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Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment

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ABSTRACT

In this paper, an inventory policy for an item is presented with inflation and selling price dependent demand under deterministic and random planning horizons allowing and not allowing shortages. In addition, there is a provision for (i) an immediate part payment (variable) to the wholesaler, (ii) borrowing some money from money lending source for the immediate part payment, (iii) earning a discount on purchasing price and relaxation on credit period from the wholesaler against the advance payment and (iv) delay in payment for the rest allowed by wholesaler. The payment to the source is made at the end of the business period with some interest charged. Against the above conjectures, inventory models under the finite (crisp) and random planning horizons have been formulated with respect to the retailer's point of view for maximum profit. The nonlinear optimization method – Generalized Reduced Gradient (GRG) method is used to find the optimal solutions and the corresponding maximum profits for the different sets of given numerical data. Some sensitivity analyses are made and presented graphically. As particular cases, the results of the crisp models and the case without shortages are obtained from those of the stochastic model and the case with shortages respectively.

1. Introduction

Normally, the payment for an order is made by the retailer to the supplier immediately just after the receipt of the consignment. Now-a-days, due to the stiff competition in the market, to attract more customers, a credit period is offered by the supplier to the retailer. Moreover, for the speedy movement of capital, a wholesaler tries to maximize his/her market through several means. For this, very often some concessions in terms of unit price, credit period, etc., are offered to the retailers against immediate full/part payment. To avail these benefits, a retailer is tempted to cash down a part of the payment immediately even making a loan from the money lending source which charges interest against this loan. Now the retailer is in dilemma for optimal procurement and also for the amount for immediate part payment. Here an amount, borrowed from the money lending source as a loan with interest, is paid to the wholesaler at the beginning on receipt of goods. In return, the wholesaler/supplier offers a relaxed credit period as permissible delay in payment of rest amount and a reduced unit purchasing price depending on the amount of immediate part payment.

Inflation also plays an important role for the optimal order policy and influences the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current

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spending. Usually, these spending are on peripherals and luxury items that give rise to demand of these items. As a result, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy.

In the present paper, an inventory control system in which immediate part payment and the delay-payment for the rest are allowed by the wholesaler for an item over a finite planning horizon or random planning horizon with selling price and inflation induced demand is considered. In addition, against an immediate part payment (variable) to the wholesaler, there is a provision for (i) borrowing money from a money lending source and (ii) earning a discount on purchasing price and relaxation on credit period from the wholesaler. The models are developed with respect to the retailer for maximum profit. The randomness in the planning horizon has been removed using the chance-constraint technique. Single objective problems incorporating immediate part payment, delay in payment for the rest, and selling price, inflation dependent demand are formulated to maximize the profit function with/without shortages and solved using Generalized Reduced Gradient (GRG) method. The decision variables for these inventory models are the immediate part payment and number of cycles. These models are illustrated with numerical examples. Finally, the sensitivity analyses for the profit function and immediate part payment with respect to some parameters are carried out and the results are presented graphically.

2. Literature review

First, Goyal [1] presented an EOQ model under the conditions of permissible delay in payments. Since then, lots of literature is available in this area of study. The interesting papers related to such studies are Chu et al. [2], Chung [3], Jamal et al. [4], Sarker et al. [5] and others. Shah [6] considered the time value of money along with the trade credit for a finite time horizon inventory model with deteriorating items. Effect of inflation and time value of money is also well established in inventory problems. Initially, Buzacott [7] used the inflation subject to different types of pricing policies. Then consequently in the subsequent years, Moon and Lee [8], Chen [9], Dey et al. [10], Padmanavan and Vrat [11], Hariga and Ben-daya [12], and others worked in this area. Jaggi et al. [13] developed an inventory model with shortages, in which units are deteriorating at constant rate and demand rate is increasing exponentially due to inflation over a finite planning horizon using discount cash flow approach. Most recently, Chen and Kang [14] and Huang [15] presented integrated inventory models considering permissible delay in payment and variant pricing strategy for determining the optimal replenishment time interval and replenishment frequency. Tripathi et al. [16] developed a cash flow oriented EOQ model under permissible delay in payments for non-deteriorating items and time-dependent demand rate under inflation and time discounting. Liao et al. [17], Chung and Liao [18] dealt with the problem of determining the EOQ for exponentially deteriorating items under permissible delay in payment depending on the ordered quantity and developed an efficient solution-finding procedure to determine the retailer's optimal ordering policy. Chang [19] extended Chung and Liao's model by taking into account the inflation and finite time horizon with large quantity of purchase orders. Yang [20] presented an inventory model with different pricing policies. Singh et al. [21] proposed a two warehouse model under inflation with large quantity of purchase orders. Chung and Huang [22] studied ordering policy with permissible delay in payments to show the convexity of total annual variable cost function. Barron et al. [23] demonstrated optimal order size to take advantage of a one-time discount offer with allowable backorders when the supplier offers a temporary fixed percentage discount and has specified a minimum quantity of additional units to purchase. Recently, Guria et al. [24] proposed a pricing model for petrol/diesel and determined the optimal ordering policy for an existing petrol/diesel retailing station under permissible delay in payment with and without fully backlogged shortages. Several authors like Panda and Maiti [25] investigated the inventory models of this type of item. Joint price and lot size determination problems for deteriorating products were studied by Kim et al. [26]. Abad [27] investigated the inventory models of this type of item. Jaggi et al. [28,29], Liang and Zhou [30] solved two warehouse inventory models for deteriorating items with price dependent demand. Dey et al. [31] developed a two-storage inventory model with shortages and lead time in which units are non-deteriorating and demand is dynamic under inflation and time-value-money. It is a fact that the demand of an item is influenced by the selling price of that item i.e. whenever the selling price of an item increases, the demand of that decreases and vice-verse. Maiti et al. [32] introduced the concept of advanced payment for determining the optimal ordering policy under stochastic lead-time and price dependent demand condition.

Though several articles are available in the area of the inventory models with permissible delay in payment, there are some lacunas in the above mentioned literature. These are:

- Though the part payment at the time of purchase is now-a-days a part of the business from both ends – i.e., to bring immediate cash to the wholesaler and to give some price and payment concessions to the retailer, this has been ignored by the researchers.
- Most of the above inventory models are developed for infinite planning horizon with the common assumptions that lifetime of the product is infinite. Due to fluctuating world economy, cost of raw materials as well as production cost of a product changes rapidly. Also, with time, fashion and liking of the customers change and the introduction of multinationals leads to change in product specifications with new features. So, in reality lifetime of a product is finite and uncertain. Very few articles (cf. [33–36] etc.) are there incorporating this assumption.

- As mentioned above, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. Formulation of inventory models with the above conjecture allowing part payment and trade credit will be a realistic one as inflation, at present, is rampant throughout the world.

Correcting the above short comings, in this paper, an attempt has been made to formulate and to solve a real-life inventory model with inflation and selling price dependent demand under finite and random planning horizon allowing trade credit against immediate part payment.

3. Mathematical model formulation

To formulate the mathematical model for the proposed inventory system, the following notations and assumptions are made.

3.1. Notations

(i) $q_i(t)$ = inventory level at time t for the ith cycle
(ii) Q_{1i} = inventory level at time $t = t_{i-1}$ for the ith cycle (t_{i-1},t_i)
(iii) Q_{2i} = shortage quantity for the ith cycle (t_{i-1},t_i)
(iv) $Q_i = (Q_{1i} + Q_{2i})$ = total inventory ordered for the ith cycle (t_{i-1},t_i)
(v) c_3 = set up cost for each cycle
(vi) A = immediate part payment for each cycle (variable)
(vii) n = number of cycles
(viii) H = finite planning horizon for the crisp model
(ix) $\hat{H} =$ random planning horizon that follows normal distribution with mean $m_{\hat{H}}$ and variance $\sigma_{\hat{H}}$ for the stochastic model
(x) T = length of each cycle
(xi) M = permissible delay period for each cycle
(xii) M' = rest period of each cycle after the credit period, i.e., $\frac{H}{n} - M$
(xiii) p_i = reduced purchasing cost per unit quantity for ith cycle
(xvi) p_0 = actual purchasing cost per unit quantity prevailing in the market
(xv) s_i = selling price per unit quantity for the ith cycle
(xvi) D_0 = original demand at $t = 0$
(xvii) $D_i(t)$ = rate of demand (variable) for the ith cycle
(xviii) I_e = rate of interest per unit to be earned by the retailer
(xix) I_b = rate of interest per unit to be paid by the retailer to money lender against immediate part payment A
(xx) c_1 = inventory holding cost per unit quantity per unit time for each cycle
(xxi) c_2 = inventory shortage cost per unit quantity per unit time for each cycle
(xxii) α = rate of inflation
(xxiii) t_s = time of beginning of shortages which is taken as $t_s = r'\frac{H}{n}, 0 < r' < 1.0$
(xxiv) β and p_r are two given real numbers where $\beta > 0, 0 \leqslant \beta \leqslant 1.0$ and $\varepsilon_{\widehat{H}}$ is a real number whose standard normal value is p_r
(xxv) $Z\,(n,A)$ is the profit function for the whole period

3.2. Assumptions

(i) Lead time is zero.
(ii) Shortages are allowed and fully backlogged.
(iii) Immediate part payment A is made and same for all the cycles and it is paid at the beginning of each cycle. For the 1st cycle, it is borrowed from a money lending source with the condition that it will be paid at the end of business period H with interest at the rate of I_b and for the remaining cycles, the immediate part payment A will be given from the revenue earned up to that time. So, for ith cycle ($i \geqslant 2$) the immediate part payment A must be less than the total revenue at t_{i-1}.
(iv) The rest payment for the wholesaler will be done at the end of the credit period M where $t_{i-1} + M \leqslant t_i$ for each cycle. There is a fixed credit period M_0 and it is assumed that it will be enhanced depending upon the amount of the immediate advanced payment A and is given in the form $M = M_0 + \frac{A}{Q_1 p_1} M'_0$ for each cycle
where, $M_0 = \frac{H}{\xi n}, M'_0 = \frac{(\xi-1)H}{\xi n}, \xi \geqslant 2$ and $A < p_1 Q_1$
Here, it is assumed that enhanced credit period is inversely proportional to the total purchasing cost during the first cycle.
(v) The length of each cycle is $T = \frac{H}{n}$ *i.e.*, $\frac{H}{n} = t_i - t_{i-1}$ and $t_0 = 0, t_i = i\frac{H}{n}$, for i = 1,2,3, ... , n for crisp model.
(vi) The rates of interest to be earned by the retailer and that to be paid to the money lender by the retailer are same for each cycle and $I_e < I_b$.

(vii) Purchasing cost per unit quantity $p_i = \frac{p_0 e^{\alpha t_{i-1}}}{A^{\gamma_1}}, \alpha > 0, \gamma_1 > 0$ and p_0 is the market cost price per unit quantity at time t_0.
(viii) Selling price per unit quantity $s_i = mp_0 e^{\alpha t_{i-1}}$, where, m (> 0) is the mark-up. Here, the retailer does not share the reduced price obtained due to advance payment with the customers.
(ix) Demand is inflation rate and selling price dependent i.e., $D_i = \frac{D_0 e^{\alpha_1 t}}{s_i^{\gamma}}, \alpha_1 = k_2\alpha, 0 < \gamma < 1.0, 0 < k_2 < 1.0$ and $t_{i-1} \leqslant t \leqslant t_i$, where D_0 is the original demand.
(x) Holding cost per unit per unit time c_1 is same for all cycles.
(xi) Set up cost c_3 is also same for each cycle.
(xii) Shortage cost per unit per unit time c_2 is same for all cycles.

4. Chance constraint method

Chance constraint programming is one of the techniques of stochastic programming which deals with a situation where some or *all* parameters of the problem are described by random variables. In this presentation, chance constraint is taken as

$$\text{prob}(|nT - \widehat{H}| \leqslant \beta) \geqslant p_r,$$

where $\widehat{H}$ is a random variable. Here, p_r is the least value of the probability with which this chance constraint is satisfied. This can be rewrite as

$$\text{prob}(nT - \beta \leqslant \widehat{H}) \geqslant p_r \text{ and } \text{prob}(\widehat{H} - nT \leqslant \beta) \geqslant p_r$$

From the first inequality

$$\text{prob}\left(\frac{nT - \beta - m_{\widehat{H}}}{\sigma_{\widehat{H}}} \leqslant \frac{\widehat{H} - m_{\widehat{H}}}{\sigma_{\widehat{H}}}\right) \geqslant p_r$$

Now, $\frac{\widehat{H} - m_{\widehat{H}}}{\sigma_{\widehat{H}}}$ represents the standard normal variant with mean 0 and variance 1, i.e.,

$$\text{prob}(nT - \beta \leqslant \widehat{H}) \geqslant p_r = 1 - F\left(\frac{nT - \beta - m_{\widehat{H}}}{\sigma_{\widehat{H}}}\right)$$

where $F(x)$ represents the continuous distribution function of standard normal distribution. Let, $\varepsilon_{\widehat{H}}$ be the standard normal value such that $F(\varepsilon_{\widehat{H}}) = p_r = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\varepsilon_{\widehat{H}}} e^{-t^2/2}\,dt$.

Then the statement $\text{prob}(nT - \beta \leqslant \widehat{H}) \geqslant p_r$ is true if and only if

$$\frac{nT - \beta - m_{\widehat{H}}}{\sigma_{\widehat{H}}} \leqslant -\varepsilon_{\widehat{H}} \text{ i.e., } nT \leqslant m_{\widehat{H}} + \beta - \varepsilon_{\widehat{H}}\sigma_{\widehat{H}}.$$

Similarly, the second inequality can be reduced to $m_{\widehat{H}} - \beta - \varepsilon_{\widehat{H}}\sigma_{\widehat{H}} \leqslant nT$.

5. Mathematical representation of the model

5.1. Case-I: Crisp time horizon

5.1.1. Model-IA: model with shortages

The retailer replenishes the item of amount Q_i at the time t_{i-1} for i-th cycle (t_{i-1}, t_i) from the wholesaler. Fully backlogged shortages are allowed towards at the end of each cycle for a $(1 - r')\frac{H}{n}$ period of time. During that period shortage level reaches Q_{2i} and then order for next is placed. Clearly, total ordered quantity for ith cycle is $Q_i = Q_{1i} + Q_{2i}$ and supply is made at the time of order.Then the inventory level decreases due to demand of the customer till it becomes zero at $t_{i-1} + t_s$. This repetition takes place for each cycle and detail of the inventory system for this case is shown in Fig. 1a. The variation of the inventory level $q_i(t)$ with respect to time t due to effect of demand $D_i(t)$ can be described by the following differential equation

$$\frac{dq_i(t)}{dt} = -D_i(t), \quad t_{i-1} \leqslant t \leqslant t_i, \quad i = 1, 2, 3, \ldots\ldots, n \tag{1}$$

with boundary conditions $q_i(t_{i-1}) = Q_{1i}$, $q_i(t_{i-1} + t_s) = 0$, $q_i(t_i) = -Q_{2i}$

The solution of Eq. (1) with the boundary conditions is given by

$$q_i(t) = \frac{D_0}{\alpha_1 s_i^{\gamma}}(e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t}) \tag{2}$$

The purchased item Q_i in the ith cycle (t_{i-1},t_i) is obtained by substituting $t = t_{i-1}$ from the above equation which is as follows

$$Q_i = \frac{D_0}{\alpha_1 s_i^{\gamma}}(e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}})$$

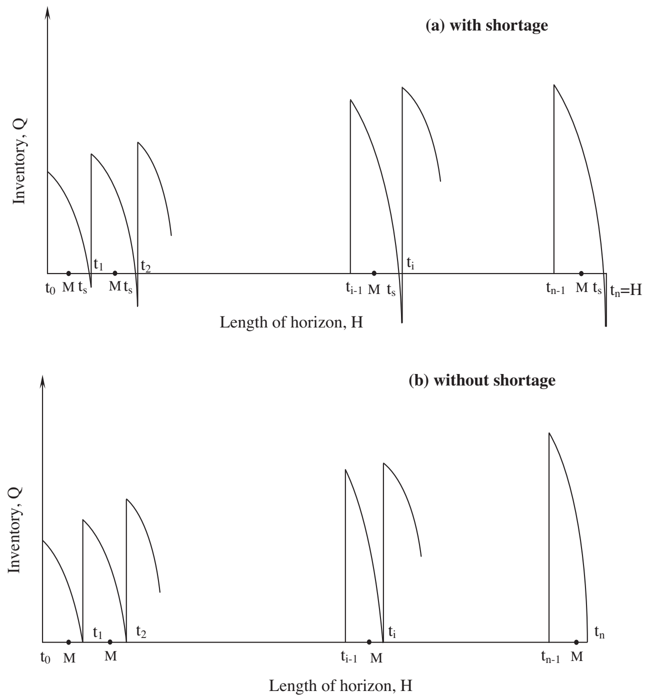

Fig. 1. Graphical representation of finite horizon inventory system: (a) with shortages and (b) without shortages.

The purchasing cost of all units purchased in *i*th cycle (t_{i-1}, t_i) is

$$PC_i = p_i Q_i = \frac{p_0^{1-\gamma} D_0}{A^{\gamma_1} \alpha_1 m^{\gamma}} e^{(1-\gamma)\alpha t_{i-1}} (e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}})$$

The total purchasing cost of all units which are purchased during the whole business period (0, H) is obtained by

$$TPC = \sum_{i=1}^{n} \frac{p_0^{1-\gamma} D_0}{A^{\gamma_1} \alpha_1 m^{\gamma}} e^{(1-\gamma)\alpha t_{i-1}} (e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}}) = \frac{p_0^{1-\lambda} D_0}{A^{\gamma_1} \alpha_1 m^{\gamma}} \sum_{i=1}^{n} [e^{\alpha_1 t_i + (1-\gamma)\alpha t_{i-1}} - e^{\{\alpha_1 + (1-\gamma)\alpha\} t_{i-1}}] \quad (3)$$

Lemma 1. *Prove that*

$$\sum_{i=1}^{n} [e^{\alpha_1 t_i + (1-\gamma)\alpha t_{i-1}} - e^{\{\alpha_1 + (1-\gamma)\alpha\} t_{i-1}}] = (e^{-S} - e^{-R}) \frac{e^R (e^{nR} - 1)}{e^R - 1}$$

where, $\frac{H}{n} = t_i - t_{i-1}, R = \{\alpha_1 + \alpha(1-\gamma)\} \frac{H}{n}$ *and* $S = \alpha(1-\gamma)\frac{H}{n}$

Proof

$$\begin{aligned}
&\sum_{i=1}^{n} \left[e^{\alpha_1 t_i + (1-\gamma)\alpha t_{i-1}} - e^{\{\alpha_1 + (1-\gamma)\alpha\} t_{i-1}}\right] \\
&= \sum_{i=1}^{n} \left[e^{\alpha_1 i \frac{H}{n} + (1-\gamma)\alpha (i-1)\frac{H}{n}} - e^{\{\alpha_1 + (1-\gamma)\alpha\}(i-1)\frac{H}{n}}\right] \\
&= \sum_{i=1}^{n} [e^{-S} e^{Ri} - e^{-R} e^{Ri}] = \sum_{i=1}^{n} (e^{-S} - e^{-R}) e^{Ri} \\
&= (e^{-S} - e^{-R}) \sum_{i=1}^{n} e^{Ri} = (e^{-S} - e^{-R}) \frac{e^R (e^{nR} - 1)}{e^R - 1}
\end{aligned}$$

Using Lemma 1 the total purchasing cost is

$$TPC = \frac{p_0^{1-\gamma} D_0}{A^{\gamma_1} \alpha_1 m^{\gamma}} (e^{-S} - e^{-R}) \frac{e^R (e^{nR} - 1)}{e^R - 1} \quad (4)$$

The total purchased units during the whole business period is given by

$$TQ = \sum_{i=1}^{n} Q_i = \frac{D_0}{\alpha_1 m^{\gamma} p_0^{\gamma}} \sum_{i=1}^{n} \left[\frac{e^{\alpha_1 t_i}}{e^{\alpha\gamma t_{i-1}}} - \frac{e^{\alpha_1 t_{i-1}}}{e^{\alpha\gamma t_{i-1}}} \right] \qquad \square$$

Lemma 2. *Prove that*

$$\sum_{i=1}^{n} e^{x(i-1)} = \frac{e^{nx} - 1}{e^x - 1}$$

Proof

$$\sum_{i=1}^{n} e^{x(i-1)} = e^0 + e^x + e^{2x} + \ldots\ldots + e^{(n-1)x} = 1 + \frac{e^x[e^{(n-1)x} - 1]}{e^x - 1} = \frac{e^x - 1 + e^{nx} - e^x}{e^x - 1} = \frac{e^{nx} - 1}{e^x - 1} \qquad \square$$

Lemma 3. *Prove that*

$$\sum_{i=1}^{n} \left[\frac{e^{\alpha_1 t_i}}{e^{\alpha\gamma t_{i-1}}} - \frac{e^{\alpha_1 t_{i-1}}}{e^{\alpha\gamma t_{i-1}}} \right] = \left(e^{\alpha_1 \frac{H}{n}} - 1 \right) \frac{e^{(\alpha_1 - \alpha\gamma)H} - 1}{e^{(\alpha_1 - \alpha\gamma)\frac{H}{n}} - 1}$$

Proof

$$\begin{aligned}
\sum_{i=1}^{n} \left[\frac{e^{\alpha_1 t_i}}{e^{\alpha\gamma t_{i-1}}} - \frac{e^{\alpha_1 t_{i-1}}}{e^{\alpha\gamma t_{i-1}}} \right] &= \sum_{i=1}^{n} e^{\alpha_1 t_i - \alpha\gamma t_{i-1}} - \sum_{i=1}^{n} e^{\alpha_1 t_{i-1} - \alpha\gamma t_{i-1}} = \sum_{i=1}^{n} e^{\alpha_1 t_i - \alpha\gamma t_{i-1}} - \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} = \sum_{i=1}^{n} e^{\alpha_1 \left(\frac{H}{n} + t_{i-1}\right) - \alpha\gamma t_{i-1}} - \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} \\
&= \sum_{i=1}^{n} e^{\alpha_1 \frac{H}{n} + (\alpha_1 - \alpha\gamma) t_{i-1}} - \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} = e^{\alpha_1 \frac{H}{n}} \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} - \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} = \left(e^{\alpha_1 \frac{H}{n}} - 1 \right) \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}} \\
&= \left(e^{\alpha_1 \frac{H}{n}} - 1 \right) \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) \frac{H}{n}}(i-1) = \left(e^{\alpha_1 \frac{H}{n}} - 1 \right) \frac{e^{(\alpha_1 - \alpha\gamma)H} - 1}{e^{(\alpha_1 - \alpha\gamma)\frac{H}{n}} - 1} \quad \left[\text{Using Lemma 2 for } \ x = (\alpha_1 - \alpha\gamma)\frac{H}{n} \right]
\end{aligned}$$

Using Lemma 3 the total inventory during the whole business period is given by

$$TQ = \frac{D_0}{\alpha_1 m^{\gamma} p_0^{\gamma}} \left(e^{\alpha_1 \frac{H}{n}} - 1 \right) \frac{e^{(\alpha_1 - \alpha\gamma)H} - 1}{e^{(\alpha_1 - \alpha\gamma)\frac{H}{n}} - 1} \tag{5}$$

The total revenue earned in the ith cycle (t_{i-1}, t_i) and during the whole business $(0, H)$ are respectively given by

$$TS_i = \int_{t_{i-1}}^{t_i} s_i D_i \, dt = \frac{(mp_0)^{1-\gamma} e^{\alpha(1-\gamma)t_{i-1}} D_0}{\alpha_1} \left[e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}} \right] \tag{6}$$

and

$$\begin{aligned}
TS = \sum_{i=1}^{n} TS_i &= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} \sum_{i=1}^{n} \left[e^{\alpha_1 t_i + \alpha(1-\gamma)t_{i-1}} - e^{(\alpha_1 + \alpha(1-\gamma))t_{i-1}} \right] \\
&= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} (e^{-S} - e^{-R}) \frac{e^R (e^{nR} - 1)}{e^R - 1} [\text{Using Lemma 1}]
\end{aligned} \tag{7}$$

The holding cost for ith cycle $(t_{i-1},\, t_i)$ and during the whole business period $(0, H)$ are respectively obtained as

$$\begin{aligned}
HC_i = c_1 \int_{t_{i-1}}^{t_{i-1}+t_s} q_i(t) \, dt &= \frac{c_1 D_0}{s_i^{\gamma} \alpha_1} \int_{t_{i-1}}^{t_{i-1}+t_s} (e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t}) \, dt = \frac{c_1 D_0}{s_i^{\gamma} \alpha_1} \left[t_s e^{\alpha_1(t_{i-1}+t_s)} - \frac{1}{\alpha_1} \{ e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t_{i-1}} \} \right] \\
&= \frac{c_1 D_0}{s_i^{\gamma} \alpha_1} \left[r' \frac{H}{n} e^{\alpha_1 \left(r' \frac{H}{n} + t_{i-1}\right)} - \frac{1}{\alpha_1} \{ e^{-\alpha_1 t_s} e^{\alpha_1 t_{i-1}} - e^{\alpha_1 t_{i-1}} \} \right] = \frac{c_1 D_0}{s_i^{\gamma} \alpha_1} \left[r' \frac{H}{n} e^{\alpha_1 r' \frac{H}{n}} e^{\alpha_1 t_{i-1}} - \frac{1}{\alpha_1} \left\{ e^{\alpha_1 r' \frac{H}{n}} - 1 \right\} e^{\alpha_1 t_{i-1}} \right] \\
&= \frac{c_1 D_0}{s_i^{\gamma} \alpha_1} \left[r' \frac{H}{n} e^{\alpha_1 r' \frac{H}{n}} - \frac{1}{\alpha_1} \left\{ e^{\alpha_1 r' \frac{H}{n}} - 1 \right\} \right] e^{\alpha_1 t_{i-1}}
\end{aligned} \tag{8}$$

$$
\begin{aligned}
\text{and } THC=\sum_{i=1}^{n} HC_i &= \frac{c_1D_0}{(mp_0)^{\gamma}\alpha_1}\left[r'\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1-\alpha\gamma)t_{i-1}}\\
&= \frac{c_1D_0}{(mp_0)^{\gamma}\alpha_1}\left[r'\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1-\alpha\gamma)(i-1)\frac{H}{n}}\\
&= \frac{c_1D_0}{(mp_0)^{\gamma}\alpha_1}\left[r'\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\frac{e^{(\alpha_1-\alpha\gamma)H}-1}{e^{(\alpha_1-\alpha\gamma)\frac{H}{n}}-1}\quad \text{[Using Lemma 3]}
\end{aligned}
\tag{9}
$$

The shortage cost for the ith cycle (t_{i-1}, t_i) and during the whole business period $(0,H)$ are respectively obtained as

$$
\begin{aligned}
SHC_i = c_2\int_{t_{i-1}+t_s}^{t_i} -q_i(t)\,\mathrm{d}t &= \frac{c_2D_0}{s_i^{\gamma}\alpha_1}\int_{t_{i-1}+t_s}^{t_i} -(e^{\alpha_1(t_{i-1}+t_s)}-e^{\alpha_1 t})\,\mathrm{d}t\\
&= \frac{-c_2D_0}{s_i^{\gamma}\alpha_1}\left[(t_i-t_{i-1}-t_s)e^{\alpha_1(t_{i-1}+t_s)}-\frac{1}{\alpha_1}\{e^{\alpha_1 t_i}-e^{\alpha_1(t_{i-1}+t_s)}\}\right]\\
&= \frac{-c_2D_0}{s_i^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1\left(r'\frac{H}{n}+t_{i-1}\right)}-\frac{1}{\alpha_1}\left\{e^{-\alpha_1\left(\frac{H}{n}+t_{i-1}\right)}-e^{\alpha_1 r'\frac{H}{n}}e^{\alpha_1 t_{i-1}}\right\}\right]\\
&= \frac{-c_2D_0}{s_i^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}e^{\alpha_1 t_{i-1}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1\frac{H}{n}}-e^{\alpha_1 r'\frac{H}{n}}\right\}e^{\alpha_1 t_{i-1}}\right]\\
&= \frac{-c_2D_0}{s_i^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1\frac{H}{n}}-e^{\alpha_1 r'\frac{H}{n}}\right\}\right]e^{\alpha_1 t_{i-1}}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
TSHC=\sum_{i=1}^{n} SHC_i &= \frac{-c_2D_0}{(mp_0)^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1\frac{H}{n}}-e^{\alpha_1 r'\frac{H}{n}}\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1-\alpha\gamma)t_{i-1}}\\
&= \frac{-c_2D_0}{(mp_0)^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1\frac{H}{n}}-e^{\alpha_1 r'\frac{H}{n}}\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1-\alpha\gamma)(i-1)\frac{H}{n}}\\
&= \frac{-c_2D_0}{(mp_0)^{\gamma}\alpha_1}\left[(1-r')\frac{H}{n}e^{\alpha_1 r'\frac{H}{n}}-\frac{1}{\alpha_1}\left\{e^{\alpha_1\frac{H}{n}}-e^{\alpha_1 r'\frac{H}{n}}\right\}\right]\frac{e^{(\alpha_1-\alpha\gamma)H}-1}{e^{(\alpha_1-\alpha\gamma)\frac{H}{n}}-1}\quad \text{[Using Lemma 3]}
\end{aligned}
\tag{11}
$$

In the finite time horizon, total interest can be earned in two ways:

(i) interest is earned from the revenue due to the continuous sale during the whole time horizon (i.e., *TIECS*) and (ii) interest is earned from the part of the revenue remaining at the end of each cycle after paying due amount of this cycle and the advance (immediate part payment) for the next cycle from the total sale revenue for the present cycle. This is followed for all cycles except the first one (i.e., *IETn*).

Therefore, for the first case, the interest earned ($IECS_i$) by continuous sale during the interval $(t_{i-1},t_{i-1}+t_s)$ is given by

$$
\begin{aligned}
IECS_i &= I_e\left[\int_{t_{i-1}}^{t_{i-1}+M} s_iD_i(t_{i-1}+M-t)\,\mathrm{d}t+\int_{t_{i-1}+M}^{t_{i-1}+t_s} s_iD_i(t_{i-1}+t_s-t)\,\mathrm{d}t\right]\\
&= I_e\left[(t_{i-1}+M)\int_{t_{i-1}}^{t_{i-1}+M} s_iD_i\,\mathrm{d}t+(t_{i-1}+t_s)\int_{t_{i-1}+M}^{t_{i-1}+t_s} s_iD_i\,\mathrm{d}t-\int_{t_{i-1}}^{t_{i-1}+t_s} s_iD_i t\,\mathrm{d}t\right]\\
&= \frac{I_e(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[(M-r'\tfrac{H}{n})e^{\alpha_1\frac{H}{n}}-M+\tfrac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]e^{(\alpha_1+\alpha(1-\gamma))t_{i-1}}
\end{aligned}
\tag{12}
$$

Details of calculations for above equation are given in Appendix A. Therefore, the total interest (*TIECS*) earned by continuous sale during the whole business period $(0,H)$ is given by

$$
\begin{aligned}
TIECS &= \frac{I_e(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[\left(M-r'\frac{H}{n}\right)e^{\alpha_1\frac{H}{n}}-M+\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1+\alpha(1-\gamma))t_{i-1}}\\
&= \frac{I_e(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[\left(M-r'\frac{H}{n}\right)e^{\alpha_1\frac{H}{n}}-M+\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\sum_{i=1}^{n} e^{(\alpha_1+\alpha(1-\gamma))(i-1)\frac{H}{n}}\\
&= \frac{I_e(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[\left(M-r'\frac{H}{n}\right)e^{\alpha_1\frac{H}{n}}-M+\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\sum_{i=1}^{n} e^{R(i-1)}\\
&= \frac{I_e(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[\left(M-r'\frac{H}{n}\right)e^{\alpha_1\frac{H}{n}}-M+\frac{1}{\alpha_1}\left\{e^{\alpha_1 r'\frac{H}{n}}-1\right\}\right]\frac{(e^{nR}-1)}{e^{R}-1}\quad \text{[Using Lemma 2]}
\end{aligned}
\tag{13}
$$

Similarly, for the second case, the interest earned (IET_n) is given by

$$IET_n = I_e \left[\begin{array}{l} M'\left\{ \int_{t_0}^{t_0+M} s_1 D_1\,dt + \int_{t_1}^{t_1+M} s_2 D_2\,dt + \int_{t_2}^{t_2+M} s_3 D_3\,dt + \ldots\ldots + \int_{t_{n-1}}^{t_{n-1}+M} s_n D_n\,dt - (Q_1 p_1 + Q_2 p_2 + \ldots. + Q_n p_n) + nA \right\} \\ +M\left\{ \begin{array}{l} (TS_1 - Q_1 p_1) + (TS_1 + TS_2 - Q_1 p_1 - Q_2 p_2) + \ldots\ldots.. \\ +(TS_1 + TS_2 + \ldots\ldots + TS_{n-1} - Q_1 p_1 - Q_2 p_2 - \ldots\ldots.. - Q_{n-1} p_{n-1}) \end{array} \right\} \\ +M'\left\{ \begin{array}{l} (TS_1 - Q_1 p_1) + (TS_1 + TS_2 - Q_1 p_1 - Q_2 p_2) + \ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots + TS_{n-1} - Q_1 p_1 - Q_2 p_2 - \ldots\ldots.. - Q_{n-1} p_{n-1}) \end{array} \right\} \end{array} \right]$$

$$= I_e \left[\begin{array}{l} M'\left\{ \sum_{j=1}^{n} \int_{t_{j-1}}^{t_{j-1}+M} s_j D_j\,dt - \sum_{j=1}^{n} Q_j p_j + nA \right\} \\ +(M+M')\left\{ \begin{array}{l} (n-1)TS_1 + (n-2)TS_2 + (n-3)TS_3 + \ldots.. + TS_{n-1} \\ -(n-1)Q_1 p_1 - (n-2)Q_2 p_2 - (n-3)Q_3 p_3 - \ldots\ldots - Q_{n-1} p_{n-1} \end{array} \right\} \end{array} \right] \tag{14}$$

$$= I_e \left[M'\left\{ \sum_{j=1}^{n} \int_{t_{j-1}}^{t_{j-1}+M} s_j D_j\,dt - \sum_{j=1}^{n} Q_j p_j + nA \right\} + (M+M')\left\{ \sum_{j=1}^{n-1} (n-j)(TS_j - Q_j p_j) \right\} \right]$$

Detailed derivation of Eq. (14) is given in Appendix B.

Now,

$$\begin{aligned} \sum_{j=1}^{n} \int_{t_{j-1}}^{t_{j-1}+M} s_j D_j\,dt &= \sum_{j=1}^{n} \frac{(mp_0)^{1-\gamma} e^{\alpha(1-\gamma)t_{j-1}} D_0}{\alpha_1} e^{\alpha_1 t_{j-1}} [e^{\alpha_1 M} - 1] \\ &= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} [e^{\alpha_1 M} - 1] \sum_{j=1}^{n} e^{\{\alpha_1 + \alpha(1-\gamma)t_{j-1}\}} \\ &= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} [e^{\alpha_1 M} - 1] \sum_{j=1}^{n} e^{\left\{\alpha_1 + \alpha(1-\gamma)(j-1)\frac{H}{n}\right\}} \\ &= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} [e^{\alpha_1 M} - 1] \sum_{j=1}^{n} e^{(j-1)R}, \text{where } R = \{\alpha_1 + \alpha(1-\gamma)\} \frac{H}{n} \\ &= \frac{(mp_0)^{1-\gamma} D_0}{\alpha_1} [e^{\alpha_1 M} - 1] \frac{(e^{nR} - 1)}{e^R - 1} \end{aligned} \tag{15}$$

and,

$$\begin{aligned} \sum_{j=1}^{n-1} (n-j)(TS_j - Q_j p_j) &= \sum_{j=1}^{n-1} (n-j)(s_j Q_j - Q_j p_j) = \sum_{j=1}^{n-1} (n-j)\left(mp_0 - \frac{p_0}{A^{\gamma_1}} \right) e^{\alpha t_{j-1}} Q_j \\ &= \frac{D_0}{\alpha_1}\left(mp_0 - \frac{p_0}{A^{\gamma_1}} \right) \sum_{j=1}^{n-1} (n-j) \frac{e^{\alpha t_{j-1}}}{(mp_0 e^{\alpha t_{j-1}})^{\gamma}} (e^{\alpha_1 t_j} - e^{\alpha_1 t_{j-1}}) \\ &= \frac{D_0 p_0}{\alpha_1 m^{\gamma} p_0^{\gamma}}\left(m - \frac{1}{A^{\gamma_1}} \right) \sum_{j=1}^{n-1} (n-j) e^{(1-\gamma)\alpha t_{j-1}} (e^{\alpha_1 t_j} - e^{\alpha_1 t_{j-1}}) \qquad \square \end{aligned} \tag{16}$$

Lemma 4. *Prove that*

$$\sum_{j=1}^{n} j e^{jx} = \frac{ne^{(n+2)x} - (n+1)e^{(n+1)x} + e^x}{(e^x - 1)^2}$$

Proof. By Lemma 2, we have

$$\sum_{j=1}^{n} e^{jx} = \frac{e^x(e^{nx} - 1)}{(e^x - 1)} \tag{17}$$

Differentiating both sides of (17) with respect to x

$$\sum_{j=1}^{n} j e^{jx} = \frac{(e^x - 1)\{e^x(e^{nx} - 1) + e^x n e^{nx}\} - e^x(e^{nx} - 1)e^x}{(e^x - 1)^2} = \frac{ne^{(n+2)x} - (n+1)e^{(n+1)x} + e^x}{(e^x - 1)^2} \qquad \square \tag{18}$$

Lemma 5. *Prove that*

$$\sum_{j=1}^{n-1}(n-j)e^{(1-\gamma)\alpha t_{j-1}}(e^{\alpha_1 t_j}-e^{\alpha_1 t_{j-1}})=(e^{-S}-e^{-R})\left\{n\frac{e^R(e^{(n-1)R}-1)}{e^R-1}-\frac{(n-1)e^{(n+1)R}-ne^{nR}+e^R}{(e^R-1)^2}\right\}$$

Proof

$$\begin{aligned}&\sum_{j=1}^{n-1}(n-j)e^{(1-\gamma)\alpha t_{j-1}}(e^{\alpha_1 t_j}-e^{\alpha_1 t_{j-1}})\\&=\sum_{j=1}^{n-1}(n-j)(e^{\alpha_1 t_j+(1-\gamma)\alpha t_{j-1}}-e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}})\\&=\sum_{j=1}^{n-1}(n-j)\left(e^{\alpha_1 j\frac{H}{n}+(1-\gamma)\alpha(j-1)\frac{H}{n}}-e^{\{\alpha_1+(1-\gamma)\alpha\}(j-1)\frac{H}{n}}\right)\\&=\sum_{j=1}^{n-1}(n-j)\left(e^{\{\alpha_1+(1-\gamma)\alpha\}j\frac{H}{n}-(1-\gamma)\alpha\frac{H}{n}}-e^{\{\alpha_1+(1-\gamma)\alpha\}(j-1)\frac{H}{n}}\right)\\&=e^{-S}\sum_{j=1}^{n-1}(n-j)e^{jR}-e^{-R}\sum_{j=1}^{n-1}(n-j)e^{jR}\\&=(e^{-S}-e^{-R})\sum_{j=1}^{n-1}(n-j)e^{jR}=(e^{-S}-e^{-R})\left\{n\sum_{j=1}^{n-1}e^{jR}-\sum_{j=1}^{n-1}je^{jR}\right\}\\&=(e^{-S}-e^{-R})\left\{n\frac{e^R(e^{(n-1)R}-1)}{e^R-1}-\frac{(n-1)e^{(n+1)R}-ne^{nR}+e^R}{(e^R-1)^2}\right\}\quad[\text{using Lemma 4}]\end{aligned}$$

Therefore,

$$\sum_{j=1}^{n-1}(n-j)(TS_j-Q_jp_j)=\frac{D_0p_0}{\alpha_1 m^\gamma p_0^\gamma}\left(m-\frac{1}{A^{\gamma_1}}\right)(e^{-S}-e^{-R})\left\{n\frac{e^R(e^{(n-1)R}-1)}{e^R-1}-\frac{(n-1)e^{(n+1)R}-ne^{nR}+e^R}{(e^R-1)^2}\right\}\tag{19}$$

Therefore, using Lemma 5, IET_n is given by

$$IET_n=I_e\left[\begin{array}{l}M'\left\{\frac{(mp_0)^{1-\gamma}D_0}{\alpha_1}[e^{\alpha_1 M}-1]\frac{(e^{nR}-1)}{e^R-1}-\frac{p_0^{1-\gamma}D_0}{A^{\gamma_1}\alpha_1 m^\gamma}(e^{-S}-e^{-R})\frac{e^R(e^{nR}-1)}{e^R-1}+nA\right\}\\+\frac{H}{n}\left[\frac{D_0p_0}{\alpha_1 m^\gamma p_0^\gamma}\left(m-\frac{1}{A^{\gamma_1}}\right)(e^{-S}-e^{-R})\left\{n\frac{e^R(e^{(n-1)R-1})}{e^R-1}-\frac{(n-1)e^{(n+1)R}-ne^{nR}+e^R}{(e^R-1)^2}\right\}\right]\end{array}\right]\tag{20}$$

The interest paid at the end of business period $(0,H)$ is given by

$$TIP=I_bAH\tag{21}$$

Total set-up cost at the end of business period $(0,H)$ is given by

$$TC_3=nc_3\tag{22}$$

Hence the total profit function for the retailer is given over the whole business period is given by

$$Z(n,A)=TS+TIECS+IET_n-TPC-THC-TSHC-TC_3-TIP\qquad\square\tag{23}$$

5.1.2. Model-IB: model without shortages

Taking $r'=1$ in $t_s=r'\frac{H}{n}$ of Model-IA, corresponding expressions of the model without shortages are obtained after simplifications and detail of the inventory system for this case is shown in Fig. 1b.

5.2. Case-II: Random time horizon

5.2.1. Chance Constraint for random time horizon

Here, the time horizon $\widehat{H}$ is considered as random but the total time horizon is partitioned into n cycles with length T for each cycle. Therefore, following Section 4, corresponding chance constraint for the whole business period is given by

$$\text{prob}(|nT-\widehat{H}|\leqslant\beta)\geqslant p_r\tag{24}$$

which is reduced to $m_{\widehat{H}}-\beta-\varepsilon_{\widehat{H}}\sigma_{\widehat{H}}\leqslant nT\leqslant m_{\widehat{H}}+\beta-\varepsilon_{\widehat{H}}\sigma_{\widehat{H}}$

where, $p_r = F(\varepsilon_{\widehat{H}}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\varepsilon_{\widehat{H}}} e^{-t^2/2} dt$ is the cumulative probability $P[t \leqslant \varepsilon]$ available in standard statistical table for different values of $\varepsilon_{\widehat{H}}$.

5.2.2. Model-IIA: model with shortages

Considering $H = m_{\widehat{H}}$ in Model-IA, the model with fully backlogged shortages in random time horizon is formulated. Hence, the total profit function for the retailer over the whole business period is given by the Eq. (23) subject to the constraint

$$m_{\widehat{H}} - \beta - \varepsilon_{\widehat{H}}\sigma_{\widehat{H}} \leqslant nT \leqslant m_{\widehat{H}} + \beta - \varepsilon_{\widehat{H}}\sigma_{\widehat{H}} \tag{25}$$

5.2.3. Model-IIB: model without shortages

As before, taking $r' = 1$ in $t_s = r' \frac{H}{n}$ of Model-IIA, expressions of the model without shortages are also obtained.

Now the profit function Eq. (23) and the corresponding expressions for the models – IB, IIA and IIB are maximized using the gradient based non-linear optimization technique – GRG for some sets of data.

6. Numerical experiments

For illustration, we use following data for Model-IA:

D_0 = 100 units/year, p_0 = \$16, γ = 0.08, γ_1 = 0.03, I_e = \$0.05/dollar/year, I_b = \$0.07/dollar/year, α = 0.02, k_2 = 0.03, m = 1.4, c_1= \$0.7/unit/year, c_2 = \$1.3/unit/year, r' = 0.8, ξ = 4, c_3 = \$100/order and H = 12 months.

Considering r' = 1, with the above numerical parameters the results of Model-IB are also obtained.

Considering $\beta = 0.1$, $m_{\widehat{H}} = 12$, $\sigma_{\widehat{H}} = 0.6$, $\varepsilon_{\widehat{H}} = 0.4$, together with the above numerical parameters the results of Model-IIA are derived. Again, with these data and taking r' = 1, outputs of Model-IIB are evaluated.

Table 1
Optimum results with and without shortages for inventory models with crisp time horizon (i.e., Model IA and Model IB).

Model	Part payment	n	A	T	M	THC	Q_1	TQ	TIE	TIP	Z
IA	With	3	3068.22	4	3.34	836.16	746.33	933.14	5791.46	2577.31	11931.49
	Without	9	0.00	1.33	0.33	278.11	744.86	931.15	1440.05	0.00	6874.00
IB	With	4	2273.89	3.00	2.47	979.31	932.40	932.40	5032.98	1910.07	11671.19
	Without	9	0.00	1.33	0.33	434.60	931.15	931.15	1641.61	0.00	6951.36

Table 2
Optimum results with and without shortages for inventory models with random time horizon (i.e., Model IIA and Model IIB).

Model	Part payment	n	A	T	M	THC	Q_1	TQ	TIE	TIP	Z
IIA	With	4	2246.26	2.97	2.44	612.10	737.11	921.56	4859.74	1864.85	11706.40
	Without	9	0.00	1.32	0.33	271.68	736.22	920.34	1405.76	0.00	6761.25
IIB	With	4	2246.26	2.97	2.44	956.63	921.56	921.56	4909.04	1864.85	11484.28
	Without	9	0.00	1.32	0.33	424.54	920.34	920.34	1602.40	0.00	6836.57

Table 3
Profit with different shortage period for the crisp inventory model (i.e., effect of r' on profit – Model IA).

r'	Z	A	n	M	THC	$TSHC$	TIE
0.73	12014.18	3068.22	3	3.34	696.16	177.00	5814.02
0.74	12003.24	3068.22	3	3.34	715.37	164.14	5809.43
0.77	11968.68	3068.22	3	3.34	774.59	128.45	5798.41
0.78	11956.58	3068.22	3	3.34	794.85	117.53	5795.64
0.80	11931.49	3068.22	3	3.34	836.16	97.13	5791.46
0.81	11918.50	3068.22	3	3.34	857.21	87.66	5790.06
0.82	11905.22	3068.22	3	3.34	878.52	78.68	5789.10
0.88	11819.39	3068.22	3	3.34	1011.89	34.94	5792.93
0.89	11807.13	2273.89	4	2.47	775.61	22.02	4987.24
0.93	11760.73	2273.89	4	2.47	846.93	8.92	4999.06
0.97	11710.86	2273.89	4	2.47	921.40	1.64	5016.39
0.98	11697.86	2273.89	4	2.47	940.51	0.73	5021.57
1.00	11761.19	2273.89	4	2.47	979.31	0.00	5032.99

Table 4
Variation of profit with different $\varepsilon_{\widehat{H}}$ and $\sigma_{\widehat{H}}$ parameters in random time horizon for stochastic inventory model (i.e., effect of $\varepsilon_{\widehat{H}}$ and $\sigma_{\widehat{H}}$ on profit – Model IIA).

$\varepsilon_{\widehat{H}}$	$\sigma_{\widehat{H}}$	Z	A	nT	M	T	TQ
0.4	0.5	11761.29	2254.16	11.90	2.45	2.96	924.65
	0.6	11706.40	2246.26	11.86	2.44	2.97	921.56
	0.8	11597.01	2230.48	11.78	2.42	2.95	915.36
	0.9	11542.46	2222.59	11.72	2.42	2.94	912.27
	1.0	11488.00	2214.70	11.70	2.41	2.93	909.17
0.6	0.5	11624.33	2234.43	11.80	2.43	2.95	916.91
	0.6	11542.46	2222.59	11.74	2.42	2.94	912.27
	0.8	11379.40	2198.93	11.62	2.39	2.91	902.97
	0.9	11298.22	2187.10	11.56	2.38	2.89	898.33
	1.0	11217.26	2175.28	11.50	2.36	2.88	893.68
0.8	0.5	11488.00	2214.70	11.70	2.41	2.93	909.17
	0.6	11379.40	2198.93	11.62	2.39	2.91	902.97
	0.8	11163.41	2167.39	11.46	2.36	2.87	890.58
	0.9	11056.02	2151.63	11.38	2.34	2.85	884.39
	1.0	10949.04	2135.88	11.30	2.32	2.83	878.19

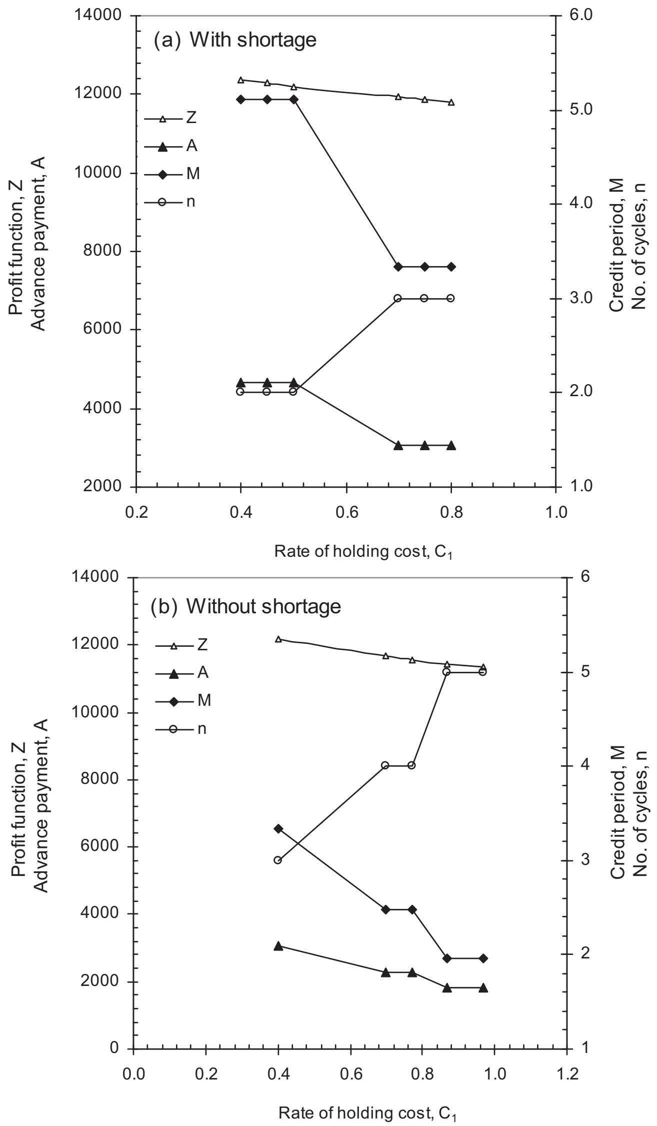

Fig. 2. Effect of holding cost per unit (c_1) on profit function, immediate part payment, number of cycle and credit period: (a) with shortages and (b) without shortages.

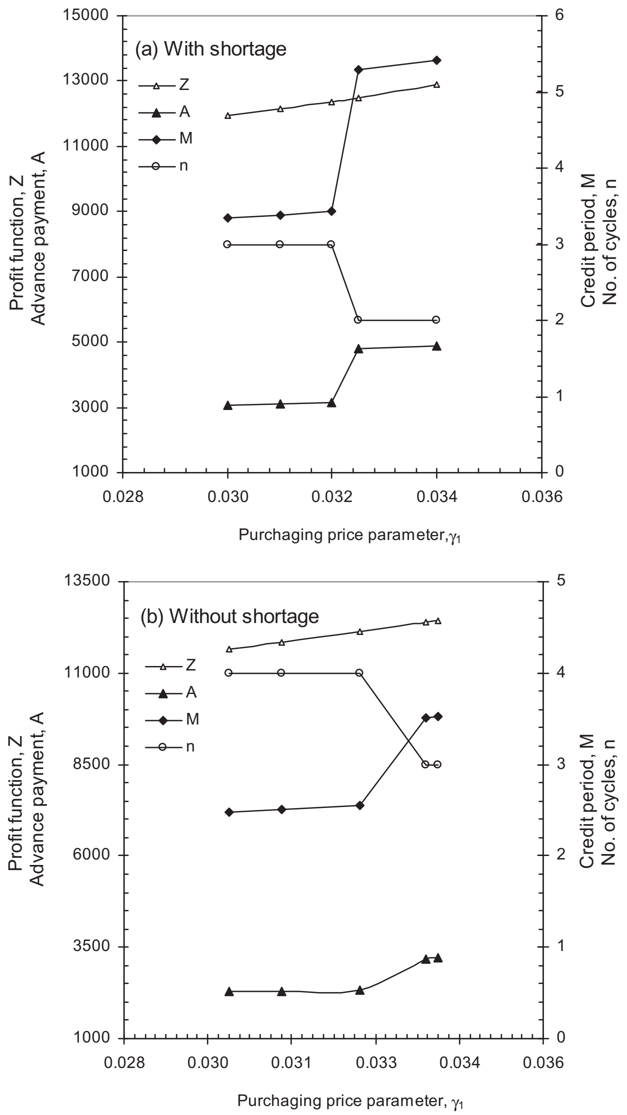

Fig. 3. Effect of purchasing price parameter (γ_1) on profit function, immediate part payment, number of cycle and credit period: (a) with shortages and (b) without shortages.

For the above numerical parameters, as mentioned earlier, the optimum results of Models-IA, IB, IIA, IIB with part payment are obtained and presented in Tables 1 and 2 respectively using the Generalized Reduced Gradient technique for optimization. LINGO optimizer tool has been used to solve all the single objective optimization problems. As particular cases, the results of the models without part payment are also presented in Tables 1 and 2.

7. Results and discussions

Optimal solutions for Model-IA and IB are given in Table 1 for above set of input parameters. From Table 1, it is observed that the maximum profit obtained for the case with immediate part payment is more than the case in which there is no advanced payment to wholesaler for both models–IA/ IB i.e., with/without shortages. In the case with immediate part payment, the optimal number of replenishment (n) = 3 and profit (Z)= $11931.49 when there is shortages and the optimal number of replenishment (n) and profit (Z) are found to be 4 and $11671.19 respectively when there is no shortages and the optimum amount given as immediate part payment for two cases are $3068.22 and $2273.90 respectively in each cycle. As expected the retailer's profit is more when there is no shortage for the case without immediate part payment. For the case with immediate part payment, as the credit period is enhanced depending on the amount of immediate part payment the retailer earns more interest and pays less amount as holding cost when there is shortages. Therefore, retailer earns more profit under fully backlogged shortages as compare to the cases where there is no allowable shortages.

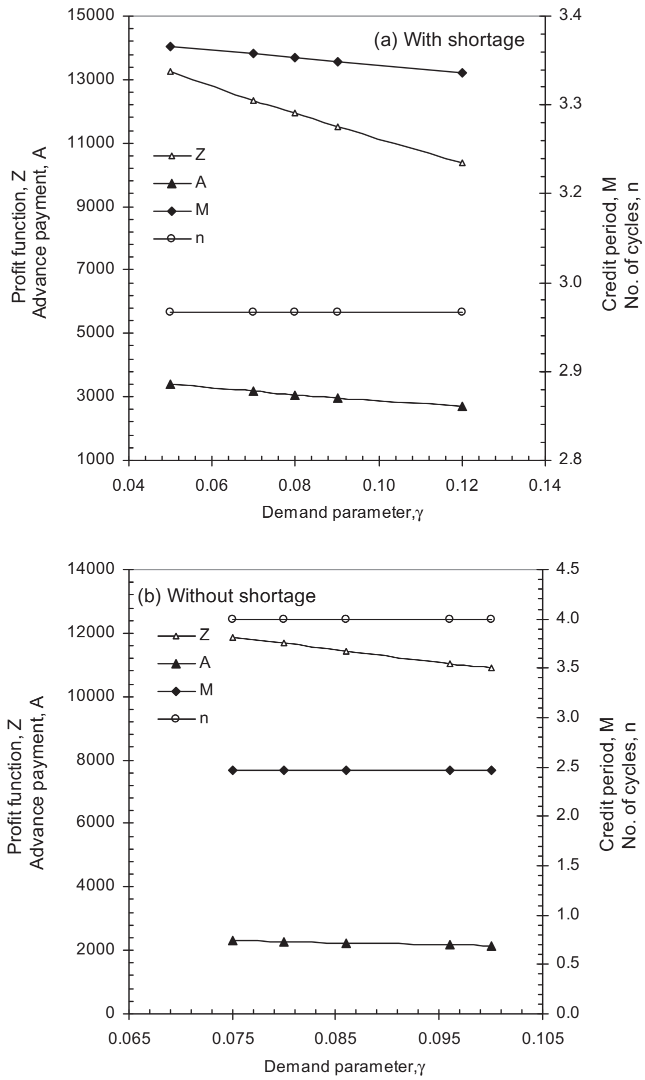

Fig. 4. Effect of demand parameter (γ) on profit function, immediate part payment, number of cycle and credit period: (a) with shortages and (b) without shortages.

Similarly optimal solution for Models-IIA and IIB are shown in Table 2. From Table 2, it is observed that in random horizon the above mentioned similar trend is followed. The maximum profit obtained for the case with immediate part payment is more than the case in which there is no advanced payment to wholesaler for both models–IIA, IIB i.e., with/without shortages. In the case with immediate part payment, the optimal number of replenishment (n) and profit (Z) are found to be 4 and $11706.40 respectively when there are shortages. If there are no shortages, the optimal number of replenishment (n) and profit (Z) are obtained as 4 and $11484.28 respectively. It is noted that the optimum amounts given as immediate part payment for both the cases are $2246.26 in each cycle. It is also seen that the retailer's profit is more when there is no shortage for both the case without immediate part payment.

In Table 3, with different shortage periods in finite time horizon crisp inventory model (i.e., effect of r' on profit function of Model-IA), values of maximum profit are presented. It is noted from Table 3 that as the period of shortages decreases (i.e., r' increases), the total profit decreases as expected. It is also noted that with the increase in r'optimal values of immediate part payment and number of cycles attain constant value i.e., $ 3068.22 and 3 respectively and remain unchanged up to $r' = 0.88$ as in Model-IA. Further increase in r' beyond 0.88, optimal values of immediate part payment and number of cycles attain $ 2273.89 and 4 respectively with gradual decrease in profit. Total profit decreases with r' and becomes $ 11671.19, when$r' = 1$, i.e., the Model-IB without allowable fully backlogged shortages. This is as per expectation for the present model.

Table 4 presents the values of profit function with different $\varepsilon_{\widehat{H}}$ and $\sigma_{\widehat{H}}$ in random time horizon (i.e., effects of $\varepsilon_{\widehat{H}}$ and $\sigma_{\widehat{H}}$ on profit: Model-IIA).

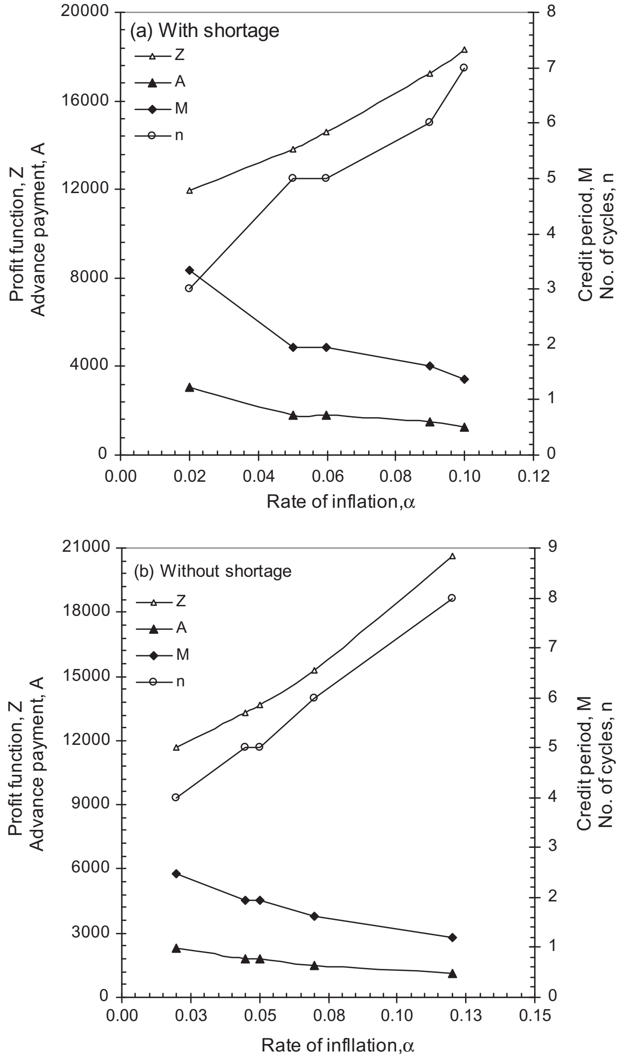

Fig. 5. Effect of inflation rate (α) on profit function, immediate part payment, number of cycle and credit period: (a) with shortages and (b) without shortages.

From Table 4, it is revealed that as the total time horizon is nearer to 12.0 (the crisp value of H), profit becomes maximum. Here, as the dispersion about the mean value of $\widehat{H}(\sigma_{\widehat{H}})$ is small, time horizon is nearer to 12.0 and hence the profit is maximum. Thus with the increase in dispersion, profit decreases. Same trend is observed with the variation of $\varepsilon_{\widehat{H}}$. Smaller $\varepsilon_{\widehat{H}}$ indicates that the chance constraint (24) is strictly satisfied i.e., the random planning horizon is approximated more nearer to the mean value (=12.0) and hence the profit is more. Therefore, all these trends are as per the formulation of the model.

8. Sensitivity analyses

The effects of change in the rate of holding cost c_1 on the number of replenishment (n), the immediate part payment (A), credit period (M) and the profit (Z) are studied and presented graphically in Fig. 2 (a: with shortage and b: without shortage). From these graphs, it is observed that with the increase in c_1, optimum number of replenishment n increases, but the immediate part payment (A), the credit period (M) and the profit (Z) decrease with the increase in holding cost. This behavior is as per expectation. As unit holding cost is high, the system will try to minimize the level of stock and as a result, if the stock level is less in each cycle, the number of cycles will be more. High holding cost also results less profit. It is seen that the immediate part payment and hence the credit period also reduce as credit period is directly proportional to the immediate part payment.

The effects of change in purchasing price parameter (γ_1) on the number of replenishment (n), the immediate part payment (A), the credit period (M) and the profit (Z) are studied and presented in Fig. 3 (a: with shortages and b: without shortages). Here, as γ_1 increases, optimum number of replenishment (n) decrease but the advance payment (A), credit period (M)

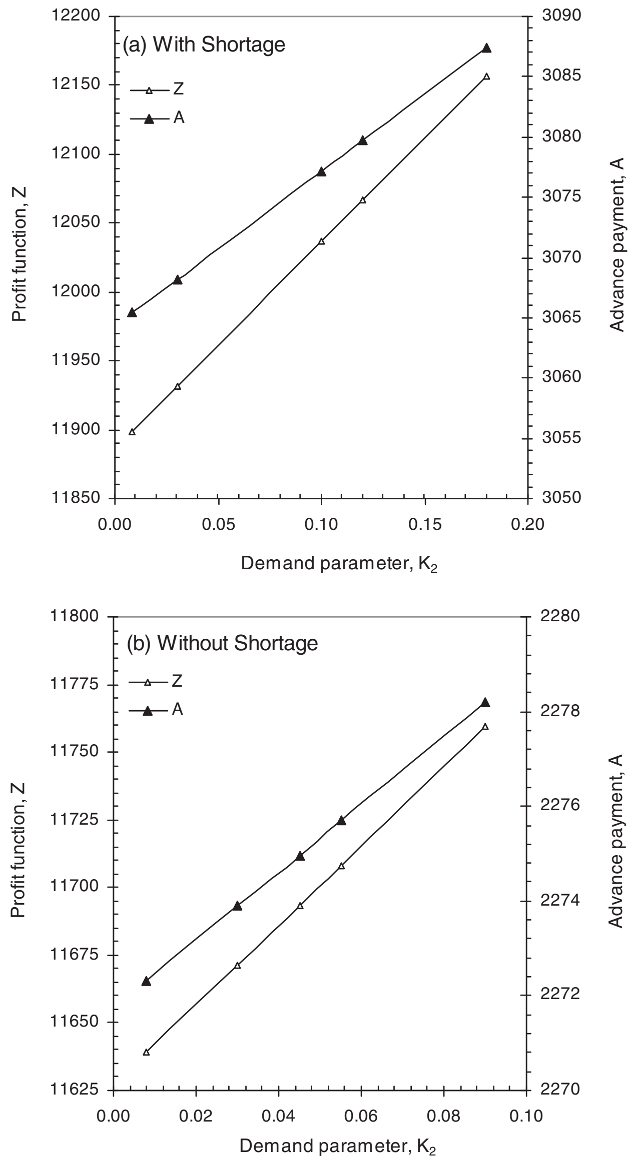

Fig. 6. Effect of demand parameter (k_2) on profit function and immediate part payment: (a) with shortages and (b) without shortages.

and the profit (Z) increase. The effect of change in γ_1 is significant when $\gamma_1 \geqslant 0.03$ [Fig. 3a, b]. As γ_1 is related to $p_i\left[p_i = \frac{p_0 e^{\alpha t_{i-1}}}{A^{\gamma_1}}\right]$, with the increase in γ_1, the reduced purchasing price including p_1 (for the first cycle) decreases and as selling price depends on the prevailing market price (p_0), the total profit is obviously more. It is observed that with the increase in γ_1 the immediate part payment increases and hence the credit period also increases as credit period is directly proportional to the immediate part payment.

The effects of change in the demand parameter (γ) on the number of replenishment (n), the immediate part payment (A), credit period (M) and the profit (Z) are studied and presented in Fig. 4 (a: with shortages and b: without shortages). From these graphs, it is observed that, when the value of γ increases, the profit and the immediate part payment decrease with same number of replenishment and almost same value of credit period, but the effect of changes in γ is significant, as with the increase in γ demand decreases and hence the optimum profit and advance payment decrease.

The effects of change in the rate of inflation (α) on the number of replenishment (n), the immediate part payment (A), credit period (M) and the profit (Z) are studied and presented in Fig. 5 (a: with shortages and b: without shortages). From these graphs, it is observed that, when the value of α increases, optimum number of replenishment (n) and profit also increase, but the immediate part payment (A) and hence credit period decreases. With the increase in α, i.e., the inflation rate, the demand increases as it directly depends on α and hence total amount of procurement will increase and as a result the No. of replenishment and total profit increase.

The effects of change in the demand parameter k_2 on the immediate part payment (A) and the profit (Z) are studied and presented in Fig. 6 (a: with shortages and b: without shortages). From these graphs, it is observed that, when the value of k_2 increases, the immediate part payment (A) and profit (Z) increases, With the increase in k_2, the demand also increases as it directly depends on α and hence total amount of procurement will increase and as a result the total profit increase.

9. Conclusions

This study introduces the concept of part payment by the retailer in an inventory control system against the concessions in credit period and unit purchasing price in both deterministic and random planning horizon under inflation allowing and not allowing shortages. It presents deterministic inventory models for inflation rate and selling price dependent consumption rate under a situation in which the wholesaler offers a relaxed credit period to settle-down the account for purchased quantities and reduced unit purchasing cost against an immediate part payment paid on the receipt of goods by the retailer who, in this process, makes loan from the money lending source. Numerical results for all models with/without immediate part payment show that immediate part payment plays a vital role for the retailer to earn more profit. The intuitive reason is that, when the credit period increases and purchasing cost per unit reduces, the retailer earns more by taking loan from money lending source and also is encouraged to hold more inventories which extends the cycle time. Hence this analysis answers to the retailer's dilemma how much to make for immediate part payment to enjoy the wholesaler's concessions for maximum profit in spite of the fact that more part payment means more loan and more interest paid. Here, the models without shortages have been derived from the models with shortages as a particular case. These general formulations may be useful in the literature. With the above type of conjectures on demand, advance payment, etc., two warehouse inventory models and supply-chain models can also be formulated and solved.

Appendix A

$$\int_{t_{i-1}}^{t_{i-1}+M} s_i D_i(t)\,dt = \int_{t_{i-1}}^{t_{i-1}+M} s_i \frac{D_0 e^{\alpha_1 t}}{s_i^{\gamma}}\,dt = \frac{s_i^{1-\gamma} D_0}{\alpha_1}\left[e^{\alpha_1(t_{i-1}+M)} - e^{\alpha_1 t_{i-1}}\right] \tag{A.1}$$

Also, $$\int_{t_{i-1}+M}^{t_{i-1}+t_s} s_i D_i(t)\,dt = \frac{s_i^{1-\gamma} D_0}{\alpha_1}\left[e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1(t_{i-1}+M)}\right] \tag{A.2}$$

and $$\int_{t_{i-1}}^{t_{i-1}+t_s} s_i D_i t\,dt = s_i^{1-\gamma} D_0 \int_{t_{i-1}}^{t_{i-1}+t_s} t e^{\alpha_1 t}\,dt = \frac{s_i^{1-\gamma} D_0}{\alpha_1}\left[(t_{i-1}+t_s)e^{\alpha_1(t_{i-1}+t_s)} - t_{i-1}e^{\alpha_1 t_{i-1}} - \frac{1}{\alpha_1}\left(e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t_{i-1}}\right)\right] \tag{A.3}$$

Therefore, using Eqs. (A.1)–(A.3) the interest earned ($IECS_i$) by continuous sale during the interval (t_{i-1}, t_i) is given by

$$\begin{aligned}
IECS_i &= I_e\left[(t_{i-1}+M)\int_{t_{i-1}}^{t_{i-1}+M} s_i D_i\,dt + (t_{i-1}+t_s)\int_{t_{i-1}+M}^{t_{i-1}+t_s} s_i D_i\,dt - \int_{t_{i-1}}^{t_{i-1}+t_s} s_i D_i t\,dt\right] \\
&= I_e\left[\begin{array}{l}(t_{i-1}+M)\frac{s_i^{1-\gamma}D_0}{\alpha_1}\left[e^{\alpha_1(t_{i-1}+M)} - e^{\alpha_1 t_{i-1}}\right] + (t_{i-1}+t_s)\frac{s_i^{1-\gamma}D_0}{\alpha_1}\left[e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1(t_{i-1}+M)}\right] \\ -\frac{s_i^{1-\gamma}D_0}{\alpha_1}\left[(t_{i-1}+t_s)e^{\alpha_1(t_{i-1}+t_s)} - t_{i-1}e^{\alpha_1 t_{i-1}} - \frac{1}{\alpha_1}\left(e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t_{i-1}}\right)\right]\end{array}\right] \\
&= I_e\frac{s_i^{1-\gamma}D_0}{\alpha_1}\left[(M-t_s)e^{\alpha_1(t_{i-1}+M)} - Me^{\alpha_1 t_{i-1}} + \frac{1}{\alpha_1}\left(e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t_{i-1}}\right)\right] \\
&= I_e\frac{s_i^{1-\gamma}D_0}{\alpha_1}\left[(M-t_s)e^{\alpha_1 M}e^{\alpha_1 t_{i-1}} - Me^{\alpha_1 t_{i-1}} + \frac{1}{\alpha_1}\left(e^{\alpha_1 t_s}e^{\alpha_1 t_{i-1}} - e^{\alpha_1 t_{i-1}}\right)\right] \\
&= I_e\frac{(mp_0)^{1-\gamma}D_0}{\alpha_1}\left[\left(M - r'\frac{H}{n}\right)e^{\alpha_1 M} - M + \frac{1}{\alpha_1}\left(e^{\alpha_1 r'\frac{H}{n}} - 1\right)\right]e^{(\alpha_1+\alpha(1-\gamma))t_{i-1}}
\end{aligned} \tag{A.4}$$

Appendix B

Here, we use the following notations to calculate the interest earned (IET_n) from the revenue deposited by already sale units after payment of wholesaler at the credit period and immediate part payment for all cycles except first cycle up to t_n.

(i) IET_i: the total interest earned up to time t_i from the revenue deposited at the credit period (M) of the ith cycle (t_{i-1},t_i).

(ii) IEM_i: the total interest earned up to the credit period M of the ith cycle (t_{i-1}, t_i) from the revenue deposited at the time t_{i-1}.

Now we have from the system that

$$IEM_1 = 0 \tag{B.1}$$

$$IET_1 = 0 + M'I_e\left[\int_{t_0}^{t_0+M} s_1 D_1\,dt - (Q_1p_1 - A)\right] \tag{B.2}$$

$$IEM_2 = IET_1 + MI_e[TS_1 - (Q_1p_1 - A) - A] = M'I_e\left[\int_{t_0}^{t_0+M} s_1 D_1\,dt - (Q_1p_1 - A)\right] + MI_e[TS_1 - Q_1p_1] \tag{B.3}$$

$$\begin{aligned} IET_2 &= IEM_2 + M'I_e\left[TS_1 - Q_1p_1 + \textstyle\int_{t_1}^{t_1+M} s_2D_2\,dt - (Q_2p_2 - A)\right] \\ &= M'I_e\left[\textstyle\int_{t_0}^{t_0+M} s_1D_1\,dt - (Q_1p_1 - A)\right] + MI_e(TS_1 - Q_1p_1) + M'I_e\left[TS_1 - Q_1p_1 + \textstyle\int_{t_1}^{t_1+M} s_2D_2\,dt - (Q_2p_2 - A)\right] \\ &= I_e\left[M'\textstyle\int_{t_0}^{t_0+M} s_1D_1\,dt + M'\textstyle\int_{t_1}^{t_1+M} s_2D_2\,dt - M'(Q_1p_1 - A) - M'(Q_2p_2 - A) + M(TS_1 - Q_1p_1) + M'(TS_1 - Q_1p_1)\right] \end{aligned} \tag{B.4}$$

$$\begin{aligned} IEM_3 &= IET_2 + MI_e[TS_1 - (Q_1p_1 - A) - A + TS_2 - (Q_2p_2 - A) - A] \\ &= I_e\begin{bmatrix} M'\int_{t_0}^{t_0+M} s_1D_1\,dt + M'\int_{t_1}^{t_1+M} s_2D_2\,dt - M'(Q_1p_1 - A) - M'(Q_2p_2 - A) + M(TS_1 - Q_1p_1) + M'(TS_1 - Q_1p_1) \\ +M[TS_1 - (Q_1p_1 - A) - A + TS_2 - (Q_2p_2 - A) - A] \end{bmatrix} \\ &= I_e\begin{bmatrix} M'\int_{t_0}^{t_0+M} s_1D_1\,dt + M'\int_{t_1}^{t_1+M} s_2D_2\,dt - M'(Q_1p_1 - A) - M'(Q_2p_2 - A) + M(TS_1 - Q_1p_1) + M'(TS_1 - Q_1p_1) \\ +M(TS_1 - Q_1p_1) + M(TS_2 - Q_2p_2) \end{bmatrix} \end{aligned} \tag{B.5}$$

$$\begin{aligned} IET_3 &= IEM_3 + M'I_e\left[TS_1 - Q_1p_1 + TS_2 - Q_2p_2 + \textstyle\int_{t_2}^{t_2+M} s_3D_3\,dt - (Q_3p_3 - A)\right] \\ &= I_e\begin{bmatrix} M'\int_{t_0}^{t_0+M} s_1D_1\,dt + M'\int_{t_1}^{t_1+M} s_2D_2\,dt - M'(Q_1p_1 - A) - M'(Q_2p_2 - A) + M(TS_1 - Q_1p_1) + M'(TS_1 - Q_1p_1) \\ +M(TS_1 - Q_1p_1) + M(TS_2 - Q_2p_2) \end{bmatrix} \\ &\quad + M'I_e\left[TS_1 - Q_1p_1 + TS_2 - Q_2p_2 + \textstyle\int_{t_2}^{t_2+M} s_3D_3\,dt - (Q_3p_3 - A)\right] \\ &= I_e\begin{bmatrix} M'\int_{t_0}^{t_0+M} s_1D_1\,dt + M'\int_{t_1}^{t_1+M} s_2D_2\,dt + M'\int_{t_2}^{t_2+M} s_3D_3\,dt - M'(Q_1p_1 - A) - M'(Q_2p_2 - A) - M'(Q_3p_3 - A) \\ +M(TS_1 - Q_1p_1) + M'(TS_1 - Q_1p_1) + M(TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + M'(TS_1 + TS_2 - Q_1p_1 - Q_2p_2) \end{bmatrix} \end{aligned} \tag{B.6}$$

In this way,

$$IET_n = I_e\begin{bmatrix} M'\begin{Bmatrix} \int_{t_0}^{t_0+M} s_1D_1\,dt + \int_{t_1}^{t_1+M} s_2D_2\,dt + \int_{t_2}^{t_2+M} s_3D_3\,dt + \ldots\ldots\ldots\ldots + \int_{t_{n-1}}^{t_{n-1}+M} s_nD_n\,dt \\ -(Q_1p_1 - A) - (Q_2p_2 - A) - \ldots\ldots\ldots\ldots - (Q_np_n - A) \end{Bmatrix} \\ +M\begin{bmatrix} (TS_1 - Q_1p_1) + (TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + \ldots\ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots\ldots + TS_{n-1} - Q_1p_1 - Q_2p_2 - \ldots\ldots\ldots - Q_{n-1}p_{n-1}) \end{bmatrix} \\ +M'\begin{bmatrix} (TS_1 - Q_1p_1) + (TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + \ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots\ldots + TS_{n-1} - Q_1p_1 - Q_2p_2 - \ldots\ldots\ldots - Q_{n-1}p_{n-1}) \end{bmatrix} \end{bmatrix}$$

$$= I_e\begin{bmatrix} M'\begin{Bmatrix} \int_{t_0}^{t_0+M} s_1D_1\,dt + \int_{t_1}^{t_1+M} s_2D_2\,dt + \int_{t_2}^{t_2+M} s_3D_3\,dt + \ldots\ldots\ldots\ldots + \int_{t_{n-1}}^{t_{n-1}+M} s_nD_n\,dt \\ -(Q_1p_1 + Q_2p_2 + Q_3p_3 + \ldots\ldots\ldots\ldots + Q_np_n) + nA \end{Bmatrix} \\ +M\begin{bmatrix} (TS_1 - Q_1p_1) + (TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + \ldots\ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots\ldots + TS_{n-1} - Q_1p_1 - Q_2p_2 - \ldots\ldots\ldots - Q_{n-1}p_{n-1}) \end{bmatrix} \\ +M'\begin{bmatrix} (TS_1 - Q_1p_1) + (TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + \ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots\ldots + TS_{n-1} - Q_1p_1 - Q_2p_2 - \ldots\ldots\ldots - Q_{n-1}p_{n-1}) \end{bmatrix} \end{bmatrix}$$

$$= I_e\begin{bmatrix} M'\begin{Bmatrix} \int_{t_0}^{t_0+M} s_1D_1\,dt + \int_{t_1}^{t_1+M} s_2D_2\,dt + \int_{t_2}^{t_2+M} s_3D_3\,dt + \ldots\ldots\ldots\ldots + \int_{t_{n-1}}^{t_{n-1}+M} s_nD_n\,dt \\ -(Q_1p_1 + Q_2p_2 + Q_3p_3 + \ldots\ldots\ldots\ldots + Q_np_n) + nA \end{Bmatrix} \\ +M\begin{bmatrix} (TS_1 - Q_1p_1) + (TS_1 + TS_2 - Q_1p_1 - Q_2p_2) + \ldots\ldots\ldots\ldots \\ +(TS_1 + TS_2 + \ldots\ldots\ldots + TS_{n-1} - Q_1p_1 - Q_2p_2 - \ldots\ldots\ldots - Q_{n-1}p_{n-1}) \end{bmatrix} \\ +M'\begin{bmatrix} (n-1)TS_1 + (n-2)TS_2 + (n-3)TS_3 + \ldots\ldots + TS_{n-1} \\ -(n-1)Q_1p_1 - (n-2)Q_2p_2 - (n-3)Q_3p_3 - \ldots\ldots - Q_{n-1}p_{n-1} \end{bmatrix} \end{bmatrix}$$

$$= I_e \left[\begin{array}{l} M' \left\{ \begin{array}{l} \int_{t_0}^{t_0+M} s_1 D_1 \, dt + \int_{t_1}^{t_1+M} s_2 D_2 \, dt + \int_{t_2}^{t_2+M} s_3 D_3 \, dt + \ldots\ldots\ldots.. + \int_{t_{n-1}}^{t_{n-1}+M} s_n D_n \, dt \\ -(Q_1 p_1 + Q_2 p_2 + Q_3 p_3 + \ldots\ldots\ldots.. + Q_n p_n) + nA \end{array} \right\} \\ +(M+M') \left\{ \begin{array}{l} (n-1)TS_1 + (n-2)TS_2 + (n-3)TS_3 + \ldots.. + TS_{n-1} \\ -(n-1)Q_1 p_1 - (n-2)Q_2 p_2 - (n-3)Q_3 p_3 - \ldots\ldots - Q_{n-1} p_{n-1} \end{array} \right\} \end{array} \right]$$

$$= I_e \left[\begin{array}{l} M' \left\{ \sum_{j=1}^{n} \int_{t_{j-1}}^{t_{j-1}+M} s_j D_j \, dt - \sum_{j=1}^{n} Q_j p_j + nA \right\} \\ +(M+M') \left\{ \begin{array}{l} (n-1)TS_1 + (n-2)TS_2 + (n-3)TS_3 + \ldots.. + TS_{n-1} \\ -(n-1)Q_1 p_1 - (n-2)Q_2 p_2 - (n-3)Q_3 p_3 - \ldots\ldots - Q_{n-1} p_{n-1} \end{array} \right\} \end{array} \right] \quad (B.7)$$

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