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A hybrid approach of goal programming for weapon systems selection ☆

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ABSTRACT

Because weapon systems are perceived as crucial in determining the outcome of a war, selecting weapon systems is a critical task for nations. Just as with other forms of decision analysis involving multiple criteria, selecting a weapon system poses complex, unstructured problems with a huge number of points that must be considered. Some defense analysts have committed themselves to developing efficient methodologies to solve weapon systems selection problems for the Republic of Korea's (ROK) Armed Forces. In the present study, we propose a hybrid approach for weapon systems selection that combines analytic hierarchy process (AHP) and principal component analysis (PCA) to determine the weights to assign to the factors that go into these selection decisions. These weights are inputted into a goal programming (GP) model to determine the best alternative among the weapon systems. The proposed hybrid approach that combines AHP, PCA and GP process components offsets the shortcomings posed by obscurity and arbitrariness in AHP and therefore can provide decision makers with more reasonable and realistic decision criteria than AHP alone. A case study on weapon system selection for the air force demonstrates the usefulness and effectiveness of the proposed hybrid AHP–PCA–GP approach.

1. Introduction

Because weapon systems are regarded as crucial to the outcome of war, the selection of weapon systems is a critical national decision. The rapid development of military technologies makes weapon systems ever more sophisticated, expensive, and quickly accelerates research on methods for selection of these systems.

The Republic of Korea (ROK) ministry of national defense (MND) has been raising its force investment budget to more than 30% of its defense budget, most of which is for weapon systems procurement (Ministry of National Defense of Republic of Korea, 2008). Moreover, along with the general trend of pursuing efficiency and rationality in budgeting and expenditures, the MND also has been under pressure to prepare its defense budget with the same transparency and efficiency of every other governmental expenditure. In fact, the ROK MND and the joint chief of staff (JCS) have been evaluating concepts and methodologies to improve efficiency in military affairs, especially in procurement of weapon systems. This effort, however, has been mainly undertaken in a way that relies on the intuition of high-level commanders for critical decisions rather than on a systematic decision making process. This has often led to subsequent changes and revisions that waste the defense budget and delay procurement of weapon systems.

These waste and delay have occurred despite significant trials and developments aimed at methods to determine force requirements efficiently and improve force structure. This is because the efforts mainly focused on conceptual, macroscopic and ambiguous projects that yielded nothing more than indications of the general direction that force development should take. This current situation demands that analysts proceed to develop concrete and tangible methods for the selection of weapon systems.

Like most real-world decision making problems, the selection of a weapon systems requires a multiple criteria decision analysis (MCDA). Ho (2007) classified MCDAs into two technical categories, multiple objective decision making (MODM) and multiple attribute decision making (MADM). MODM is mathematical programming that has multiple objective functions and constraints. When an MCDA involves a number of independent or competing objectives, a multi-criteria mathematical programming approach is useful because it forces the simultaneous resolution of various objectives. Goal programming (GP) is an example of MODM.

MADM selects the best alternative among the various attributes that are to be considered. One of the most popular MADM techniques includes AHP. AHP structurally combines tangible and intangible criteria with alternatives in decision making. AHP logically integrates the judgment, experience, and intuition of decision makers. Because of its usability and flexibility, AHP has

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been widely applied to complex and unstructured decision making problems such as resource allocation, alternative selection, manufacturing, and military decision making. Recently, the analytic network process has been developed to handle decision problems that are not hierarchically structured (Saaty, 2005). Further, the fuzzy AHP is introduced to facilitate decisions under fuzzy situations (Kong & Liu, 2005).

A number of studies have integrated MADM and MODM. These studies have included a combined AHP-mathematical programming approach. Vaidya and Kumar (2006) performed a comprehensive review of integrating AHP with a variety of applications. Further, Ho (2008) surveyed numerous studies that investigated AHP-mathematical programming approaches. Many articles are available on combining AHP and GP. Some applied an AHP–GP combination to university resource allocation (Kwak & Lee, 1998), media selection in the consumer/industrial market (Kwak, Lee, & Kim, 2005), software architecture selection (Reddy, Naidu, & Govindarajulu, 2007), resource planning for health-care system (Kwak & Lee, 2002; Lee & Kwak, 1999), computer-integrated manufacturing technology selection (Yurdakul, 2004), selection of cost drivers (Schniederjans & Garvin, 1997), nuclear fuel cycle scenario selection (Kim, Lee, & Lee, 1999), maintenance selection problems (Bertolini & Bevilacqua, 2006), and supply chain selection (Wang, Huang, & Dismukes, 2004, 2005). On weapon system projects, some researchers applied combined approaches such as a hybrid AHP-integer programming approach to screen weapon systems projects (Greiner, Fowler, Shunk, Carlyle, & McNutt, 2003), an AHP approach based on linguistic variable weights (Cheng & Lin, 2002; Cheng, Yang, & Hwang, 1999), and an approach that integrated AHP with a technique for ordering performance by comparing alternatives to an ideal solution under a fuzzy environment (Dagdeviren, Yavuz, & Kılın, 2009).

Despite the usefulness of AHP, its limitation lies in its overreliance on the intuition of decision makers. Existing AHP–GP models are limited in their ability to cope with both tangible data and intangible intuitive factors. Weapon systems selection, considered the terminal stage of a force requirement decision process, has an enormous spectrum of criteria and data that should be taken into account. Until now, however, analysts have made limited use of these data. It may be unreasonable to use AHP with only intangible factors and weighted decision elements when tangible real data are available that can be incorporated into the decision making.

To overcome the shortcomings posed by the obscurity and arbitrariness in AHP, the present study integrates principal component analysis (PCA) and AHP to determine the weights in a GP model. In contrast with AHP, the weights obtained from PCA are derived from actual data, and more weight is assigned to criteria that have more information. In our proposed hybrid AHP–PCA–GP model, tangible real data are elevated to an equal position with intangible intuition; thus, both have the same importance in producing the weights integrated into decision elements. This scheme pursues reality and rationality by actively using the real data in the decision process.

2. Analytical methods

2.1. Analytic hierarchy process

AHP, introduced by Saaty (1980), designs general decision problems based on a multilevel hierarchy of goals, criteria, subcriteria, and alternatives. AHP is characterized by three basic principles: hierarchical structure, the relative priority of decision criteria; and consistent judgment. It uses a pairwise comparison technique to derive the relative importance (or weight) of each criterion that reflects reasonable human judgment on elements in the same category. A pairwise comparison allows conversion of linguistic judgments into numerical scales. When the importance of one element to another can be expressed as a scale of 1–9, scale 1 means the two elements are of equal importance, and scale 9 means one is extremely more important than the other. Pairwise comparison helps decision makers simplify a complex problem by focusing their interest on the comparison of just two criteria and improves their consistency across the decision process (Badri, 1999, 2001).

Judgment by pairwise comparison produces a reciprocal matrix A, represented as follows:

$$A = \begin{pmatrix} a_{11} & a_{12}\dots & a_{1n} \\ a_{21} & \ddots & \vdots \\ \vdots & & \\ a_{n1} & \dots & a_{nn} \end{pmatrix}.$$

Each entry of A represents the relative importance of decision elements. For example, a_{ij} is the relative importance in decision element i against decision element j, and vice versa. It satisfies $a_{ij} = 1/a_{ji}$. The actual relative weights of decision elements can be obtained by computing the normalized eigenvector of A that satisfies the following equation:

$$A \cdot w = \lambda \cdot w, \tag{1}$$

where λ is the eigenvalue associated with eigenvector w. Saaty (1980) recommended using the eigenvector, $w_{max} = [w_1, w_2, \dots, w_n]^T$ corresponding to the maximum eigenvalue, λ_{max}, to represent the relative weights of each of the n criteria.

This process should be performed at all levels of the criteria to obtain all the relative weights of the decision elements. During the process of deducing the weights, a consistency test can be performed to verify the reasonability of the decision makers' pairwise comparison. The measure of consistency is obtained by a consistency index (CI) and a consistency ratio (CR), which are defined as follows:

$$CI = \frac{\lambda_{\max} - n}{n - 1}, \tag{2}$$

$$CR = \frac{CI}{RI}, \tag{3}$$

where n is the number of decision elements, and the random consistency index (RI) is an experimental value provided by Saaty (1990) as shown in Table 1.

It can be seen that the RI increases in proportion to the order of matrix A. λ_{max} equals to n if the judgments by comparison are perfectly consistent. If the CR is less than 0.1, the judgment is consistent; if the CR is greater than 0.2, the judgment is not consistent. If the value of the CR is between 0.1 and 0.2, the judgment is acceptable (Saaty, 1990).

2.2. Principal component analysis

We apply PCA, one of the most widely used multivariate statistical techniques for dimensionality reduction, to determine the weights of the variables (Jolliffe, 2002). PCA identifies a lower dimensional variable set that can explain most of the variability of the original variable set. Given the random vector that has a sample of observations (n) for a set of p variables (i.e., $\boldsymbol{X}^T = [X_1, X_2, \dots, X_p]$) and its covariance (or correlation) matrix Σ, the first step of PCA is to transform the original variables X into linear combinations $\boldsymbol{Z} = \alpha^T\boldsymbol{X}$ that are uncorrelated, where $\boldsymbol{Z}^T = [Z_1, Z_2, \dots, Z_p]$.

Table 1
Random consistency index according to number of elements.

n	3	4	5	6	7	8	9	10
RI	0.58	0.90	1.12	1.32	1.41	1.45	1.49	1.51

Using eigenvalue analysis of Σ, these Z_i ($i = 1, \ldots p$) are ordered such that the Z_i with the highest eigenvalue of Σ corresponds to the first PC and describes the largest amount of the variability in the original data; the second highest is the second PC, etc. Significant dimension reduction can be achieved if only the first few PCs are needed to represent most of the variability in the original high-dimensional space. This is common when there is high multicollinearity among the original variables. It should be noted that if the units of the original variables are different, the data should be normalized before performing PCA (Shumueli, Patel, & Bruce, 2007).

2.3. Goal programming

GP is a multi-objective decision making approach that can effectively handle multiple independent or conflicting objectives simultaneously. As a modification and extension of linear programming, GP serves to minimize an objective function that can be defined as a combination of multidimensional absolute deviations from the target value. GP has wide usability and flexibility because of its capability to admit nonhomogeneous units of measure (Bertolini & Bevilacqua, 2006).

GP can be classified into two widely used variants that are distinguished by the way they determine weights (or priorities) and objective functions (Bertolini & Bevilacqua, 2006; Ignizio, 1980). First, the preemptive GP model, also known as the lexicographic GP model, is formed when goals are clearly ranked and deviation variables are ranked. Second, the weighted GP model, also called the nonpreemptive GP model, attempts to minimize the total weighted deviations from all goals. This latter model is useful when the relative weights of decision elements for goals are available.

In the present study, we used the weighted GP model with additional hard constraints and integer decision variables that can be formulated as follows (Min & Storbeck, 1991):

$$\begin{aligned}
&\mathit{Min} \quad Z = \sum_{i=1}^{m} (w_i^+ \cdot d_i^+ + w_i^- \cdot d_i^-) \\
&\text{Subject to} \quad \sum\nolimits_{i=1}^{m} a_{ij} \cdot x_j - d_i^+ + d_i^- = g_i; \quad j = 1, 2, \ldots, n \\
&x_j \geqslant 0; \quad j = 1, 2, \ldots, r \\
&x_j \in Z; \quad j = r+1, r+2, \ldots, n \\
&d_i^+, d_i^- \geqslant 0; \quad i = 1, 2, \ldots, m
\end{aligned}$$

where Z is the sum of weighted deviational variables; w_i^+, w_i^-, the positive and negative relative weight of ith deviation; d_i^-, d_i^+, the positive and negative deviation variable from the ith goal; a_{ij}, the jth decision coefficient of the ith goal or hard constraint; x_j, the jth decision variable and g_i is the ith goal or target value. The deviation variables can also be represented as follows:

$$d_i^- = \left[g_i - \sum_{j=1}^{n} a_{ij} \cdot x_j \right]^+,$$

$$d_i^+ = \left[\sum_{j=1}^{n} a_{ij} \cdot x_j - g_i \right]^+,$$

where $[x]^+ = \max(x, 0)$.

The relative weights (w_i^+, w_i^-) are nonnegative real numbers in which the greater their value, the greater their relative importance of the ith goal. The weighted sum of deviation variables should be minimized to achieve an optimal solution of the problem. When the target value of a goal should not be exceeded, then it is necessary to minimize the respective positive deviation variable. Similarly, the GP problem minimizes the negative deviation variable of goals that should not be underachieved. If it is desirable to have a goal exactly match its target value, the summation of both of the deviation variables must be minimized. The GP model is in the class of NP-hard problems, so the best algorithms to solve it require exponential computational time in the worst case scenario. Branch-and-bound is the most prevalent of such algorithms and is used within most integer programming commercial software. In the special case in which there are no integer decision variables ($r = n$), the GP model can be solved using linear programming methods. Such methods include interior point methods, some of which have polynomial complexity, as well as the simplex method, which works well in practice and is also prevalent in commercial software. When solving the GP model, there may be cases in which multiple equally optimal solutions exist. Although the decision maker would likely be satisfied with any such equally optimal solution, she/he may wish to reconsider the weights in the pairwise comparison matrix of the AHP component of the method to break ties.

3. The proposed hybrid approach

Fig. 1 displays an overview of the proposed AHP–PCA–GP hybrid approach.

Setting up the problem requires decomposing all of the complex multi-criteria into a hierarchical structure in which all of the decision elements can be arranged. Decision elements consist of all the goals, criteria, subcriteria, and alternatives identified as necessary to arrive at an optimal solution to the problem. Each element should be defined and hierarchically designed so that it describes the problem realistically and helps reach a proper decision.

Data of the terminal subcriteria are then collected. All of the data should be in quantitative or metric form. If quantitative data are not available, conversion of the qualitative data into quantitative data by a suitable technique is recommended. The conversion methods include Delphi method and AHP. It should be stated that when applying this approach to a real problem, the data collection is often time-consuming and challenging.

Next, we determine weights by the AHP and PCA. For AHP, all decision makers use their experience and knowledge to conduct pairwise comparisons with those decision elements in the same

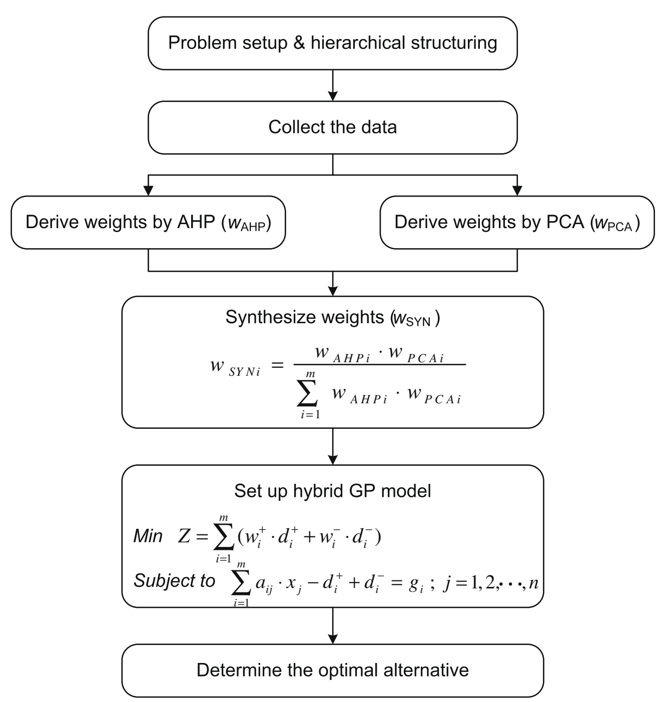

Fig. 1. Hybrid model development procedure.

cluster. Then comparison matrices are formed with each cluster to determine the relative importance of the components at the criteria and subcriteria levels. To check for consistency of judgments, consistency tests are performed in every cluster. If the consistency ratio exceeds the limit, the pairwise comparisons should be revised. After all the pairwise comparisons and consistency tests are performed at every level of criteria, the final relative weights of criteria and subcriteria are obtained by aggregating all of the decision makers' relative weights that have been derived through the above process.

In PCA, the reduced dimensions, Z_i's are each a linear combination of the original variables with the loading values (α_{ij}, $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, p$). The Z_i's can be represented as follows:

$$
\begin{aligned}
Z_1 &= \alpha_{11}X_1 + \alpha_{12}X_2 + \cdots + \alpha_{1p}X_p \\
Z_2 &= \alpha_{21}X_1 + \alpha_{22}X_2 + \cdots + \alpha_{2p}X_p \\
&\vdots \\
Z_p &= \alpha_{p1}X_1 + \alpha_{p2}X_2 + \cdots + \alpha_{pp}X_p
\end{aligned}
\tag{4}
$$

The loading values represent the importance of each variable to form a PC. For example, α_{ij} indicates the degree of importance of the jth variable in the ith PC. This idea can be extended to k PCs of interest. Determination of the number of PCs (i.e., k) to retain is subjective. Typically, the scree plot that effectively visualizes the variability of each PC is used (Johnson & Wichern, 2002). A PCA loading value for the jth original variable can be computed from the first k PCs. Thus, an overall PCA weight for jth variable can be represented as follows:

$$w_{PCAj} = \sum_{i=1}^{k} |\alpha_{ij}|\omega_i, \quad j = 1, 2, \ldots, p, \tag{5}$$

where ω_i represents the proportion of total variance explained by the ith PC. In other words, w_{PCAj} is a linear combination of k loading values weighted by their variability and thus reflect the contribution of jth variable to form k PCs.

The weights obtained from the AHP and PCA can be synthesized by the following equation to calculate the overall weight of the ith criterion, w_{SYNi}:

$$w_{SYNi} = \frac{w_{AHPi} \cdot w_{PCAi}}{\sum_{i=1}^{m} w_{AHPi} \cdot w_{PCAi}} \tag{6}$$

where m is the number of criteria.

We built a weighted zero–one GP model with the synthesized weights in an effort to select the best weapon system. This process includes defining the decision variables and their parameters and constructing constraints and objective functions. Because the deviations are measured with different units or scales, the objective functions must be normalized to ensure the comparability of any deviations from the goals.

Finally, the optimal solution (alternative) for the GP problem is obtained by using the LINGO, version 11.0 (Lindo systems, Chicago, IL), mathematical programming solution package. The optimal solution determines the best weapon system alternative that satisfies the constraints.

4. Case study

To illustrate the procedure involved in the proposed hybrid approach and to demonstrate its effectiveness, we present a case study on surface-to-air missile system selection.

4.1. Problem setup and data

The problem is designed as a hierarchical structure of four levels: First the goal of the decision problem, followed by the criteria, subcriteria, and alternative levels. As shown in Fig. 2, to select an optimal alternative, we considered six candidate missile systems as decision variables ($x_1, x_2, \ldots, x_6$) and evaluated them based on three criteria and 19 subcriteria.

Each subcriterion, identified and structured in the previous stage, has its own characteristic data about the candidate missile systems (Table 2). The criteria and characteristic data were identified by the research team on the basis of confidential materials on missile systems and Ahn's study (2003). Because of the confidentiality issue, part of the data was arbitrary but meaningfully generated.

We also have target values, or goals, for each subcriterion that should be achieved in the decision making process. Usually the JCS and Armed Forces determine the target values in the form of requirements for operational capability that describe the capabilities demanded for successful operational performance.

4.2. Hybrid AHP–PCA model

In the process of obtaining the weights by the AHP in this case study, five defense procurement experts were consulted to derive individual relative preferences through pairwise comparison. Each weight was determined by relative preference matrices that used the eigenvector method. The relative preference matrices produced by each expert were input to Expert Choice version 11.5 (Expert choice, Arlington, VA) software to establish the weights of the criteria and subcriteria. These weights were then aggregated to establish a single set of weights. A consistency test is required to verify the decision makers' judgment. Because all the CRs were less than 0.1 in the case study, the pairwise judgments were found to be reasonable.

The PCA weights of individual variables were also calculated by (5). We selected the first 5 PCs (i.e., $k = 5$) that account for 99% of the variability in the entire dataset. We used MATLAB (MathWork Inc., Natick, MA) to generate PCA weights. The resulting weights for AHP (w_{AHP}), PCA (w_{PCA}) and AHP + PCA (w_{SYN}) are shown in Table 3. For example, the synthesized weight for the acquisition cost variable (14th variable) was calculated as follows:

$$w_{SYN14} = \frac{w_{AHP14} \cdot w_{PCA14}}{\sum_{i=1}^{19} w_{AHPi} \cdot w_{PCAi}} = \frac{0.029 \cdot 0.044}{0.045} = 0.028$$

4.3. Hybrid GP model

A weighted integer GP model can be formulated with a decision variable of x_j (0 or 1) to indicate whether missile system j is selected. Because we have 19 goals to satisfy, 19 goal constraints

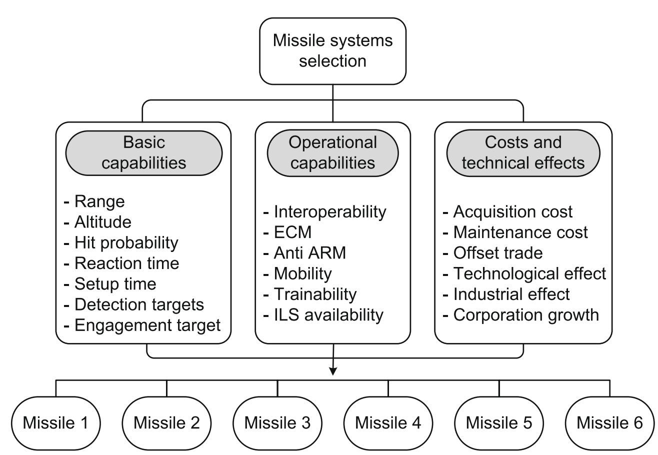

Fig. 2. Hierarchical structure for missile systems selection.

Table 2
Characteristic data on alternative missile systems.

Criteria	Subcriteria	Target values	Alternative missile systems					
			Missile 1	Missile 2	Missile 3	Missile 4	Missile 5	Missile 6
Basic capabilities	Range	150	150	160	135	140	155	170
	Altitude	25	24	28	22	24	28	30
	Hit probability	0.8	0.75	0.8	0.75	0.75	0.8	0.8
	Reaction time	10	12	9	13	12	10	9
	Setup time	5	5.5	5	6	5.5	5	5
	Detection targets	100	95	110	85	95	100	100
	Engagement targets	8	6	9	6	6	8	8
Operational capabilities	Interoperability	0.7	0.75	0.8	0.65	0.65	0.7	0.7
	ECM	0.7	0.65	0.75	0.65	0.65	0.75	0.75
	Anti–ARM	0.8	0.75	0.8	0.65	0.7	0.7	0.8
	Mobility	0.7	0.65	0.65	0.75	0.75	0.75	0.7
	Trainability	0.7	0.75	0.75	0.7	0.65	0.65	0.65
	ILS availability	0.8	0.8	0.8	0.75	0.75	0.75	0.75
Cost and technical effects	Acquisition cost	1100	1100	1250	950	1050	1050	1100
	Maintenance cost	11	12	14	8	8	9	12
	Offset trade	0.6	0.5	0.45	0.75	0.6	0.6	0.45
	Technological effect	1	0.9	0.9	1.1	1	0.9	0.9
	Industrial effect	1	0.8	0.8	1.2	1.1	1	0.8
	Corporation growth	1	0.9	0.9	1.1	1	1	0.8

Table 3
Weight-deriving process for criteria.

Criteria	w_{AHP}	w_{PCA}	w_{SYN}
Basic capabilities (0.410)			
Range (0.119)	0.049	0.044	0.047
Altitude (0.089)	0.037	0.062	0.051
Hit probability (0.260)	0.107	0.017	0.039
Reaction time (0.143)	0.059	0.086	0.111
Setup time (0.112)	0.046	0.041	0.042
Detection targets (0.107)	0.044	0.040	0.039
Engagement targets (0.169)	0.069	0.083	0.127
Operational capabilities (0.449)			
Interoperability (0.181)	0.081	0.042	0.076
ECM (0.274)	0.123	0.038	0.104
Anti–ARM (0.254)	0.114	0.037	0.093
Mobility (0.107)	0.048	0.034	0.036
Trainability (0.067)	0.030	0.024	0.016
ILS availability (0.118)	0.053	0.014	0.017
Cost and technical effects (0.141)			
Acquisition cost (0.204)	0.029	0.044	0.028
Maintenance cost (0.124)	0.018	0.116	0.045
Offset trade (0.096)	0.014	0.097	0.029
Technological effect (0.262)	0.037	0.039	0.032
Industrial effect (0.145)	0.021	0.089	0.040
Corporation growth (0.168)	0.024	0.051	0.027

are also present. The constraints on the basic capabilities are expressed as follows:

$$150x_1 + 160x_2 + 135x_3 + 140x_4 + 155x_5 + 170x_6 - d_1^+ + d_1^- = 150 \quad (7)$$

$$24x_1 + 28x_2 + 22x_3 + 24x_4 + 28x_5 + 30x_6 - d_2^+ + d_2^- = 25 \quad (8)$$

$$0.75x_1 + 0.8x_2 + 0.75x_3 + 0.75x_4 + 0.8x_5 + 0.8x_6 - d_3^+ + d_3^- = 0.8 \quad (9)$$

$$12x_1 + 9x_2 + 13x_3 + 12x_4 + 10x_5 + 9x_6 - d_4^+ + d_4^- = 10 \quad (10)$$

$$5.5x_1 + 5x_2 + 6x_3 + 5.5x_4 + 5x_5 + 5x_6 - d_5^+ + d_5^- = 5 \quad (11)$$

$$95x_1 + 110x_2 + 85x_3 + 95x_4 + 100x_5 + 100x_6 - d_6^+ + d_6^- = 100 \quad (12)$$

$$6x_1 + 9x_2 + 6x_3 + 6x_4 + 8x_5 + 8x_6 - d_7^+ + d_7^- = 8 \quad (13)$$

The constraints on operational capabilities are:

$$0.75x_1 + 0.8x_2 + 0.65x_3 + 0.65x_4 + 0.7x_5 + 0.7x_6 - d_8^+ + d_8^- = 0.7 \quad (14)$$

$$0.65x_1 + 0.75x_2 + 0.65x_3 + 0.65x_4 + 0.75x_5 + 0.75x_6 - d_9^+ + d_9^- = 0.7 \quad (15)$$

$$0.75x_1 + 0.8x_2 + 0.65x_3 + 0.7x_4 + 0.7x_5 + 0.8x_6 - d_{10}^+ + d_{10}^- = 0.8 \quad (16)$$

$$0.65x_1 + 0.65x_2 + 0.75x_3 + 0.75x_4 + 0.75x_5 + 0.7x_6 - d_{11}^+ + d_{11}^- = 0.7 \quad (17)$$

$$0.75x_1 + 0.75x_2 + 0.7x_3 + 0.65x_4 + 0.65x_5 + 0.65x_6 - d_{12}^+ + d_{12}^- = 0.7 \quad (18)$$

$$0.8x_1 + 0.8x_2 + 0.75x_3 + 0.75x_4 + 0.75x_5 + 0.75x_6 - d_{13}^+ + d_{13}^- = 0.8 \quad (19)$$

A set of the constraints on cost and technical effects are:

$$1100x_1 + 1250x_2 + 950x_3 + 1050x_4 + 1050x_5 + 1100x_6 - d_{14}^+ + d_{14}^- = 1100 \quad (20)$$

$$12x_1 + 14x_2 + 8x_3 + 8x_4 + 9x_5 + 12x_6 - d_{15}^+ + d_{15}^- = 11 \quad (21)$$

$$0.5x_1 + 0.45x_2 + 0.75x_3 + 0.6x_4 + 0.6x_5 + 0.45x_6 - d_{16}^+ + d_{16}^- = 0.6 \quad (22)$$

$$0.9x_1 + 0.9x_2 + 1.1x_3 + x_4 + 0.9x_5 + 0.9x_6 - d_{17}^+ + d_{17}^- = 1 \quad (23)$$

$$0.8x_1 + 0.8x_2 + 1.2x_3 + 1.1x_4 + x_5 + 0.8x_6 - d_{18}^+ + d_{18}^- = 1 \quad (24)$$

$$0.9x_1 + 0.9x_2 + 1.1x_3 + x_4 + x_5 + 0.8x_6 - d_{19}^+ + d_{19}^- = 1 \quad (25)$$

where decision variables are the missile system alternatives.

$$x_j = \begin{cases} 1 & \text{if the } j\text{th alternative is selected,} \\ 0 & \text{otherwise,} \end{cases}; \quad j = 1, 2, \ldots, 6.$$

The model also includes the following hard constraint:

$$\sum_{j=1}^{6} x_j = 1$$

The objective function is to minimize the total weighted deviations from the goals that satisfy the above constraints. It can be expressed as follows:

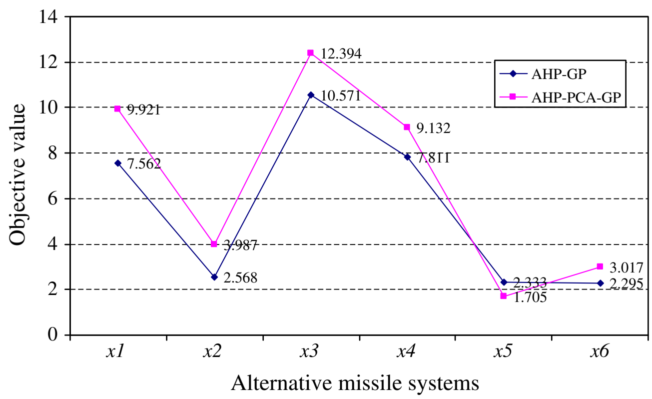

Fig. 3. Comparison of the decision results between the AHP–PCA–GP and AHP–GP models. AHP–GP approach chose the alternative 6 with the objective value of 2.295 and AHP–PCA–GP approach chose the alternative 5 with the objective value of 1.705.

$$\begin{aligned}\text{Min } Z = {} & 0.047d_1^- + 0.051d_2^- + 0.039d_3^- + 0.111d_4^+ \\ & + 0.042d_5^+ + 0.039d_6^- + 0.127d_7^- + 0.076d_8^- \\ & + 0.104d_9^- + 0.093d_{10}^- + 0.036d_{11}^- + 0.016d_{12}^- \\ & + 0.017d_{13}^- + 0.028d_{14}^+ + 0.045d_{15}^+ + 0.029d_{16}^- \\ & + 0.032d_{17}^- + 0.04d_{18}^- + 0.027d_{19}^-. \end{aligned} \tag{26}$$

The weights of the deviation variables in the objective function are w_{SYN} as shown in Table 3. The objective function of the GP problem is a combination of the heterogeneous units of measure. Thus, the constraints should be normalized before solving the problem so that the deviation variables in the objective function are adjusted to the same unit of measure. We used Lingo to solve the GP model. Because the purpose of the problem is to select the best missile system, the optimal alternative in our case study was missile system 5.

Because of the conflicts that typically accompany any selection result, whatever it is, more information, such as the degree of preference the optimal selection represents, is necessary. A selection chosen as an overwhelming preference of the decision makers is likely to gain wide support and follow-up measures will be accelerated in the execution stage. On the other hand, a solution selected by a slight margin may provoke controversy and questions about the reliability and validity of the decision process.

Fig. 3 shows the values of the objective function with the selection of each alternative, with x_5 shown as the best choice with an objective value of 1.705, and x_3 is the least favorable choice with an objective value of 12.394. For comparison, we constructed the GP model with AHP alone (AHP–GP) and solved the same problem. The AHP–GP model yielded a different decision result from the AHP–PCA–GP model in that the optimal solution of the AHP–GP model is x_6 with an objective value of 2.295.

5. Conclusion

We have proposed a hybrid AHP–PCA–GP model and applied this model to the decision making process for selection of a weapon system. In deriving the relative weights to assign to various decision elements necessary for the GP model, we used PCA to offset the shortcomings of AHP when used alone. The proposed model for decision making goes beyond the previous AHP–GP combined model. This improvement is achieved by identifying the attributes affecting weapon systems selection and by reflecting the real data characteristics of weapon systems as well as the intuition of experts in the decision process.

As with other MCDA, weapon systems selection presents tremendous challenges because such systems and the decisions they involve are complex, unstructured, and detailed. Because the decision involves both tangible and intangible factors and both quantitative and qualitative scales, decision makers have difficulty in structuring the problem and evaluating each criterion under the same conditions. The proposed approach will help to solve these problems.

Although we have restricted ourselves to the problem of weapon systems selection, the proposed hybrid model has so much flexibility that with slight modification, it could be applied widely in fields other than military ordnance.

Confronted with the trend of increased demand for rationality and transparency in defense budget expenditures, decision makers in the ROK MND and JCS place increasing emphasis on methods, the validity of the decision process, and the reliability of data used in decisions on military affairs. This trend will only intensify with the increase in the national defense expenditures and the shorter life cycle of weapon systems that is a result of the pace of military technology development. The proposed hybrid approach may contribute to satisfying the demands for rationality and transparency in defense expenditures by strengthening the underlying rationale behind military procurement decisions.

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