.1]),\ \hat{\tilde{c}}_{r1} = (\beta_1 - 0.05, \beta_1, \beta_1 + 0.05),\ \beta_1 = ([0.8,1.0][0.65,1.15]),\ \hat{\tilde{c}}_{r2} = (\beta_2 - 0.005, \beta_2, \beta_2 + 0.007),\ \beta_2 = ([0.03,0.05][0.025,0.053]),\ \hat{\tilde{c}}_z = (\gamma - 0.03, \gamma, \gamma + 0.04),\ \gamma = ([0.08,0.12][0.05,0.16]),\ \hat{\tilde{s}}_u = (\gamma_1 - 0.5, \gamma_1, \gamma_1 + 0.5),\ \gamma_1 = ([4.5,5][4,5.5])$.

5.2. Results and discussion

For these input data optimal values of $u_0,\ u_1,\ u_2, p_0,\ p_1,\ E_j$ and production time duration t'_{ui} of each cycle i for different m and n are obtained for $\rho = 0.5$ and presented in Table 1.

From Table 1 it is found that for the assumed parametric values that one production cycle with out repairing and two production cycles with repairing gives maximum expected profit. It is also observed that as number of cycles increases initially profit increases and reaches a maximum value and then again it decreases. It happens because as the number of cycles increases expected holding cost decreases but expected set up cost increases. But initially gain due to decrease of expected holding cost dominates loss due to expected set up cost, as a result total expected profit increases with n. But after some cycles ($n = 3$) loss due to increase of expected set up cost dominates over the gain due to decrease of expected holding cost and as a result profit decreases with n for $n > 3$. It is interesting to note from Table 1 that for any value of $n, m = 1$ gives maximum expected profit, i.e., profit will be maximum if repair process starts from second cycle. It happens because holding cost of repairable units increases if repair starts after few cycles, which decreases expected profit. This observation agrees with reality. So for the assumed parametric values, $m = 1$ and $n = 3$ gives maximum expected profit. Due to this reason, further numerical studies are made for $m = 1$ and $n = 3$.

A parametric study is made with different optimism parameter (ρ) and the optimum results are presented in Table 2. As per expectation, it is found that expected profit is increased with increase of ρ. All these observations agree with reality.

A parametric study is made with different resultant effect of inflation and time value of money (R) and the optimum results are presented in Table 3. Results are also obtained ignoring the inflationary effect and results are presented in Table 3. As per expectation, it is found that expected profit is maximum when there is no inflationary effect. It is also observed that expected profit decreases with increase of R. All these observations agree with reality.

Another parametric investigation is made for the different values of shape parameter of the demand function i.e. d_1. These results are shown in the Table 4. It is found that expected profit increases with d_1. It happens because increase of d_1, increases demand of the item as a whole, which forces to increase production rate, production duration and repairing rate and ultimately fetches more expected profit.

Due to the assumptions made in 3.1., the serviceable stock, repairable stock (i.e. accumulated defective stock for repair), demand, production and repairing rates are dynamic, i.e. function of t implicitly or explicitly. These values vary with t within the fixed time horizon, $T = 25$ units. Values of the parameters for different t's are given in Table 5 for $R = 0.0757$ and $d_1 = 0.2$. Pictorial representation of the optimal stocks, demand, production and repairing rates are presented in Fig. 3. From Table 5 as well as from Fig. 3 it is found that the nature of behavior of production rate, serviceable stock, repairable stock, repairing rate and demand are as per expectation.

6. Particular cases

In this section, the results of several particular production repairing inventory models with different demand, production and repairing rates and costs in fuzzy-rough, fuzzy and rough environments are presented.

6.1. With different production and repairing rates

In model formulation (cf. Section 4), production and repairing rates are considered as $u(t) = u_0 + u_1 t + u_2 t^2$, $p(t) = p_0 + p_1 t$. As particular cases,following six subcases are considered depending upon the different functional form of production and repairing rates

Subcase-S1 : $u(t) = u_0 + u_1 t + u_2 t^2$, $p(t) = p_0 + p_1 t$ (General Model).
Subcase-S2 : $u(t) = u_0 + u_1 t + u_2 t^2,\ p(t) = p_0$.
Subcase-S3 : $u(t) = u_0 + u_1 t,\ p(t) = p_0 + p_1 t$.
Subcase-S4 : $u(t) = u_0 + u_1 t,\ p(t) = p_0$.
Subcase-S5 : $u(t) = u_0,\ p(t) = p_0 + p_1 t$.
Subcase-S6 : $u(t) = u_0,\ p(t) = p_0$.

Table 1
Optimum results of fuzzy rough model.

m	$n-m$	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}	t'_{u4}	t'_{u5}
1	1	18.000	1.332	−0.0344	2.337	0.150	678.03	11.33	9.83	–	–	–
1	2	23.714	1.164	−0.0330	1.679	0.164	684.30	7.02	6.02	5.80	–	–
2	1	21.207	1.955	−0.0768	3.932	0.212	652.74	7.02	6.37	5.88	–	–
1	3	28.863	0.587	−0.0094	1.394	0.182	682.54	4.85	4.30	4.04	3.86	–
2	2	27.123	1.224	−0.0457	2.655	0.193	658.27	4.89	4.49	3.96	4.04	–
3	1	26.157	1.497	−0.0625	5.576	0.279	630.37	4.91	4.48	4.47	3.86	–
1	4	32.317	−0.485	0.0578	1.275	0.208	666.54	3.68	3.43	3.17	2.82	2.44
2	3	30.528	0.860	−0.0304	2.111	0.192	650.16	3.64	3.42	3.03	2.97	3.00
3	2	29.607	1.186	−0.0497	3.639	0.228	629.94	3.67	3.39	3.31	2.90	3.02
4	1	29.999	1.003	−0.0431	7.138	0.342	603.95	3.66	3.43	3.37	3.45	2.71

Please cite this article in press as: M. Mondal et al., A production-repairing inventory model with fuzzy rough coefficients under inflation and time value of money, Appl. Math. Modell. (2012), http://dx.doi.org/10.1016/j.apm.2012.07.024

Table 2
Optimal results of fuzzy rough model due to different optimism parameter ρ.

ρ	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}
0.0	23.807	1.139	−0.0321	1.676	0.163	659.27	7.02	6.02	5.81
0.1	23.788	1.144	−0.0323	1.677	0.163	664.27	7.02	6.02	5.80
0.2	23.769	1.149	−0.0325	1.677	0.163	669.28	7.02	6.02	5.80
0.3	23.751	1.154	−0.0326	1.678	0.163	674.28	7.02	6.02	5.80
0.4	23.732	1.159	−0.0328	1.678	0.164	679.29	7.02	6.02	5.80
0.5	23.714	1.164	−0.0330	1.679	0.164	684.30	7.02	6.02	5.80
0.6	23.696	1.169	−0.0331	1.679	0.164	689.30	7.02	6.01	5.80
0.7	23.678	1.174	−0.0332	1.679	0.164	694.31	7.02	6.01	5.80
0.8	23.660	1.178	−0.0334	1.680	0.164	699.31	7.02	6.01	5.80
0.9	23.643	1.183	−0.0336	1.680	0.164	704.32	7.02	6.01	5.79
1.0	23.626	1.188	−0.0338	1.681	0.164	709.33	7.02	6.01	5.79

Table 3
Optimum results of fuzzy rough model due to different R (resultant effect of inflation and discount rate).

R	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}
0.00	36.726	0.285	−0.0081	2.042	0.201	1676.98	5.94	5.43	5.28
0.05	28.239	0.778	−0.0203	1.796	0.176	948.64	6.64	5.83	5.61
0.08	22.966	0.232	−0.0353	1.660	0.162	650.84	7.08	6.05	5.83
0.11	18.283	1.682	−0.0505	1.532	0.149	476.15	7.52	6.25	6.07
0.14	18.066	1.101	−0.0237	1.437	0.139	369.57	7.82	6.59	6.17
0.17	18.000	0.562	−0.0030	1.369	0.132	301.16	8.14	6.90	6.18
0.20	18.000	0.626	−0.0055	1.311	0.126	254.22	8.15	6.98	6.42

Table 4
Optimum results of fuzzy rough model due to different d_1.

d_1	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}
0.15	22.061	1.349	−0.0407	1.650	0.162	669.51	7.01	5.78	5.57
0.175	22.989	1.250	−0.0366	1.667	0.163	677.77	7.01	5.90	5.68
0.20	23.714	1.164	−0.0330	1.679	0.164	684.30	7.02	6.02	5.80
0.225	24.315	1.088	−0.0297	1.687	0.164	689.70	7.04	6.13	5.91
0.25	24.837	1.020	−0.0268	1.694	0.164	694.37	7.09	6.25	6.03

Table 5
Optimum values of $x(t), D(t), u(t), x_R(t)$ and $p(t)$ for fuzzy rough model due to different t.

t	0	3.5	7.02	7.75	12.0	14.35	15.5	20.0	22.47	23.5	25
$x(t)$	0	13.20	27.12	11.14	37.34	53.06	23.65	41.96	59.39	31.49	0
$D(t)$	18	20.64	23.42	20.23	25.47	28.61	22.73	26.39	29.39	24.30	18
$u(t)$	23.71	27.38	30.26	–	32.94	33.63	–	33.81	33.23	–	–
$x_R(t)$	0	7.62	16.26	16.26	14.01	11.65	11.65	5.67	0	–	–
$p(t)$	–	–	–	–	3.64	4.03	–	4.95	5.35	–	–

For the assumed parametric values, results of all the subcases due to $m = 1,\ n = 3$, are obtained and presented in Table 6. It is observed that general quadratic production and linear repairing rates draws maximum expected profit i.e., general model gives maximum expected profit compared to other subcases. In fact in subcase-S1, production and repairing rate functions are more general compared to other subcases. As production and repairing rates are control variables in general case, these rates are more effective compared to others in maximizing profit. Actually more appropriate production rate decreases holding cost as well as production and repairing cost which intern increases profit.

6.2. With constant demand:

The production-repairing model with quadratic dynamic production rate, dynamic linear repairing rate and constant demand ($D(t) = d_0$) is formulated following earlier process. In this case also different subcases are considered depending upon the production and repairing rates, which are as follows-.

Subcase-S7: $u(t) = u_0 + u_1 t + u_2 t^2,\ p(t) = p_0 + p_1 t.$

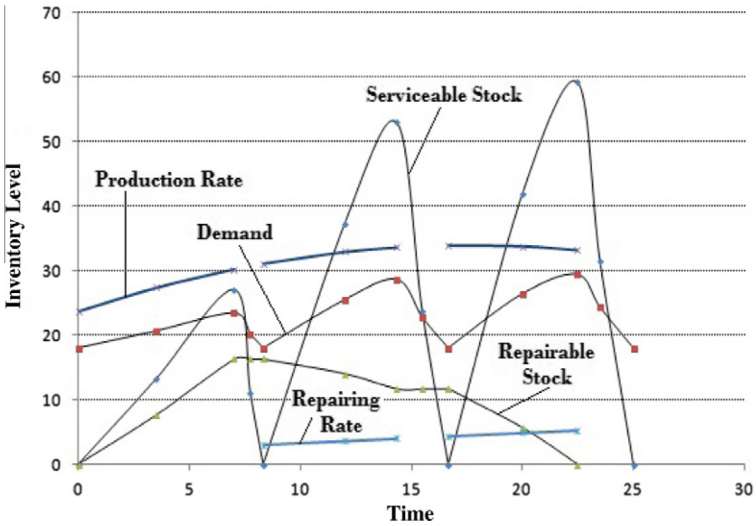

Fig. 3. Optimal production, repairing, demand, serviceable and repairable stocks.

Table 6
Optimal results of fuzzy rough model for different production and repairing rates.

Subcases	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}
S1	23.714	1.164	−0.0330	1.679	0.164	684.30	7.02	6.02	5.80
S2	23.027	1.154	−0.0316	4.122	–	672.90	7.10	6.02	5.92
S3	25.754	0.491	–	1.700	0.165	683.45	7.03	6.12	5.64
S4	25.018	0.504	–	4.150	–	669.80	7.12	6.13	5.75
S5	29.807	–	–	1.558	0.150	678.26	6.78	6.35	6.20
S6	29.211	–	–	3.82	–	666.45	6.86	6.36	6.36

Subcase-S8: $u(t) = u_0 + u_1 t + u_2 t^2$, $p(t) = p_0$.
Subcase-S9: $u(t) = u_0 + u_1 t,\ p(t) = p_0 + p_1 t$.
Subcase-S10: $u(t) = u_0 + u_1 t,\ p(t) = p_0$.
Subcase-S11: $u(t) = u_0,\ p(t) = p_0 + p_1 t$.
Subcase-S12: $u(t) = u_0,\ p(t) = p_0$.

The numerical results of these models are presented in Table 7. From this table it is found that as usual, optimum profit decreases gradually with quadratic, linear and constant production forms and also with linear and constant repairing forms. Moreover, from Tables 6,7, it can be concluded that for all production forms, stock dependent demand draws much more expected profit than the constant demand.

6.3. General model with cost and coefficients as fuzzy parameters:

In this case total profit $\tilde{J}$ from the planning horizon is fuzzy in nature and is given by

$$\tilde{J}(u_0, u_1, u_2, p_0, p_1) = \tilde{c_s}A - \tilde{c_{h1}}B_1 - \tilde{c_{h2}}B_2 - \tilde{c_0}P_1 - \tilde{c_1}P_2 - \tilde{c_2}P_3 - \tilde{c_{r1}}R_1 - \tilde{c_{r2}}R_2 - \tilde{c_z}D_1 - \tilde{s_u}S = (j_{f1}, j_{f2}, j_{f3}), \tag{25}$$

where $A,\ B_1,\ B_2,\ P_1,\ P_2,\ P_3,\ R_1,\ R_2,\ D_1$ are S are given in Appendix (cf. Eq. (30) to (39)).

Let $\tilde{c_s} = (c_{fs1}, c_{fs2}, c_{fs3})$, $\tilde{c_{h1}} = (c_{fh11}, c_{fh12}, c_{fh13})$, $\tilde{c_{h2}} = (c_{fh21}, c_{fh22}, c_{fh23})$, $\tilde{c_0} = (c_{f01}, c_{f02}, c_{f03})$, $\tilde{c_1} = (c_{f11}, c_{f12}, c_{f13})$, $\tilde{c_2} = (c_{f21}, c_{f22}, c_{f23})$, $\tilde{c_{r1}} = (c_{fr11}, c_{fr12}, c_{fr13})$, $\tilde{c_{r2}} = (c_{fr21}, c_{fr22}, c_{fr23})$, $\tilde{c_z} = (c_{fz1}, c_{fz2}, c_{fz3})$ and $\tilde{s_u} = (s_{fu1}, s_{fu2}, s_{fu3})$. therefore

$$j_{f1} = c_{fs1}A - c_{fh13}B_1 - c_{fh23}B_2 - c_{f03}P_1 - c_{f13}P_2 - c_{f23}P_3 - c_{fr13}R_1 - c_{fr23}R_2 - c_{fz3}D_1 - s_{fu3}S, \tag{26}$$

$$j_{f2} = c_{fs2}A - c_{fh12}B_1 - c_{fh22}B_2 - c_{f02}P_1 - c_{f12}P_2 - c_{f22}P_3 - c_{fr12}R_1 - c_{fr22}R_2 - c_{fz2}D_1 - s_{fu2}S, \tag{27}$$

$$j_{f3} = c_{fs3}A - c_{fh11}B_1 - c_{fh21}B_2 - c_{f01}P_1 - c_{f11}P_2 - c_{f21}P_3 - c_{fr11}R_1 - c_{fr21}R_2 - c_{fz1}D_1 - s_{fu1}S. \tag{28}$$

Please cite this article in press as: M. Mondal et al., A production-repairing inventory model with fuzzy rough coefficients under inflation and time value of money, Appl. Math. Modell. (2012), http://dx.doi.org/10.1016/j.apm.2012.07.024

Table 7
Optimal results of fuzzy rough model for different production and repairing rates with constant demand.

Subcases	u_0	u_1	u_2	p_0	p_1	E_j	t'_{u1}	t'_{u2}	t'_{u3}
S7	21.437	−0.393	0.0085	1.000	0.0803	552.52	8.33	8.27	8.33
S8	21.278	−0.349	0.0076	2.376	–	552.03	8.33	8.01	8.33
S9	20.35	−0.103	–	1.232	0.0900	548.90	8.33	7.39	7.14
S10	20.37	−0.121	–	2.676	–	548.55	8.33	7.27	7.23
S11	20.00	–	–	1.211	0.0800	546.71	8.33	7.38	7.13
S12	20.00	–	–	2.673	–	546.55	8.33	7.25	7.25

Table 8
Optimum results of fuzzy production inventory model.

m	$n-m$	u_0	u_1	u_2	p_0	p_1	$E[\widetilde{J}]$	t'_{u1}	t'_{u2}	t'_{u3}	t'_{u4}	t'_{u5}
1	1	18.508	1.396	−0.0377	2.390	0.154	717.49	11.21	9.75	–	–	–
1	2	24.602	1.111	−0.0312	1.710	0.167	726.61	6.93	5.96	5.74	–	–
2	1	22.056	1.925	−0.0764	4.002	0.216	694.28	6.94	6.31	5.83	–	–
1	3	29.579	0.545	−0.0078	1.414	0.185	725.69	4.79	4.26	4.01	3.82	–
2	2	27.776	1.213	−0.0459	2.692	0.195	701.01	4.82	4.44	3.92	4.01	–
3	1	26.859	1.471	−0.0618	5.656	0.283	672.68	4.85	4.43	4.43	3.82	–
1	4	32.884	−0.526	0.0602	1.292	0.211	709.40	3.64	3.40	3.15	2.79	2.41
2	3	31.077	0.852	−0.0305	2.137	0.195	692.97	3.60	3.39	3.00	2.95	2.97
3	2	30.184	1.170	−0.0493	3.684	0.231	672.53	3.62	3.36	3.28	2.87	2.99
4	1	30.569	0.988	−0.0427	7.232	0.346	646.25	3.62	3.40	3.33	3.42	2.68

So, using Lemma 1 expected profit of $\widetilde{J}$, $E[\widetilde{J}]$ is given by

$$E[\widetilde{J}] = (1-\rho)j_{f1} + j_{f2} + \rho j_{f3}.$$

To illustrate this production-repairing model numerically, following input data are considered:

$d_0 = 18$, $d_1 = 0.2$, $T = 25$, $\delta = 0.1$, $\mu = 0.85$, $R = 0.0757$, $\tilde{c_s} = (6,7,8)$, $\tilde{c_{h1}} = (0.13, 0.2, 0.25)$, $\tilde{c_{h2}} = (0.1, 0.15, 0.2)$, $\tilde{c_0} = (1.3, 2.0, 2.5)$, $\tilde{c_1} = (0.5, 1.0, 1.3)$, $\tilde{c_2} = (0.027, 0.075, 0.105)$, $\tilde{c_{r1}} = (0.6, 1, 1.20)$, $\tilde{c_{r2}} = (0.025, 0.04, 0.055)$, $\tilde{c_z} = (0.05, 0.09, 0.16)$, $\tilde{s_u} = (4,5,6)$.

For these input data optimal values of u_0, u_1, u_2, p_0, p_1, $E[\widetilde{J}]$ and production time duration t'_{ui} of each cycle i for different m and n are obtained for $\rho = 0.7$ and presented in Table 8.

From Table 8 it is found that for the assumed parametric values, one production cycle with out repairing and two production cycles with repairing gives maximum expected profit from the planning horizon. It is also observed that as number of cycles increases initially profit increases and reaches a maximum value (for $n = 3$) and then again it decreases. This observation agrees with the results obtained in general model (cf. Table 1). As per expectation, it is also noticed that from Table 8, for any value of n, $m = 1$ gives maximum expected profit, i.e., profit will be maximum if repair process starts from second cycle, because holding cost of repairable units increases if repair starts after few cycles, which decreases expected profit. Due to this reason for further study of this model it is assumed that $m = 1$ and $n = 3$.

A parametric study is made on the optimism parameter (ρ) for $m = 1$ and $n = 3$ and the optimum results are presented in Table 9. As per expectation, it is found that the expected profit increases with increase in degree of optimism ρ, which agrees with reality.

6.3.1. Model with "about d% of mid values" of fuzzy parameters:

Here, a parametric study with the fuzzy costs and coefficients taken as "about d% of the mid-values" of the fuzzy numbers where $d = 5, 10, 20, 30, 40, 50$ are made. These results are presented in Table 10. Here $m = 1$, $n = 3$, $d_0 = 18$, $d_1 = 0.2$, $T = 25$, $\delta = 0.1$, $\mu = 0.85$, $R = 0.0757$, the mid values of different costs and coefficients such as for $c_s = 7.0$, $c_{h1} = 0.2$, $c_{h2} = 0.15$, $c_0 = 2.0$, $c_1 = 1.0$, $c_2 = 0.075$, $c_{r1} = 1.0$, $c_{r2} = 0.04$, $c_z = 0.09$, $s_u = 5.0$ are considered.

From Lemma 1 it is clear that ρ is degree of optimism for expected profit $E[\widetilde{J} = (j_{f1}, j_{f2}, j_{f3})] = (1-\rho)j_{f1} + j_{f2} + \rho j_{f3}$. $\rho = 0$ means least possible profit (j_{f1}) maximized and $\rho = 1$ means most optimistic profit (j_{f3}), which is least feasible, is maximized. Increase of d increase j_{f3}, decrease j_{f1} but j_{f2} remains unchanged. Again amount of increase of j_{f1} = amount of decrease of j_{f3}. As a result $E[\widetilde{J}]$ remains unchanged for $\rho = 0.5$. i.e. for $\rho = 0.5$, there is no effect on optimal expected profit. But for pessimistic DM $(\rho < 0.5)$ if fuzziness increases i.e. d increases expected profit decreases and for optimistic DM $(\rho > 0.5)$ expected profit increases with increase of fuzziness. But for pessimistic DMs' much risk involves as they give weightage on least feasible profit (j_{f3}). So increase of fuzziness has a very bad impact on inventory control problems.

Table 9
Optimal results of fuzzy model due to different optimism parameter ρ.

ρ	u_0	u_1	u_2	p_0	p_1	$E[\widetilde{J}]$	t'_{u1}	t'_{u2}	t'_{u3}
0.0	18.117	0.592	−0.0195	1.128	0.106	249.08	8.22	7.42	7.36
0.1	18.000	0.605	−0.0160	1.177	0.111	308.66	8.21	7.31	7.09
0.2	18.000	0.573	−0.0101	1.231	0.116	369.28	8.20	7.20	6.81
0.3	18.000	0.807	−0.0179	1.303	0.124	431.73	8.04	6.96	6.60
0.4	18.000	1.157	−0.0315	1.387	0.133	497.87	7.81	6.67	6.42
0.5	19.602	1.229	−0.0348	1.484	0.143	568.36	7.54	6.42	6.20
0.6	21.992	1.175	−0.0332	1.592	0.154	644.43	7.23	6.19	5.97
0.7	24.602	1.111	−0.0312	1.710	0.167	726.61	6.93	5.96	5.74
0.8	27.464	1.035	−0.0285	1.840	0.181	815.47	6.63	5.73	5.51
0.9	30.619	0.946	−0.0251	1.984	0.196	911.68	6.32	5.50	5.27
1.0	34.118	0.842	−0.0209	2.145	0.214	1016.05	6.02	5.26	5.03

Table 10
Optimal results of fuzzy model due to different d.

ρ	$d\%$	$E[\widetilde{J}]$	$d\%$	$E[\widetilde{J}]$	$d\%$	$E[\widetilde{J}]$	$d\%$	$E[\widetilde{J}]$	$d\%$	$E[\widetilde{J}]$	$d\%$	$E[\widetilde{J}]$
0.0		454.03		385.66		254.50		127.34		4.59		−117.22
0.1		468.07		412.65		306.51		203.08		102.48		4.59
0.2		482.22		440.12		359.10		280.43		203.08		127.34
0.3		496.49		468.06		412.65		359.10		306.51		254.50
0.4		510.88		496.49		468.06		440.12		412.65		385.66
0.5	5	525.39	10	525.39	20	525.39	30	525.39	40	525.39	50	525.39
0.6		540.03		554.83		584.90		615.61		646.95		678.93
0.7		554.83		584.90		646.95		711.55		778.72		848.48
0.8		569.79		615.61		711.55		813.28		920.86		1034.40
0.9		584.90		646.95		778.72		920.86		1073.60		1237.22
1.0		600.18		678.93		848.48		1034.40		1237.22		1457.62

6.4. General model with cost and coefficients as rough parameters:

In this case total profit $\widehat{J}$ from the planning horizon is rough in nature and is given by

$$\widehat{J}(u_0,u_1,u_2,p_0,p_1) = \hat{c_s}A - \hat{c_{h1}}B_1 - \hat{c_{h2}}B_2 - \hat{c_0}P_1 - \hat{c_1}P_2 - \hat{c_2}P_3 - \hat{c_{r1}}R_1 - \hat{c_{r2}}R_2 - \hat{c_z}D_1 - \hat{s_u}S, \quad (29)$$

where $A,\ B_1,\ B_2,\ P_1,\ P_2,\ P_3,\ R_1,\ R_2,\ D_1$ and S are given in Appendix (cf. Eq. (30) to (39)).

Let $\hat{c_s} = ([c_{s1},c_{s2}][c_{s3},c_{s4}])$, $\hat{c_{h1}} = ([c_{h11},c_{h12}][c_{h13},c_{h14}])$, $\hat{c_{h2}} = ([c_{h21},c_{h22}][c_{h23},c_{h24}])$, $\hat{c_0} = ([c_{01},c_{02}][c_{03},c_{04}])$, $\hat{c_1} = ([c_{11},c_{12}][c_{13},c_{14}])$, $\hat{c_2} = ([c_{21},c_{22}][c_{23},c_{24}])$, $\hat{c_{r1}} = ([c_{r11},c_{r12}][c_{r13},c_{r14}])$, $\hat{c_{r2}} = ([c_{r21},c_{r22}][c_{r23},c_{r24}])$, $\hat{c_z} = ([c_{z1},c_{z2}][c_{z3},c_{z4}])$ and $\hat{s_u} = ([s_{u1},s_{u2}][s_{u3},s_{u4}])$.

So, using Lemma 2 expected profit of $\widehat{J},\ E[\widehat{J}]$ is given by

$$E[\widehat{J}] = \frac{1}{4}[j_1 + j_2 + j_3 + j_4],$$

where $j_1,\ j_2,\ j_3$ and j_4 are given by Eq. (16) to (19).

To illustrate the above production-repairing model numerically, following input data are considered:

$d_0 = 18,\ d_1 = 0.2,\ T = 25,\ \delta = 0.1,\ \mu = 0.85,\ R = 0.0757,\ \hat{c_s} = ([7,7.3][6.8,7.5]),\ \hat{c_{h1}} = ([0.18,0.2][0.15,0.22]),\ \hat{c_{h2}} = ([0.13,0.16][0.11,0.18]),\ \hat{c_0} = ([1.8,2.1][1.5,2.2]),\ \hat{c_1} = ([0.8,1.2][0.5,1.3]),\ \hat{c_2} = ([0.04,0.08][0.03,0.1]),\ \hat{c_{r1}} = ([0.8,1.0][0.65,1.15]),\ \hat{c_{r2}} = ([0.03,0.05][0.025,0.053]),\ \hat{c_z} = ([0.08,0.12][0.05,0.16]),\ \hat{s_u} = ([4.5,5][4,5.5])$.

For these input data the optimal values of $u_0,\ u_1,\ u_2,\ p_0,\ p_1,\ E[\widehat{J}]$ and production time duration of each cycle are found and these optimal values are presented in Table 11.

From this subcase also, it is observed that one production cycle with out repairing and two production cycles with repairing gives maximum expected profit from the planning horizon (cf. Table 11).

7. Conclusion

In this paper, a production-repairing decision-making model in fuzzy rough environment is considered. Till now, no production inventory model has been formulated in such an environment. Here the cost and other parameters are assumed to be fuzzy rough variables. Production and repairing rates are dynamic control variables, i.e. function of t, where coefficients are decision variable. From the results obtained in numerical illustration section, following conclusions can be drawn

Table 11
Optimum results of rough production inventory model.

m	$n-m$	u_0	u_1	u_2	p_0	p_1	$E[\widehat{J}]$	t'_{u1}	t'_{u2}	t'_{u3}	t'_{u4}	t'_{u5}
1	1	18.000	1.333	−0.0344	2.338	0.150	685.87	11.33	9.83	–	–	–
1	2	23.712	1.168	−0.0331	1.680	0.164	692.88	7.01	6.01	5.80	–	–
2	1	21.202	1.959	−0.0770	3.935	0.212	660.93	7.02	6.36	5.88	–	–
1	3	28.876	0.589	−0.0094	1.395	0.183	691.77	4.85	4.30	4.04	3.86	–
2	2	27.137	1.225	−0.0458	2.657	0.193	667.18	4.88	4.48	3.96	4.04	–
3	1	26.169	1.499	−0.0626	5.580	0.279	638.94	4.91	4.47	4.47	3.86	–
1	4	32.339	−0.484	0.0578	1.276	0.209	676.24	3.68	3.43	3.17	2.82	2.44
2	3	30.549	0.861	−0.0305	2.112	0.192	659.63	3.64	3.42	3.03	2.97	3.00
3	2	29.630	1.187	−0.0497	3.641	0.228	639.15	3.66	3.39	3.31	2.89	3.02
4	1	30.022	1.003	−0.0431	7.143	0.342	612.85	3.66	3.43	3.36	3.45	2.71

- Increase of uncertainties of parameters pushes the DMs into much uncertain situations specially who belongs to optimistic class. On the other hand, increase of uncertainties decreases expected profit for pessimistic DMs.
- When production rate is dynamic control variable, more generalized production rate fetches more profit i.e. linear production rate is better than constant production rate, quadratic production rate is better than linear production rate.
- When repairing rate is dynamic control variable, repairing should start from second cycle.
- Presence of inflation has a negative impact on expected profit for any production-repairing model.

Though the model is considered with displayed inventory level dependent demand, time dependent production rate and repairing rate with out shortages, the present analysis can be extended to various inventory problems and production models with time dependent demand, variable rate of reworking, price discount etc. In this paper, the fuzzy/rough variables have been transformed to equivalent crisp value in general form, using degree of optimism/pessimism $\rho/(1-\rho)$. So DM can find decisions for his/her organization according to their optimism/pessimism. Hence, the problem in other areas such as transportation, portfolio selection etc. can be solved with fuzzy rough parameters, using this approach.

Acknowledgment

This research is supported by University Grants Commission of India by Grant No. PSW-089/11-12. The authors thank the honorable Reviewers for their valuable comments to improve the quality of the paper.

Appendix A

$$A=\int_0^T (d_0+d_1x(t))e^{-Rt}dt = d_0\frac{1-e^{-RT}}{R}+d_1B_1, \tag{30}$$

where

$$\begin{aligned}B_1 &= \int_0^T x(t)e^{-Rt}dt\\ &=\sum_{i=1}^{m}\left[\int_{(i-1)t_u}^{(i-1)t_u+t'_{ui}} e^{-Rt}\left\{\frac{((1-\delta)u_0-d_0)(1-e^{-d_1(t-(i-1)t_u)})}{d_1}+(1-\delta)\left[u_1\left\{\frac{(t-(i-1)t_u e^{-d_1(t-(i-1)t_u)})}{d_1}-\frac{(1-e^{-d_1(t-(i-1)t_u)})}{d_1^2}\right\}\right.\right.\right.\\ &\quad\left.\left.\left.+u_2\left\{\frac{(t^2-(i-1)^2t_u^2e^{-d_1(t-(i-1)t_u)})}{d_1}-\frac{2(t-(i-1)t_u e^{-d_1(t-(i-1)t_u)})}{d_1^2}+\frac{2(1-e^{-d_1(t-(i-1)t_u)})}{d_1^3}\right\}\right]\right\}dt+\int_{(i-1)t_u+t'_{ui}}^{it_u} e^{-Rt}\left(\frac{d_0(e^{d_1(it_u-t)}-1)}{d_1}\right)dt\right]\\ &\quad+\sum_{j=m+1}^{n}\left[\int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}\left\{\frac{((1-\delta)u_0-d_0+p_0)(1-e^{-d_1(t-(j-1)t_u)})}{d_1}+(1-\delta)\left[\left(u_1+\frac{p_1}{1-\delta}\right)\left\{\frac{(t-(j-1)t_u e^{-d_1(t-(j-1)t_u)})}{d_1}-\frac{(1-e^{-d_1(t-(j-1)t_u)})}{d_1^2}\right\}\right.\right.\right.\\ &\quad\left.\left.\left.+u_2\left\{\frac{(t^2-(j-1)^2t_u^2e^{-d_1(t-(j-1)t_u)})}{d_1}-\frac{2(t-(j-1)t_u e^{-d_1(t-(j-1)t_u)})}{d_1^2}+\frac{2(1-e^{-d_1(t-(j-1)t_u)})}{d_1^3}\right\}\right]\right\}dt+\int_{(j-1)t_u+t'_{uj}}^{jt_u} e^{-Rt}\left\{\frac{d_0(e^{d_1(jt_u-t)}-1)}{d_1}\right\}dt,\right]\end{aligned} \tag{31}$$

$$\begin{aligned}B_2 &= \int_0^T x_R(t)e^{-Rt}dt\\ &=\sum_{i=1}^{m}\left[\int_{(i-1)t_u}^{(i-1)t_u+t'_{ui}} e^{-Rt}\left\{x_R((i-1)t_u)+\mu\delta[u_0(t-(i-1)t_u)+\frac{u_1(t^2-(i-1)^2t_u^2)}{2}+\frac{u_2(t^3-(i-1)^3t_u^3)}{3}]\right\}dt\right.\\ &\quad\left.+\int_{(i-1)t_u+t'_{ui}}^{it_u} e^{-Rt}x_R((i-1)t_u+t'_{ui})dt\right]+\sum_{j=m+1}^{n}\left[\int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}\left\{x_R((j-1)t_u)-p_0(t-(j-1)t_u)-\frac{p_1(t^2-(j-1)^2t_u^2)}{2}+\mu\delta[u_0(t-(j-1)t_u)\right.\right.\\ &\quad\left.\left.+\frac{u_1(t^2-(j-1)^2t_u^2)}{2}+\frac{u_2(t^3-(j-1)^3t_u^3)}{3}]\right\}dt+\int_{(j-1)t_u+t'_{uj}}^{jt_u} e^{-Rt}x_R((j-1)t_u+t'_{uj})dt,\right]\end{aligned} \tag{32}$$

$$P_1 = \int_0^T u(t)e^{-Rt}dt = \sum_{i=1}^{m} \int_{(i-1)t_u}^{(i-1)t_u+t'_{ui}} e^{-Rt}\{u_0 + u_1 t + u_2 t^2\}dt + \sum_{j=m+1}^{n} \int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}\{u_0 + u_1 t + u_2 t^2\}dt, \tag{33}$$

$$P_2 = \int_0^T e^{-Rt}dt = \sum_{i=1}^{m} \int_{(i-1)t_u}^{(i-1)t_u+t'_{ui}} e^{-Rt}dt + \sum_{j=m+1}^{n} \int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}dt, \tag{34}$$

$$P_3 = \int_0^T \{u(t)\}^2 e^{-Rt}dt = \sum_{i=1}^{m} \int_{(i-1)t_u}^{(i-1)t_u+t'_{ui}} e^{-Rt}\{u_0 + u_1 t + u_2 t^2\}^2 dt + \sum_{j=m+1}^{n} \int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}\{u_0 + u_1 t + u_2 t^2\}^2 dt, \tag{35}$$

$$R_1 = \int_0^T e^{-Rt}dt = \sum_{j=m+1}^{n} \int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}dt, \tag{36}$$

$$R_2 = \int_0^T \{p(t)\}^2 e^{-Rt}dt = \sum_{j=m+1}^{n} \int_{(j-1)t_u}^{(j-1)t_u+t'_{uj}} e^{-Rt}\{p_0 + p_1 t\}^2 dt, \tag{37}$$

$$D_1 = \delta(1-\mu)P_1 + x_R(nt_u)\int_0^T e^{-Rt}dt, \tag{38}$$

$$S = n\int_0^T e^{-Rt}dt. \tag{39}$$

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