Swarm and Evolutionary Computation 6 (2012) 25–38

Contents lists available at SciVerse ScienceDirect

Swarm and Evolutionary Computation

journal homepage: www.elsevier.com/locate/swevo

Regular Paper

Optimum load shedding based on sensitivity to enhance static voltage stability using DE

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ARTICLE INFO

Article history:
Received 14 July 2011
Received in revised form
1 June 2012
Accepted 1 June 2012
Available online 16 June 2012

Keywords:
Schur's inequality
Sensitivity
Load shedding
Differential Evolution
Voltage stability

ABSTRACT

This paper proposes a methodology to optimize the load curtailments necessary to restore the equilibrium of operating point by accounting for operating and stability inequality constraints. To get desired stability margin Schur's inequality based proximity indicator has been selected whose threshold value along with minimization of load shedding assures desired static voltage stability margin. The methodology anticipates the risk of voltage instability in a time frame using sensitivity of proximity indicator of load flow Jacobian with respect to load. If the normal controls are exhausted, the proposed algorithm based on sensitivity, sheds, required amount of low priority loads in advance. This makes the system to survive voltage instability threat even during worst system period. The buses which are having large sensitivity are selected for load shedding. A computational algorithm for minimum load shedding at selected load buses has been developed using Differential Evolution (DE), Self-adaptive Differential Evolution (SaDE) and Ensemble of Mutation and Crossover Strategies and Parameters in Differential Evolution (EPSDE). Developed algorithm accounts inequality constraints not only in present operating conditions (after load shedding) but also for predicted next interval load (with load shedding). Proposed methodology has been implemented on IEEE 14-bus and 25-bus test systems. Performance of the methodology has been compared with Davidon–Fletcher–Powell's (DFP), Particle Swarm Optimization (PSO), Co-ordinated Aggregation based Particle Swarm Optimization (CAPSO) and Genetic Algorithm (GA) techniques based on statistical inference. Simulation results have been obtained which confirm that the proposed methodology provide considerable mitigation in the load shedding and enhancement in voltage stability. By using this methodology various power system blackouts can be prevented.

1. Introduction

When an extended period of reduced generation is anticipated, generation limits may be forecasted for the entire period. During this emergency, system operators have to decide in a very short time which load circuits are to be shed when overloading occurs either due to increased demand or circuit restoration, so that power balance can be achieved and the nominal value of frequency and voltage can be regained. Well planned preventive actions are required for this purpose. Load shedding is initialized as last line of defense. Load shedding is a coordinated set of controls which results in decrease of the electric load in the system [1]. It is one of the possible corrective actions aimed at forcing perturbed system to a new stable equilibrium state. Load-shed criterion may be based on some proximity indicator whose magnitude indirectly reflects the stability margin and provides information for initialization of load shedding. Under such situations the magnitude of the indicator may be monitored during normal operating conditions and when it falls below a threshold value an alarm is actuated. If the indicator continues to decline and reaches to another lower value, load-shed is to be initiated. Such situations may arise due to (i) sudden loss of generation/increase in load which may result in decrease in frequency, (ii) outage of one or more transmission line thus reducing network loadability and may cause load bus limit violations and (iii) overloading of transmission line. In view of this load shedding may be adopted based on (i) under-frequency consideration, (ii) overload alleviation of transmission lines and (iii) voltage limit violation/voltage stability consideration. Tuan et al. [2] presented viable load shedding algorithm based on indicators of risk for voltage instability, based on sensitivities of these indicators to

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http://dx.doi.org/10.1016/j.swevo.2012.06.002

Nomenclature

J	objective function
τ	Schur's inequality based proximity indicator of load flow Jacobian
λ_{min}	minimum eigen value of load flow Jacobian
J'	load flow Jacobian
NL	number of load buses
ls_i	total load (real and reactive power) shedding at ith load bus
NLS	number of load buses selected for load shedding
τ_{th}	threshold value of proximity indicator of load flow Jacobian
τ_o	proximity indicator of load flow Jacobian under current operating condition accounting load shed
τ_p	proximity indicator of load flow Jacobian under predicted load condition accounting load shed
$\underline{P}_{gk}, \underline{Q}_{gk}$	lower bound on active and reactive power generation at kth bus
$\overline{P}_{gk}, \overline{Q}_{gk}$	upper bound on active and reactive power generation at kth bus
P^o_{gk}, Q^o_{gk}	active and reactive power generation at kth bus under current operating condition accounting load shed
P^p_{gk}, Q^p_{gk}	active and reactive power generation at kth bus under predicted load condition accounting load shed
NG	total number of generator buses
V^o_i	load bus voltage at ith load bus under current operating condition accounting load shed
V^p_i	load bus voltage at ith load bus under predicted load condition accounting load shed
$\underline{V}_i, \overline{V_i}$	lower and upper bound on ith load bus voltage
σ	standard deviation
μ	mean

changes in load to be shed. Balanathan et al. [3] presented a technique for practically calculating the shedding necessary to assure the power system voltage stability following a disturbance. A computational method is based on the Monte Carlo simulation approach [4,5] can be used for comparing and selecting the most appropriate load shedding strategies. Wiszniewski [6] presented a methodology which gives new criteria of voltage stability margin for the purpose of load shedding. Girgis and Mathure [7] presented a methodology that shows the rate of change of frequency can be utilized to determine the magnitude of generation–load imbalance, while the rate of change of voltage with respect to active power can be utilized to identify the sensitive bus for load shedding. Fu and Wang [8] presented an algorithm which was developed for studying the load shedding problem in emergencies, where an ac power flow solution cannot be found for the stressed system. Amraee et al. [9] proposed an adaptive under-voltage load shedding scheme using model predictive control to protect power system against voltage instability. A specified outage from a set of multiple contingencies was modeled with a homotopy function including a parameter representing the outage. Outage-continuation power flow traces the path of solutions satisfying the power-flow equations with respect to variations of the parameters. At the nose point, it performs a sensitivity analysis with a normal vector to identify the most effective control variables. With the sensitivity information, location of load shedding is determined; then, an adequate amount of control is decided by applying a searching method [10]. In this paper a new algorithm has been developed for optimum load shedding based on voltage stability consideration. Schur's inequality has been used as proximity indicator. A threshold value of this indicator can be assumed for a specific system. During emergency load shedding is required, if the value of proximity indicator falls below the threshold value. The value of Schur's inequality proximity indicator is very small or close to zero at collapse point. The proposed algorithm consists of two parts, one of it identifies load buses for load shedding using sensitivity of proximity indicator with respect to real and reactive load, the other determines the optimum load to be shed at selected load buses using Differential Evolution and improved DE variants (SaDE and EPSDE) subject to operating and stability constraints. Results have been obtained using proposed methodology are compared with PSO, CAPSO, DFP and GA. Section 2 presents sensitivity derivation of proximity indicator with respect to load shedding at load buses. Section 3 presents problem formulation. Section 4 presents an overview of DE technique, bounce back technique and handling of inequality constraints. Section 5 presents implementation of the developed algorithm (DE, SaDE and EPSDE) for optimizing objective function. Section 6 gives results and discussions. Section 7 presents conclusions and highlights of the paper.

2. Sensitivity derivation of proximity indicator with respect to load shedding at load buses

Schur's inequality is given as follows [11]:

$$\lambda_{max} \leq \sqrt{\sum_{ij} a_{ij}^2} \tag{1}$$

where, a_{ij}—ijth element of load flow Jacobian $[J']$; λ_{max}—greatest eigen value of load flow Jacobian.

Magnitude of greatest eigen value is less than or equal to square root of sum of square of each element of the matrix. Eq. (1) is used to derive lower bound on the minimum eigen value of load flow Jacobian. Sensitivity matrix $[S]$ is given as follows:

$$[S] = [J']^{-1} \tag{2}$$

Now using inequality (1) upper bound on maximum eigen value of $[S]$ is given as follows:

$$S\lambda_{max} \leq \sqrt{\sum_{ij} s_{ij}^2} \tag{3}$$

where, $S\lambda_{max}$ denotes maximum eigen value of $[S]$; and s_{ij} is its element. It is known from matrix theory that:

$$J'\lambda_{min} = 1/(S\lambda_{max}) \tag{4}$$

where, $J'\lambda_{min}$ is the minimum eigen value of load flow Jacobian. Using Eq. (4), inequality Eq. (3) can be written as:

$$J'\lambda_{min} \geq 1/\left(\sqrt{\sum_{ij} s_{ij}^2}\right) = \tau \tag{5}$$

where, $J'\lambda_{min}$ is the minimum eigen value of load flow Jacobian.

In fact, right hand side of Eq. (5) is lower bound on the minimum eigen value of load flow Jacobian and defined as a proximity indicator (τ). Magnitude of this proximity indicator reflects the distance to voltage collapse from the current operating point and has been used for voltage stability monitoring.

It is assumed that load flow Jacobian at current operating point is known. If one of the disturbance variable (active and reactive power on load bus) is changed then modified Jacobian can be written as follows:

$$J'_m = J' + \Delta J' \tag{6}$$

New sensitivity matrix is written as follows:

$$S' = [J'_m]^{-1} = [J' + \Delta J']^{-1} \tag{7}$$

Using Maclaurin's series expansion and retaining only first order term, following expression is obtained:

$$S' \cong S - S.\Delta J' S \tag{8}$$

where, $\Delta J'$ is the change in Jacobian matrix due to change in disturbance (load) variables. New elements of sensitivity matrix are given as follows:

$$s'_{ij} = s_{ij} - \sum_{l,k} s_{il}.s_{kj}.\Delta J'_{lk} \tag{9}$$

where, $\Delta J'_{lk}$ denotes change in lth and kth element due to change in disturbance variables. Now, expression for each element of load flow Jacobian is known and sensitivity relating Jacobian element and disturbance variables can be evaluated and written as [12]:

$$\sigma_{lk-p} = \partial J'_{lk} / \partial D_p \tag{10}$$

because $D_p = P_p + jQ_p$

Expressing $\Delta J'_{lk}$ in terms of change in load ΔD_p via sensitivity as given in Eq. (10), following expression is obtained:

$$s'_{ij} = s_{ij} - \sum (s_{il} s_{kj} \sigma_{lk-p}).\Delta D_p$$
$$s'_{ij} = s_{ij} + \rho_{ij-p}.\Delta D_p \tag{11}$$

where, ρ_{ij-p} is expressed as follows:

$$\rho_{ij-p} = -\sum_{l,k} s_{il}.s_{kj}.\sigma_{lk-p}$$

New value of proximity indicator is written as follows:

$$\tau' = 1 \Big/ \sqrt{\sum_{i,j} (s_{ij} + \rho_{ij-p}.\Delta D_p)^2} \tag{12}$$

Expression (12) can be written using series expression as:

$$\tau' = \tau - \sum_{i,j} s_{ij}.\rho_{ij-p}.\tau^3.\Delta D_p$$
$$\tau' = \tau + SI_p.\Delta D_p \tag{13}$$

Sensitivity vector is evaluated as follows using Eq. (13):

$$SI = [SI_1, SI_2, \ldots, SI_{NL}]^T \tag{14}$$

If the value of proximity indicator drops below threshold value then sensitivity vectors SI as given in Eq. (14) is evaluated. Disturbance vector with large sensitivity magnitude is selected for load shedding:

$$SI_{ls} = \overset{max}{ls}[SI_1, SI_2, \ldots, SI_{NL}]^T \tag{15}$$

For static voltage stability studies of a power system, the loading of the system is increased incrementally and slowly (in certain direction) to the point of voltage collapse. The MW-distance to this point is a good measure of system voltage stability limit. The voltage profile of the system is shown by the PV-curves which are plotted using continuation power flow programs as the loading varies from the base values to the point of collapse. The only way to save the system from voltage collapse is to reduce the reactive power load or add additional reactive power prior to reaching the point of voltage collapse.

3. Problem formulation

It is understood that load shedding should take place at minimum number of buses. These buses should be selected and then upper limits of load shed must be decided from operating and stability constraints view point. Buses which are selected having large sensitivity (ranked in descending order); is the change in proximity indicator with respect to load shed (real and reactive power at load buses). Once buses are selected for load shedding the following objective function is minimized at current operating condition.

$$J = \sum_{i=NLS} ls_i \tag{16}$$

Above objective function is optimized subject to following constraints:

(i) Power flow constraints under current operating condition as well as next predicted loading condition accounting load shed:

$$\underline{P} = \underline{f}(\underline{V}, \delta)$$
$$\underline{Q} = \underline{g}(\underline{V}, \delta) \tag{17}$$

It is stressed that power flow is performed under current operating condition as well as at next predicted loading condition after the load shed.

(ii) Inequality constraint on Schur's inequality proximity indicator of load flow Jacobian at current operating point as well as next predicted load condition accounting load shed:

$$\tau_o \geq \tau_{th}$$
$$\tau_p \geq \tau_{th} \tag{18}$$

(iii) Active power generation constraint under base case condition as well as at next predicted loading condition accounting load shed:

$$\underline{P}_{gk} \leq P^o_{gk} \leq \overline{P}_{gk}$$
$$\underline{P}_{gk} \leq P^p_{gk} \leq \overline{P}_{gk}, \quad k = 1, 2, \ldots, NG \tag{19}$$

(iv) Reactive power generation constraint under base case condition as well as at next predicted loading condition accounting load shed:

$$\underline{Q}_{gk} \leq Q^o_{gk} \leq \overline{Q}_{gk}$$
$$\underline{Q}_{gk} \leq Q^p_{gk} \leq \overline{Q}_{gk}, \quad k = 1, 2, \ldots, NG \tag{20}$$

(v) Inequality constraint on load bus voltages in present as well as for next predicted interval load that is load bus voltages are within limit, accounting load shed:

$$\underline{V}_i \leq V^o_i \leq \overline{V}_i$$
$$\underline{V}_i \leq V^p_i \leq \overline{V}_i, \quad i = NG+1 \ldots NB \tag{21}$$

(vi) $\underline{ls}_i \leq ls_i \leq \overline{ls}_i,\ i \in NLS$ (22)

In fact permissible load shed is a fraction of total load at selected bus. It may be say 80% of the total load and rest 20% may be a load, which is required in emergency condition.

It is stressed that load shedding is performed at current loading condition. Further, constraints as in Eqs. (17)–(21) are ascertained by performing load flow solution at current operating condition (after load shedding) and predicted load condition (accounting load shed).

4. Differential Evolution: an overview

Differential Evolution (DE) is a very simple population based, stochastic function minimizer and has been found very powerful to solve various natures of engineering problems [13,14,26,27]. DE optimizes the problem by sampling the objective function at multiple randomly chosen initial points. Preset parameter bounds define the region from which '*M*' vectors in this initial population are chosen. DE generates new solution points in '*D*' dimensional space that are perturbations of existing points. It perturbs vectors with the scaled difference of two randomly selected population vectors. To produce a mutated vector, DE adds the scaled, random vector difference to a third selected population vector (base vector). Further, DE also employs a uniform crossover to produce trial vector from target vector and mutated vector. The four fundamental steps are explained below:

Step-(a) initialization: initial population of size '*M*' is generated as follows:

$$ls^0 = [x_1^0, x_2^0, x_3^0, \ldots, x_M^0]$$
$$x_i^0 = [ls_{i,1}^0, ls_{i,2}^0, ls_{i,3}^0, \ldots, ls_{i,NLS}^0]^T \tag{23}$$

ls_{ij} i.e. jth parameter of x_i vector is obtained from uniform distribution as follows:

$$ls_{i,j}^0 = \underline{ls}_j + (\overline{ls}_j - \underline{ls}_j) rand_j \tag{24}$$

$\underline{ls}_j$ and $\overline{ls}_j$ are lower and upper bounds on variable ls_j. $rand_j$ is a random digit in the range [0,1].

Step-(b) mutation: DE mutates and recombines the population to produce a population of '*M*' trial vectors. Differential mutation adds a scaled, randomly sampled, vector difference to a third vector as follows:

For each target vector $ls_i^{(k+1)}$ at generation M, an associated mutant vector $\underline{\rho}_i^{(k)} = ls_{1i}, ls_{2i}, ls_{3i}, \ldots, ls_{ni}$ can usually be generated by using one of the following five strategies as shown in the online available code [16]:

"DE/rand/1":

$$\underline{\rho}_i^{(k)} = ls_{r1}^{(k)} + \alpha(ls_{r2}^{(k)} - ls_{r3}^{(k)}) \tag{25}$$

"DE/best/1":

$$\underline{\rho}_i^{(k)} = ls_{base}^{(k)} + \alpha(ls_{r1}^{(k)} - ls_{r2}^{(k)}) \tag{26}$$

"DE/current to best/1":

$$\underline{\rho}_i^{(k)} = ls_i^{(k)} + \alpha(ls_{base}^{(k)} - ls_i^{(k)}) + \alpha(ls_{r1}^{(k)} - ls_{r2}^{(k)}) \tag{27}$$

"DE/best/2":

$$\underline{\rho}_i^{(k)} = ls_{base}^{(k)} + \alpha(ls_{r1}^{(k)} - ls_{r2}^{(k)}) + \alpha(ls_{r3}^{(k)} - ls_{r4}^{(k)}) \tag{28}$$

"DE/rand/2":

$$\underline{\rho}_i^{(k)} = ls_{r1}^{(k)} + \alpha(ls_{r2}^{(k)} - ls_{r3}^{(k)}) + \alpha(ls_{r4}^{(k)} - ls_{r5}^{(k)}) \tag{29}$$

is known as scale factor usually lies in the range [0,1]; $ls_{base}^{(k)}$ is known as base vector; $\underline{\rho}_i^{(k)}$ is a mutant vector; $ls_{r1}^{(k)}, ls_{r2}^{(k)}, ls_{r3}^{(k)}, ls_{r4}^{(k)}$ and $ls_{r5}^{(k)}$ are five randomly selected vectors (*r1? r2? r3? r4? r5*). The base vector index '*b*' may be determined in variety of ways. This may be a randomly chosen vector (*base? r1? r2? r3? r4? r5*).

Step-(c) crossover: DE employs a uniform crossover strategy. Crossover generates trial vectors $t_i^{(k)}$ as follows:

$$t_{i,j}^{(k)} = \begin{cases} \rho_{i,j}^{(k)}, & \text{if } (rand_j \le C_r \ or.\ j = j_{rand}) \\ ls_{i,j}^{(k)}, & \text{Otherwise} \end{cases} \tag{30}$$

C_r is a crossover probability lies in the range [0,1]. C_r is a user defined value which controls the number of parameter values which are copied from the mutant. If the random number $rand_j$ is less than or equal to C_r, the trial parameter is adopted from the mutant $\rho_i^{(k)}$. Further, the trial parameter with randomly chosen index, j_{rand} is taken from the mutant to ensure that trial vector does not duplicate target vector $ls_i^{(k)}$. Otherwise the parameter is adopted from the target vector $ls_i^{(k)}$.

Step-(d) selection: objective function is evaluated for target vector and trial vector, trial vector is selected if it provides better value of the function than target vector as follows:

$$ls_i^{(k+1)} = \begin{cases} t_i^{(k)}, & \text{if } [f(t_i^{(k)}) \le f(ls_i^{(k)})] \\ ls_i^{(k)}, & \text{Otherwise} \end{cases} \tag{31}$$

The process of mutation, crossover and selection is executed for all target vector index '*i*' and a new population is created till the optimal solution is obtained. The procedure is terminated if a maximum number of generations (k) have been executed or no improvement in objective function is noticed in a pre-specified generations. In this paper DE/best/1/bin has been selected. The first term after DE i.e. 'best' specifies the way base vector is chosen. In this selected scheme the base vector is the current best so far vector. '1' After best denotes that one vector difference contributes to the differential. Last term 'bin' denotes binomial distribution that result because of uniform crossover. Number of parameters donated by mutant vector closely follows binomial distribution. It is to be noted that best, target and difference vector indices are all different.

4.1. Bounce back technique for handling bounds on decision variables

Some of the variables may cross the lower or upper bounds in a mutant vector $\rho_i^{(k)}$ in executing differential as governed by relations (25)–(29). Bounce back mechanism is adopted to bring such decision variables within limit. The bounce-back method replaces element which has violated limit by the new element whose value lies between the base parameter value and the bound being violated. The following relations are used for violated mutant vector elements [13].

$$\rho_{i,j}^{(k)} = \begin{cases} ls_{base,j} + rand.(\underline{ls}_j - ls_{base,j}), & \text{if } (\rho_{i,j}^{(k)} \le \underline{ls}_j) \\ ls_{base,j} + rand.(\overline{ls}_j - ls_{base,j}), & \text{if } (\rho_{i,j}^{(k)} > \overline{ls}_j) \end{cases} \tag{32}$$

4.2. Handling of inequality constraints

A direct inequality constraint handling technique devised by Lampinen [15] has been adopted in this paper. In its simple form Lampinen's criterion selects {step-(d)} the trial vector $t_i^{(k)}$ under following conditions:

(i) $t_i^{(k)}$ satisfies all constraints and has a lower or equal value of objective function than $ls_i^{(k)}$,

(ii) $t_i^{(k)}$ is feasible and $ls_i^{(k)}$ is not feasible,

(iii) $t_i^{(k)}$ and $ls_i^{(k)}$ are both infeasible, but $t_i^{(k)}$ does not violet any constraint more than $ls_i^{(k)}$. Otherwise $ls_i^{(k)}$ is retained in the new population.

5. Problem solution

5.1. Implementation of Differential Evolution algorithm to solve formulated problem

Step-1: *data input*; reactive power control variables and system parameters (resistance, reactance, and susceptance etc.).
Step-2: base case load flow solution is obtained using continuation power flow methodology.
Step-3: next interval load is predicted.
Step-4: obtain load flow solution for the predicted next interval load.
Step-5: obtain sensitivities using relation (15) in Section 2, for selection of most critical load bus.
Step-6: *initialization*; generate population of size '*M*' for load shedding. Generated population is uniformly distributed in the range $[0,\overline{ls_i}]$

$$x_i^{(0)} = [ls_{i,1}^{(0)}, ls_{i,2}^{(0)}, \ldots, ls_{i,NLS}^{(0)}]^T, \quad i = 1,2,\ldots,M$$

Step-7: run continuation power flow program for each vector of the population and monitor all inequality constraints (17)–(21). If, a vector satisfies the constraints call it '*F*' (feasible). Otherwise, call it '*NF*' (not-feasible).
Step-8: calculate objective function for the feasible vectors.
Step-9: based on the value of objective function, identify the best solution vector ls_{best}. This is selected as a base vector.
Step-10: set generation count $k=1$.
Step-11: select target vector $i=1$.
Step-12: select two vectors ls_{r1} and ls_{r2} such that $best \neq i \neq r1 \neq r2$.
Step-13: generate a mutated vector $\rho_i^{(k)}$ using relation (26).
Step-14: if any component of mutated vector i.e. $\rho_i^{(k)}$ violets the bounds on decision variable ls_j then apply bounce back technique using relation (32) and bring the violated variables within limit.
Step-15: apply uniform crossover using relation (30) to get trial vector $t_i^{(k)}$. If the trial vector satisfy load shedding inequality constraint (22) call it '*F*' otherwise, '*NF*'.
Step-16: apply Lampinen's criteria as explained in Section 4.2 to select $t_i^{(k)}$ in new population or reject it to retain $ls_i^{(k)}$ in new population.
Step-17: increase target vector $i=i+1$, if $i \le M$, repeat from step-12. Otherwise increase generation count $k=k+1$.
Step-18: if $k \le k_{max}$ repeat from step-11. Otherwise stop.

The implementation described above is applied in sequence to solve formulated problem. Solution of the problem gives anticipatory optimum load shedding at critical load buses based on sensitivity.

5.2. Implementation of Self-adaptive Differential Evolution (SaDE) algorithm to solve formulated problem [16]

The performance of the original DE algorithm is highly dependent on the strategies and parameter settings. Selection of strategy and the corresponding control parameters settings for a specific problem require a huge amount of computation time. These problems are rectified using SaDE algorithm. SaDE algorithm can automatically adapt the learning strategies and the parameters settings during evolution. In SaDE algorithm a probability P_f to control which mutation strategy to use, and P_f is gradually self-adapted according to the learning experience. Additionally, SaDE utilizes two methods to adapt and self-adapt DE's parameters α and C_r. SaDE selects mutation strategies Eqs. (26) and (28) as candidates, and produces the trial vector based on:

$$t_{i,j}^{(k)} = \begin{cases} \text{Eq. (26)}, & \text{if } (rand_j \le P_f.j = j_{rand}) \\ \text{Eq. (28)}, & \text{Otherwise} \end{cases} \tag{33}$$

P_f is set to 0.5 initially. After evaluation of all offspring, the number of offspring successfully entering the next generation while generated by Eqs. (26) and (28) are recorded as ns_1 and ns_2 respectively and the numbers of offspring discarded while generated by Eqs. (26) and (28) are recorded as nf_1 and nf_2. Those two pairs of numbers are accumulated within a specified number of generations (40), called the "learning period". Then, the probability P_f updated as:

$$P_f = \frac{ns_1.(ns_2 + nf_2)}{ns_2.(ns_1 + nf_1) + ns_1.(ns_2 + nf_2)} \tag{34}$$

ns_1, ns_2, nf_1 and nf_2 will be reset once P_f is updated after each learning period.

5.3. Implementation of Ensemble of Mutation and Crossover strategies and parameters in Differential Evolution (EPSDE) algorithm to solve formulated problem [17]

The effectiveness of conventional DE in solving a numerical optimization problem depends on the selected mutation and crossover strategy and its associated parameter values are:

Mutation strategies: EPSDE selects mutation strategies Eqs. (25)–(29).
Crossover strategies: normal distributions crossover.
The pool of the parameter: scale factor (α) with normal distributions of $\mu=0.5$ and $\sigma=0.3$.

Crossover probability (C_r) with normal distributions of $\mu=0.5$ and $\sigma=0.1$.

Step-1: *initialization*; generate population of size '*M*' for load shedding. Generated population is uniformly distributed in the range $[0,\overline{ls_i}]$

$$x_i^{(0)} = [ls_{i,1}^{(0)}, ls_{i,2}^{(0)}, \ldots, ls_{i,NLS}^{(0)}]^T, \quad i = 1,2,\ldots,M$$

Step-2: run continuation power flow program for each vector of the population and monitor all inequality constraints (17)–(21). If, a vector satisfies the constraints call it '*F*' (feasible). Otherwise, call it '*NF*' (not-feasible).
Step-3: calculate objective function for the feasible vectors.
Step-4: based on the value of objective function, identify the best solution vector ls_{best}.
Step-5: set generation count $k=1$.
Step-6: select target vector $i=1$.
Step-7: select four vectors ls_{r1}, ls_{r2}, ls_{r3} and ls_{r4} such that $best \neq i \neq r1 \neq r2 \neq r3 \neq r4$.
Step-8: select a pool of mutation strategies and a pool of values for each associated parameters corresponding to each mutation strategy.
Step-9: each population member is randomly assigned with one of the mutation strategy from the pool Eqs. (26) and (28) and the associated parameter values are chosen randomly from the corresponding pool of values.
Step-10: stop, criterion is not satisfied.
Step-11: *mutation step*; generate a mutated vector $\underline{\rho}_i^{(k)} = ls_{1i}, ls_{2i}, ls_{3i}, \ldots, ls_{ni}$ for each target vector $ls_i^{(k+1)}$ using the mutation strategy and (α) value associated with the target vector.

Step-12: *crossover step*; generate a trial vector $t_i^{(k)}$ using Eq. (30) for each target vector $ls_i^{(k+1)}$ using the crossover strategy and (C_r) value associated with the target vector.
Step-13: *selection step*; best of among the target (parent) and trial (offspring) vectors survive and enter the next generation.
Step-14: *updating step*; the mutation strategy and parameter values that could not produce successful offspring individuals are replaced by randomly selected mutation strategy and parameter values from the respective pools.
Step-15: increase target vector $i=i+1$, if $i \le M$, repeat from step-7. Otherwise increase
generation count $k=k+1$.
Step-16: if $k \le k_{max}$ repeat from step-6. Otherwise stop.

6. Results and discussions

The developed algorithms have been implemented for generating load shedding strategies on IEEE 14-bus and 25-bus test systems [18]. For this purpose system has been stressed by uniform loading such that proximity indicator has been reduced to very small value and there is severe violation of bus voltages. Under this simulated stressed systems condition, all other control means are exhausted. Other constraints as defined in Section 3 can be easily incorporated in algorithm by limiting load shedding to prescribed condition.

6.1. 14-Bus system

This system consists of three generator buses and eleven load buses. The desired range of load bus voltage is 0.95 pu–1.05 pu. Table 1 shows system load, PV-bus voltages, load bus voltages, value of proximity indicator under simulated stressed condition and static voltage stability limit. Because of network overloading buses automatically gets switched into PQ buses after hitting the maximum limit of their reactive power generation. PV-bus no. 3 has crossed the limit and other PV buses operating very close to their limits. Because of switching from PV bus to PQ bus, the dimension of system Jacobian undergoes a change (increase in dimension). This change in dimension results a drastic change in proximity indicator (sudden decrease). It imposes a challenging task of setting threshold value in algorithm such that desired threshold value of indicator is achieved while considering the fact that these PQ buses will switch back to PV buses after load shedding.

In the present work a piecewise method is adopted to handle this situation. Set value of indicator is increased in small steps till drastic change (switching back to PV bus) in proximity indicator is observed. From operational experience of the system threshold value of proximity indicator is selected as τ_{th}=0.3200. Table 2 shows evaluated sensitivities using Eq. (15) at all PQ buses (except at no load PQ buses). Load buses which having large sensitivity are selected for load shedding. Selected buses are shown as star marked in Table 2. Initially, 3000 populations (0–80% of individual load bus capacity) for each selected load bus has been generated randomly using excel software according to probability distribution of disturbance variables. Continuation power flow was carried out and only 33 sets of particles (total load to be shed) of all selected load bus were selected which satisfied all inequality constraints (Eqs. (17)–(22)) and ranked in ascending order on the basis of values of objective function. Best initial solution (particles) are selected as, Pd_{14}=0.0031 pu, Qd_{14}=0.0783 pu, Pd_{13}=0.1525 pu, Qd_{13}=0.0373 pu, Pd_{11}=0.0009 pu, Qd_{11}=0.0003 pu, Pd_{10}=0.0291 pu, Qd_{10}=0.0631 pu, Pd_9=0.2028 pu, Qd_9=0.1530 pu, objective function (J)=0.5109 pu and τ=0.3523. Objective function was evaluated at selected buses using Eq. (16). Table 3 gives the objective function with various combinations of DE parameters (α and C_r). Values of DE parameters lie in the range [0,1]. Maximum numbers of iterations

Table 1
Load flow solution for 14-bus test system under stressed condition. Total load (S_{di})=5.3923 pu, proximity indicator (τ)=0.2228, static voltage stability limit=6.2381 pu.

Bus no.	Bus voltage (pu)		Load (pu)	
1	V_1	1.0929	S_{d1}	0.0000
2	V_2	1.0379	S_{d2}	0.5154
3	V_3	1.0323	S_{d3}	1.9699
4	V_4	0.8722	S_{d4}	0.2763
5	V_5	0.8389	S_{d5}	0.0000
6	V_6	0.9289	S_{d6}	0.9831
7	V_7	0.8389	S_{d7}	0.0000
8	V_8	0.9262	S_{d8}	0.1601
9	V_9	0.8080	S_{d9}	0.6939
10	V_{10}	0.7994	S_{d10}	0.2195
11	V_{11}	0.8262	S_{d11}	0.0806
12	V_{12}	0.8412	S_{d12}	0.1292
13	V_{13}	0.8235	S_{d13}	0.3012
14	V_{14}	0.7804	S_{d14}	0.1681

Table 2
Load bus ranking based on sensitivity of Schur's inequality proximity indicator with respect to system load for 14-bus system.

Sr. no.	Load bus	Sensitivity
1	11	0.3938[a]
2	14	0.3154[a]
3	10	0.3011[a]
4	13	0.2548[a]
5	9	0.2135[a]
6	12	0.1875
7	4	0.1853
8	8	0.1434
9	6	0.0532

[a] Load buses selected for load shedding.

Table 3
Effect of DE parameters on optimization of objective function and number of iteration required for convergence for 14-bus test system.

Case	α	c_r	J	No. of iterations for convergence
1	0.95	0.55	0.4053	759
2	0.95	0.50	0.4129	932
3	0.95	0.45	0.4085	734
4	0.90	0.70	0.4098	941
5	0.90	0.65	0.4106	918
6	0.90	0.60	0.4141	798
7	0.85	0.60	0.4021	841
8	0.85	0.55	0.4051	823
9	0.85	0.50	0.4034	954
10	0.85	0.45	0.4018	944
11	**0.80**	**0.70**	**0.4009**	**438**
12	0.80	0.60	0.4039	823
13	0.80	0.55	0.4074	948
14	0.80	0.50	0.4026	975
15	0.80	0.45	0.4100	944
16	0.75	0.60	0.4043	661
17	0.75	0.55	0.4032	827
18	0.75	0.50	0.4014	769
19	0.75	0.45	0.4075	811
20	0.75	0.40	0.4025	702

Bold face number indicates minimum value of objective function has been achieved in 438 iterations with Cr=0.70 and α=0.80.

were set equal to 1000. Minimum value of objective function (J) has been obtained for scale factor (α)=0.80 and crossover probability (C_r)=0.70 [28,29]. This process is terminated after 438iterations and no improvement in objective function is noticed. It is observed that all inequality constraints are satisfied and value of proximity indicator is τ=0.3324. It is obvious that all PV buses switched, all load bus voltages and line flow are within limit. Table 4 shows comparison of load on load buses and bus voltages; (i) before and after load shedding without optimization (ii) after optimized load shedding using DE and its improved variants, DFP, PSO, CAPSO and GA techniques [19,20,25]. Table 5 gives comparison of optimal load shedding at different load buses (bus nos. 11, 14, 10, 13, 9) using DE and its variants, have been compared with DFP, PSO, CAPSO and GA techniques (Appendix A). Table 6 gives the comparison of DE and its variants with DFP, PSO, CAPSO and GA based on objective function's mean value, standard deviation, best value, worst value, frequency of convergence, standard error, length of confidence interval and confidence interval (Appendix B). Table 7 gives the comparison of class interval and proportionate frequency of objective function (Appendix B) have been evaluated using DE, SaDE, EPSDE, DFP, PSO, CAPSO and GA techniques. Fig. 1 shows a plot between objective function (J) and computational time for convergence of DE, SaDE, EPSDE, DFP, PSO, CAPSO and GA techniques for best objective function accounting the effect of variation in parameters. Variation in values of parameters was within the specified range. CPU time is with respect to P-IV intel Core 2 Duo machine which includes the time for load flow program and total number of iterations. Static voltage stability limit obtained at the end of optimization process using EPSDE, SaDE, DE, GA, CAPSO, PSO and DFP techniques are 7.0598 pu, 7.0295 pu, 7.0237 pu, 7.0189 pu, 6.9956 pu, 6.9776 pu and 6.9663 pu respectively. It is observed that DE and its variants (SaDE and EPSDE) give much better global optimal results than genetic algorithm, PSO, CAPSO and classical technique DFP.

6.2. 25-Bus system

This system consists of five generator buses and twenty load buses. The desired range of load bus voltage is 0.95 pu–1.05 pu. Table 8 shows system load, PV-bus voltages, all other load bus voltages, value of proximity indicator under simulated stressed condition, and static voltage stability limit. Because of network overloading buses got switched into PQ buses after hitting the maximum limit of their reactive power generation. PV-bus nos. 1, 2, 3, 4 and 5 has crossed the limit. From operational experience of the system threshold value of indicator is selected as τ_{th}=0.3100. Table 9 shows sensitivities at all PQ buses (except at no load PQ buses). Load buses which having large sensitivities are selected for load shedding. Selected buses are shown as star marked in Table 9. Initially, 3000 populations for each selected load bus have been generated randomly. Continuous power flow was carried out and only 43 sets of particles of all selected load bus were selected which satisfied all inequality constraints and ranked in ascending order on the basis of values of objective function. Best initial solution (particles) are selected as Pd_{15}=0.0867 pu, Qd_{15}=0.0983 pu, Pd_{14}=0.1483 pu, Qd_{14}=0.1174 pu, Pd_{22}=0.4242 pu Qd_{22}=0.0950 pu, Pd_{16}=0.0259 pu, Qd_{16}=0.1072 pu, Pd_{18}=0.2828 pu, Qd_{18}=0.0178 pu, Pd_{9}=0.1873 pu, Qd_{9}=0.0154 pu, Pd_{17}=0.8574 pu, Qd_{17}=0.4296 pu, Objective function (J)=2.1969 pu and τ=0.3271. Table 10 gives the objective function with various combinations of DE parameters (α and C_r). Values of DE parameters lie in the range [0,1]. Maximum numbers of iterations were set equal to1000. Minimum value of objective function (J) has been obtained for scale factor (α)=0.90 and crossover probability (C_r)=0.60 [28,29]. This process is terminated after 813 iterations and no improvement in objective function is noticed. It is observed that all inequality constraints are satisfied and value of proximity indicator is τ=0.3188. It is obvious that all PV buses

Table 4
Bus voltages and load on load bus after load shedding with and without optimization techniques for 14-bus test system.

Bus no.	Without optimization				After optimized load shedding													
	Base case		Best initial solution based load shedding		DE		EPSDE		SaDE		CAPSO		PSO		DFP		GA	
	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)
1	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929	0.0000	1.0929
2	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379	0.5154	1.0379
3	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323	1.9699	1.0323
4	0.2763	0.8722	0.2763	1.0224	0.2763	1.0042	0.2763	0.9988	0.2763	1.0030	0.2763	1.0046	0.2763	1.0054	0.2763	1.0060	0.2763	1.0060
5	0.0000	0.8389	0.0000	0.9843	0.0000	0.9630	0.0000	0.9618	0.0000	0.9616	0.0000	0.9651	0.0000	0.9659	0.0000	0.9662	0.0000	0.9663
6	0.9831	0.9289	0.9831	1.0134	0.9831	0.9964	0.9831	0.9951	0.9831	0.9960	0.9831	0.9993	0.9831	0.9998	0.9831	1.0004	0.9831	0.9970
7	0.0000	0.8389	0.0000	0.9843	0.0000	0.9630	0.0000	0.9618	0.0000	0.9616	0.0000	0.9651	0.0000	0.9659	0.0000	0.9662	0.0000	0.9663
8	0.1601	0.9262	0.1601	1.0058	0.1601	0.9902	0.1601	0.9884	0.1601	0.9899	0.1601	0.9930	0.1601	0.9935	0.1601	0.9940	0.1601	0.9903
9	0.6939	0.8080	0.4435	0.9744	0.5687	0.9524	0.5739	0.9512	0.5868	0.9505	0.5073	0.9536	0.4979	0.9544	0.4887	0.9545	0.5631	0.9567
10	0.2195	0.7994	0.1651	0.9719	0.1830	0.9507	0.1816	0.9507	0.1848	0.9501	0.1706	0.9510	0.1706	0.9522	0.1625	0.9523	0.1693	0.9525
11	0.0806	0.8262	0.0797	0.9894	0.0746	0.9708	0.0515	0.9716	0.0721	0.9708	0.0807	0.9698	0.0809	0.9707	0.0683	0.9725	0.0631	0.9724
12	0.1292	0.8412	0.1292	1.0031	0.1292	0.9840	0.1292	0.9762	0.1292	0.9824	0.1292	0.9847	0.1292	0.9856	0.1292	0.9860	0.1292	0.9863
13	0.3012	0.8235	0.1487	0.9972	0.1870	0.9763	0.2844	0.9653	0.2128	0.9742	0.1563	0.9781	0.1509	0.9792	0.1702	0.9791	0.2521	0.9777
14	0.1681	0.7804	0.1227	0.9691	0.1181	0.9502	0.0738	0.9510	0.0752	0.9515	0.1178	0.9500	0.1208	0.9500	0.1123	0.9500	0.1088	0.9538

Table 5
Optimum load shedding at selected load buses using DE, EPSDE, SaDE, GA, CAPSO, PSO and DFP techniques for 14-bus system.

Sr. no.	Methodology	Amount of load shed at selected load bus (pu)										Objective function (pu)
		Bus no. 14		Bus no. 13		Bus no. 11		Bus no. 10		Bus no. 9		
		Pd_{14}	Qd_{14}	Pd_{13}	Qd_{13}	Pd_{11}	Qd_{11}	Pd_{10}	Qd_{10}	Pd_9	Qd_9	
1	EPSDE	0.0452	0.1149	0.0180	0.0010	0.0203	0.0386	0.0072	0.0796	0.0782	0.1122	0.3854
2	SaDE	0.0477	0.0947	0.0831	0.0308	0.0012	0.0219	0.0026	0.0862	0.0672	0.1049	0.3940
3	DE	0.0030	0.0963	0.1091	0.0361	0.0006	0.0143	0.0078	0.0714	0.0757	0.1315	0.4009
4	GA	0.0127	0.0951	0.0295	0.0698	0.0191	0.0020	0.0394	0.0317	0.0645	0.1815	0.4144
5	CAPSO	0.0062	0.0846	0.1463	0.0331	0.0002	0.0002	0.0239	0.0612	0.1430	0.1301	0.4447
6	PSO	0.0046	0.0797	0.1511	0.0353	0.0002	0.0011	0.0217	0.0679	0.1547	0.1271	0.4553
7	DFP	0.0160	0.0738	0.1247	0.0425	0.0099	0.0079	0.0357	0.0536	0.1653	0.1264	0.4649

Table 6
Comparison of DE and its variants with GA, CAPSO, PSO and DFP techniques based on statistical inference for 14-bus test system.

Optimization methods	Arithmetic mean value of the objective function ($\bar{J}$)	Standard deviation of objective function (σ)	Best value of objective function (J_{best})	Worst value of objective function (J_{worst})	Frequency of convergence	Confidence level (γ)	Determined value for the engg. application (c)	Standard error of the mean objective function (ε)	Confidence interval of the objective function (μ)	Length of confidence interval of the objective function (L)
EPSDE	0.3890	0.0031	0.3854	0.3973	13	0.95	2.0452	0.0014	$0.3876 \leq \mu \leq 0.3904$	0.0058
SaDE	0.3983	0.0035	0.3940	0.4066	12	0.95	2.0452	0.0016	$0.3967 \leq \mu \leq 0.3999$	0.0066
DE	0.4059	0.0040	0.4009	0.4141	12	0.95	2.0452	0.0018	$0.4041 \leq \mu \leq 0.4077$	0.0075
GA	0.4275	0.0079	0.4144	0.4416	9	0.95	2.0452	0.0036	$0.4239 \leq \mu \leq 0.4311$	0.0148
CAPSO	0.4592	0.0087	0.4447	0.4699	8	0.95	2.0452	0.0039	$0.4552 \leq \mu \leq 0.4632$	0.0162
PSO	0.4718	0.0097	0.4553	0.4846	7	0.95	2.0452	0.0044	$0.4674 \leq \mu \leq 0.4762$	0.0180
DFP	0.4873	0.0104	0.4649	0.4995	5	0.95	2.0452	0.0048	$0.4825 \leq \mu \leq 0.4920$	0.0194

switched, all load bus voltages and line flow are within limit. Table 11 shows comparison of load on load buses and bus voltages; (i) before and after load shedding without optimization, (ii) after optimized load shedding using DE and its variants (SaDE and EPSDE), DFP, PSO, CAPSO and GA techniques. Table 12 gives comparison of optimal load shedding at different load buses (bus nos. 9, 14, 15, 16, 17, 18, 22) using DE and its variants, have been compared with DFP, PSO, CAPSO and GA techniques. Table 13 gives the comparison of DE and its variants, with DFP, PSO, CAPSO and GA based on mean value, standard deviation, best value, worst value, frequency of convergence, standard error, length of confidence interval and confidence interval of objective function. Table 14 gives the comparison of class interval and proportionate frequencies of objective function using DE, SaDE, EPSDE, DFP, PSO, CAPSO and GA techniques. Fig. 2 shows a plot between objective function (J) and computational time for convergence of DE, SaDE, EPSDE, DFP, PSO, CAPSO and GA techniques for best objective function accounting the effect of variation in parameters. Variation in values of parameters was within the specified range. CPU time is with respect to P-IV intel Core 2 Duo machine which includes the time for load flow program and total number of iterations. Static voltage stability limit obtained at the end of optimization process using EPSDE, SaDE, DE, GA, CAPSO, PSO and DFP techniques are 22.4775 pu, 22.2680 pu,22.2448 pu, 22.1572 pu, 22.0997 pu, 21.9807 pu and 21.9640 pu respectively. Similarly it is observed that DE and its variants (SaDE and EPSDE) give much better global optimal results than genetic algorithm, PSO, CAPSO and classical technique DFP.

7. Conclusions

A new algorithm has been developed considering operating and stability inequality constraints for anticipatory optimum load shedding at selected load buses. A computational algorithm for minimum load shedding has been developed using DE and its improved variants. Load buses have been ranked based on sensitivity of Schur's inequality proximity indicator of load flow Jacobian with respect to the load (real and reactive load). Load flow Jacobian is obtained using continuation power flow method. Load buses with large sensitivities for load shedding have been selected. The amount of load to be shed at load bus has been optimized using the proposed algorithm. It is stressed that load shedding has been performed at current loading condition. Further, constraints as in Eqs. (17)–(21) are ascertained by performing load flow solution at current operating condition (after load shed) and predicted load condition (accounting load shed). Performance of the developed algorithm has been compared based on mean, standard deviation, best value, worst value, frequency of convergence, standard error of mean, confidence interval, length of confidence interval, class interval and proportionate frequency of objective function with GA, CAPSO, PSO and DFP techniques. The satisfactory results of the simulation (DE and its improved variants SaDE and EPSDE) on 14-bus and 25-bus systems are good enough to accept and validate the proposed methodology.

Appendix A. Overview of PSO, CAPSO and GA

A1. Particle swarm optimization (PSO)

The PSO is a population-based optimization algorithm. Its population is called swarm and each individual is called a particle [21]. Each particle flies through the solution space to search for global optimal solution. The mechanization of the PSO procedure is explained in following steps:

Step-1: a current position is an n-dimensional search space, which represents a potential solution (particle or agent).

$$ls_i^k = (ls_{i,1}^k, ls_{i,2}^k, \ldots, ls_{i,n}^k)$$

where, i denotes ith particle and k denotes the iteration.

Table 7
Comparison of DE and its variants with GA, CAPSO, PSO and DFP techniques based on class interval and proportionate frequency for 14-bus test system.

No. of class	DE			EPSDE			SaDE			GA			CAPSO			PSO			DFP		
	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*	Class interval of objective function		*Proff*
	$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$	
1	0.4009	0.4036	0.40	0.3854	0.3878	0.45	0.3940	0.3965	0.40	0.4144	0.4198	0.30	0.4447	0.4497	0.25	0.4553	0.4616	0.15	0.4649	0.4718	0.40
2	0.4036	0.4062	0.20	0.3878	0.3902	0.25	0.3965	0.3991	0.25	0.4198	0.4253	0.05	0.4497	0.4547	0.15	0.4616	0.4670	0.30	0.4718	0.4787	0.00
3	0.4062	0.4088	0.15	0.3902	0.3925	0.15	0.3991	0.4016	0.15	0.4253	0.4307	0.10	0.4547	0.4598	0.00	0.4670	0.4729	0.20	0.4787	0.4857	0.30
4	0.4088	0.4115	0.15	0.3925	0.3950	0.10	0.4016	0.4041	0.10	0.4307	0.4362	0.40	0.4598	0.4648	0.25	0.4729	0.4787	0.00	0.4857	0.4926	0.20
5	0.4115	0.4141	0.10	0.3950	0.3973	0.05	0.4041	0.4066	0.10	0.4362	0.4416	0.15	0.4648	0.4699	0.35	0.4787	0.4846	0.35	0.4926	0.4995	0.40

Proff—proportionate frequency.

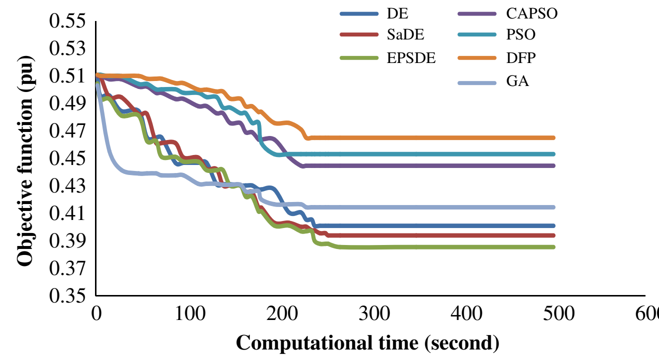

Fig. 1. Plot of convergence of objective function with respect to computational time using DE, SaDE, EPSDE, GA, CAPSO, PSO and DFP algorithm for 14-bus system.

Table 8
Load flow solution for 25-bus test system under stressed condition. Total load (S_{di})=20.1877 pu, proximity indicator (τ)=0.1326, static voltage stability limit=20.6763 pu.

Bus no.	Bus voltage (pu)		Load (pu)	
1	V_1	1.0949	S_{d1}	5.7832
2	V_2	1.0557	S_{d2}	0.2871
3	V_3	1.0437	S_{d3}	1.4523
4	V_4	1.0287	S_{d4}	0.3131
5	V_5	1.0237	S_{d5}	0.7218
6	V_6	0.9079	S_{d6}	0.4348
7	V_7	0.9126	S_{d7}	0.4348
8	V_8	0.9054	S_{d8}	0.6875
9	V_9	0.8792	S_{d9}	0.4348
10	V_{10}	0.9178	S_{d10}	0.4348
11	V_{11}	0.8898	S_{d11}	0.1375
12	V_{12}	0.8872	S_{d12}	0.2750
13	V_{13}	0.9108	S_{d13}	0.7218
14	V_{14}	0.9029	S_{d14}	0.4674
15	V_{15}	0.9032	S_{d15}	0.6976
16	V_{16}	0.9422	S_{d16}	0.8696
17	V_{17}	0.8683	S_{d17}	1.7392
18	V_{18}	0.8841	S_{d18}	0.4348
19	V_{19}	0.9750	S_{d19}	0.4348
20	V_{20}	0.9980	S_{d20}	0.7218
21	V_{21}	0.9043	S_{d21}	0.5827
22	V_{22}	0.8162	S_{d22}	0.5827
23	V_{23}	0.8991	S_{d23}	0.4348
24	V_{24}	0.8189	S_{d24}	0.4348
25	V_{25}	0.8236	S_{d25}	0.7218

Step-2: a current velocity (p_i^k) for ith agent in kth iteration is given as:

$$p_i^k = (p_{i1}^k, p_{i2}^k, \ldots, p_{in}^k)$$

Step-3: the fitness of each particle is evaluated using objective function. The best position of a particle is in terms of objective function and is denoted as $P_{best(i)}$. The position of the best individual of the whole swarm is named as global best position, G_{best}. At each iteration the particle is updated by the following relation:

$$p_i^{(k+1)} = w.p_i^k + c_1.rand_1.(P_{best(i)}^k - ls_i^k) + c_2.rand_2.(G_{best(i)}^k - ls_i^k) \quad (A-1)$$

$$ls_i^{(k+1)} = ls_i^k + p_i^{k+1} \quad (A-2)$$

where, $P_{best(i)}^K$ is the best previous position of ith particle, G_{best}^K is the global best position among all the participation in the swarm

Table 9
Load bus ranking based on sensitivity of Schur's inequality proximity indicator with respect to system load for 25-bus system.

Sr. no.	Load bus	Sensitivity
1	14	0.0686[a]
2	15	0.0668[a]
3	22	0.0657[a]
4	17	0.0653[a]
5	16	0.0649[a]
6	9	0.0643[a]
7	18	0.0641[a]
8	21	0.0619
9	12	0.0611
10	13	0.0602
11	25	0.0601
12	6	0.0583
13	11	0.0507
14	8	0.0401
15	7	0.0392
16	24	0.0291
17	23	0.0192
18	10	0.0051
19	19	0.0040
20	20	0.0021

[a] Load buses selected for load shedding.

Table 10
Effect of DE parameters on optimization of objective function and number of iteration required for convergence for 25-bus test system.

Case	α	c_r	J	No. of iterations for convergence
1	0.80	0.75	1.9556	796
2	0.80	0.65	1.9775	827
3	0.80	0.60	1.9270	867
4	0.85	0.50	1.9504	734
5	0.85	0.45	1.9286	785
6	0.85	0.40	1.9442	849
7	0.90	0.65	1.9515	773
8	**0.90**	**0.60**	**1.9248**	**813**
9	0.90	0.55	1.9280	833
10	0.95	0.50	1.9983	847
11	0.95	0.45	1.9391	718
12	0.95	0.40	1.9285	757
13	0.70	0.65	1.9599	819
14	0.70	0.60	1.9374	799
15	0.70	0.55	1.9907	726
16	0.75	0.50	1.9601	782
17	0.75	0.45	1.9521	826
18	0.75	0.40	1.9563	739
19	0.65	0.45	1.9785	787
20	0.65	0.40	1.9511	841

Bold face number indicates minimum value of objective function has been achieved in 813 iterations with Cr=0.60 and α=0.90.

from objective function viewpoint, $r_1(.)$ and $r_2(.)$ are random digit generated from uniform distribution [0,1]. The process of updating is repeated until a user defined stopping criterion is satisfied. 'w' Is an inertia weight that is typically chosen in the range [0,1]. A large inertia weight facilitates global exploration and a smaller inertia weight tends to facilitate local exploration to fine-tune the current search area. Therefore the inertia weight 'w' is an important parameter for the PSO's convergence behavior. A suitable value for the inertia weight usually provides balance between global and local exploration abilities and consequently, results in a better optimum solution. In view of this, it has been suggested that iteration wise the weight 'w' is varied according to the following relation [22]:

$$w = w_{max} - ((w_{max} - w_{min})/NIT).NIT_{max} \quad (A-3)$$

where, NIT_{max} is the maximum number of iteration supplied and NIT denotes number of iteration. w_{max} and w_{min} denote maximum and minimum values of inertia weights. Thus, as iteration increases w varies from w_{max} say 2.0 to w_{min} say 0.5. c_1 And c_2 are acceleration constant selected in the range from 1 to 2. Inequality constraints are accounted by a fly back mechanism [23]. It assumed that the population is initialized in feasible region. Hence flying back to its previous position will assure that particle is always in feasible region.

A2. Co-ordinated aggregation based particle swarm optimization (CAPSO)

A co-ordinated aggregation (CA) based approach has been proposed and applied to solve steady state optimization problem related to power network [24]. Achievement weight factor $AF_{i,j}$ is the ratio of differences between the achievement of particle-i, $J(ls_i^{k-1})$ and better achievements by particles-j $J(ls_j^{k-1})$ to the sum of all these differences. AF_{ij} (achievement factor) is calculated as:

$$AF_{i,j}^{(k-1)} = J(ls_j^{k-1}) - J(ls_i^{k-1}) \Bigg/ \left[\sum_{j \in T} J(ls_j^{k-1}) - J(ls_i^{k-1}) \right] \quad (A-4)$$

'T' represents the set of particles-j with better achievements in terms of objective function. The velocities of the particles except the particle with best achievement are updated using following relation:

$$p_i^k = w^{k-1}.p_i^{k-1} + \sum_j rand_j.AF_{i,j}^{k-1}[ls_j^{k-1} - ls_i^{k-1}] \quad (A-5)$$

Velocity of the best particle (ls_b^{k-1}) in the swarm is updated using relation as given below:

$$p_b^k = w^{k-1}.p_l^{k-1} + rand[ls_p^{k-1} - ls_b^{k-1}] \quad (A-6)$$

rand is random digits from interval [0,1]. (ls_p^{k-1}) Is the position of randomly selected particle from the swarm. Positions of all the particles are updated using relation (A-2). The iterative process is continued for a maximum number of iterations or no improvement in objective function is observed for a specific number of iterations.

A3. Genetic Algorithm (GA)

A3.1. Implementation of developed algorithm to solve formulated problem [25]

Genetic Algorithm (GA) is a generalized search and optimization technique inspired by the theory of biological evolution. GA maintains a population of individuals that represent candidate solutions. Each individual is evaluated to give some measure of its fitness to the problem from the objective function. In each generation, a new population is formed by selecting the more fit individuals based on a particular selection strategy. Some members of the new population undergo genetic operations to form new solutions. The two commonly used genetic operations are crossover and mutation. Crossover is a mixing operator that combines genetic material from selected parents. Mutation acts as a background operator and is used to search the unexplored search space by randomly changing the values at one or more positions of the selected chromosome. Various components of the proposed algorithm used to solve the load shedding problem, the details of which are presented in the following sections.

Table 11
Bus voltages and load on load bus after load shedding with and without optimization techniques for 25-bus test system.

No of buses	Without optimization				After optimized load shedding													
	Base case		Best initial solution based load shedding		DE		EPSDE		SaDE		CAPSO		PSO		DFP		GA	
	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)	Load (pu)	Voltage (pu)
1	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949	5.7832	1.0949
2	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557	0.2871	1.0557
3	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437	1.4523	1.0437
4	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287	0.3131	1.0287
5	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737	0.7218	1.0737
6	0.4348	0.9079	0.4348	0.9654	0.4348	0.9644	0.4348	0.9636	0.4348	0.9638	0.4348	0.9647	0.4348	0.9653	0.4348	0.9653	0.4348	0.9640
7	0.4348	0.9126	0.4348	1.0088	0.4348	1.0049	0.4348	1.0002	0.4348	1.0020	0.4348	1.0069	0.4348	1.0080	0.4348	1.0080	0.4348	1.0059
8	0.6875	0.9054	0.6875	1.0105	0.6875	1.0058	0.6875	1.0002	0.6875	1.0016	0.6875	1.0078	0.6875	1.0092	0.6875	1.0094	0.6875	1.0069
9	0.4348	0.8792	0.2562	0.9928	0.3358	0.9888	0.2984	0.9802	0.3397	0.9796	0.2948	0.9885	0.2824	0.9900	0.2648	0.9910	0.2820	0.9885
10	0.4348	0.9178	0.4348	1.0154	0.4348	1.0119	0.4348	1.0070	0.4348	1.0078	0.4348	1.0131	0.4348	1.0141	0.4348	1.0144	0.4348	1.0126
11	0.1375	0.8898	0.1375	1.0127	0.1375	1.0067	0.1375	1.0001	0.1375	1.0029	0.1375	1.0099	0.1375	1.0115	0.1375	1.0114	0.1375	1.0084
12	0.2750	0.8872	0.2750	1.0077	0.2750	1.0016	0.2750	0.9952	0.2750	0.9985	0.2750	1.0053	0.2750	1.0068	0.2750	1.0066	0.2750	1.0035
13	0.7218	0.9108	0.7218	0.9633	0.7218	0.9624	0.7218	0.9616	0.7218	0.9618	0.7218	0.9626	0.7218	0.9632	0.7218	0.9632	0.7218	0.9619
14	0.4674	0.9029	0.2952	0.9860	0.4067	0.9811	0.4165	0.9867	0.4086	0.9855	0.3356	0.9797	0.3421	0.9786	0.3109	0.9817	0.3316	0.9816
15	0.6976	0.9032	0.5879	0.9832	0.6482	0.9807	0.6559	0.9872	0.6520	0.9834	0.6072	0.9761	0.5989	0.9755	0.6115	0.9784	0.5933	0.9794
16	0.8696	0.9422	0.8165	1.0049	0.8333	1.0024	0.8125	1.0094	0.8333	1.0043	0.8091	0.9997	0.8300	0.9981	0.8259	1.0008	0.8118	1.0023
17	1.7392	0.8683	0.8017	1.0171	0.8831	1.0087	1.0309	0.9999	0.9008	1.0053	0.8437	1.0141	0.8077	1.0162	0.8236	1.0157	0.9688	1.0113
18	0.4348	0.8841	0.1765	1.0026	0.1611	0.9976	0.1633	0.9972	0.2132	0.9952	0.1747	1.0004	0.1783	1.0016	0.1797	1.0012	0.1653	0.9997
19	0.4348	0.9750	0.4348	1.0199	0.4348	1.0186	0.4348	1.0181	0.4348	1.0178	0.4348	1.0192	0.4348	1.0197	0.4348	1.0196	0.4348	1.0190
20	0.7218	0.9980	0.7218	1.0318	0.7218	1.0327	0.7218	1.0332	0.7218	1.0334	0.7218	1.0321	0.7218	1.0316	0.7218	1.0317	0.7218	1.0321
21	0.5827	0.9043	0.5827	0.9894	0.5827	0.9917	0.5827	0.9928	0.5827	0.9935	0.5827	0.9904	0.5827	0.9888	0.5827	0.9891	0.5827	0.9901
22	0.5827	0.8162	0.1591	0.9663	0.1530	0.9701	0.1176	0.9712	0.1189	0.9725	0.1648	0.9680	0.1591	0.9651	0.1601	0.9656	0.1338	0.9667
23	0.4348	0.8991	0.4348	0.9977	0.4348	0.9978	0.4348	0.9981	0.4348	0.9986	0.4348	0.9972	0.4348	0.9973	0.4348	0.9973	0.4348	0.9976
24	0.4348	0.8189	0.4348	0.9801	0.4348	0.9818	0.4348	0.9825	0.4348	0.9834	0.4348	0.9804	0.4348	0.9792	0.4348	0.9795	0.4348	0.9801
25	0.7218	0.8236	0.7218	0.9656	0.7218	0.9660	0.7218	0.9665	0.7218	0.9671	0.7218	0.9652	0.7218	0.9650	0.7218	0.9651	0.7218	0.9655

Table 12
Optimum load shedding at selected load buses using DE, EPSDE, SaDE, CAPSO, PSO, DFP and GA techniques for 25-bus system.

Sr. no.	Selected load buses for load-shedding		Amount of load-shed in per unit (pu) using optimization techniques						
			DE	EPSDE	SaDE	CAPSO	PSO	DFP	GA
1	Bus no. 15	Pd_{15}	0.0157	0.0074	0.0126	0.0728	0.0775	0.0633	0.0823
		Qd_{15}	0.1691	0.1754	0.1600	0.0729	0.0893	0.0951	0.0934
2	Bus no. 14	Pd_{14}	0.0363	0.0258	0.0326	0.1066	0.1006	0.1324	0.1120
		Qd_{14}	0.1161	0.1235	0.1522	0.1282	0.1219	0.1186	0.1143
3	Bus no. 22	Pd_{22}	0.4045	0.4416	0.4367	0.3975	0.4322	0.4244	0.4577
		Qd_{22}	0.1451	0.1469	0.1564	0.1300	0.0856	0.0933	0.0956
4	Bus no. 16	Pd_{16}	0.0065	0.0190	0.0061	0.0353	0.0176	0.0190	0.0301
		Qd_{16}	0.1179	0.1723	0.1206	0.0990	0.0827	0.0948	0.1104
5	Bus no. 18	Pd_{18}	0.3259	0.2659	0.2224	0.2907	0.2850	0.2826	0.2992
		Qd_{18}	0.0017	0.0656	0.0411	0.0123	0.0128	0.0133	0.0171
6	Bus no. 9	Pd_{9}	0.0890	0.1463	0.1019	0.1457	0.1600	0.1797	0.1584
		Qd_{9}	0.0476	0.0026	0.0001	0.0121	0.0111	0.0114	0.0152
7	Bus no. 17	Pd_{17}	0.7850	0.6579	0.7703	0.8142	0.8498	0.8356	0.6878
		Qd_{17}	0.3720	0.2699	0.3559	0.4345	0.4398	0.4270	0.4368
8	**Objective function (pu)**		**1.9248**	**1.8330**	**1.8648**	**2.0640**	**2.0993**	**2.1167**	**2.0295**

Table 13
Comparison of DE and its variants with GA, CAPSO, PSO and DFP techniques based on statistical inference for 25-bus test system.

Optimization methods	Arithmetic mean value of the objective function ($\bar{J}$)	Standard deviation of objective function (σ)	Best value of objective function (J_{best})	Worst value of objective function (J_{worst})	Frequency of convergence	Confidence level (γ)	Determined value for the Engg. Application (c)	Standard error of the mean objective function (ε)	Confidence interval of the objective function (μ)	Length of confidence interval of the objective function (L)
EPSDE	1.8495	0.0155	1.8330	1.8881	13	0.95	2.0452	0.0071	$1.8424 \le \mu \le 1.8566$	0.0291
SaDE	1.8858	0.0185	1.8648	1.9280	12	0.95	2.0452	0.0084	$1.8774 \le \mu \le 1.8942$	0.0346
DE	1.9520	0.0213	1.9248	1.9983	11	0.95	2.0452	0.0097	$1.9422 \le \mu \le 1.9617$	0.0398
GA	2.0708	0.0223	2.0295	2.1078	08	0.95	2.0452	0.0102	$2.0606 \le \mu \le 2.0810$	0.0417
CAPSO	2.1196	0.0229	2.0640	2.1447	07	0.95	2.0452	0.0105	$2.1090 \le \mu \le 2.1301$	0.0430
PSO	2.1504	0.0234	2.0993	2.1815	06	0.95	2.0452	0.0107	$2.1397 \le \mu \le 2.1611$	0.0438
DFP	2.1668	0.0238	2.1167	2.1903	06	0.95	2.0452	0.0109	$2.1560 \le \mu \le 2.1777$	0.0445

A3.2. Representation

Each individual in the genetic population represents a candidate solution. In the binary-coded GA, the solution variables are represented by a string of binary alphabets. The size of the string depends on the precision of the solution required. For problems with more than one decision variables, each variable is usually represented by a sub-string. All the sub-strings are concatenated together to form a bigger string. In the load shedding problem, the elements of the solution consist of all the disturbance variables namely real power (P_{di}), and the reactive power (Q_{di}) at selected load buses. These variables are represented as binary strings in the GA population. With binary representation, an individual in the GA population for the load shedding optimization problem will look like the following.

100001,	…,	111111,	001110,	…,	100101
P_{d1}	…,	P_{di}	Q_{d1}	…,	Q_{di}

A3.3. Fitness function

In the load shedding problem under consideration the objective is to minimize the total real and reactive power curtailment at selected load buses. Run continuation power flow program for each vector of the population and monitor all inequality constraints (17)–(21). If a vector satisfies the constraints call it '*F*' (feasible). Otherwise call it '*NF*' (not-feasible).

(i) Calculate objective function for the feasible vectors.

(ii) Calculate the decoded value of a binary substring s_i.

$$s_i = \sum_{i=0}^{l-1} 2^i.\gamma_i; \quad \gamma_i \in (0,1) \qquad (A-7)$$

(iii) The variable ls_i is coded in a substring s_i of length l_i.

$$ls_i = \underline{ls}_i + \frac{\overline{ls}_i - \underline{ls}_i}{2^{l_i}-1} \sum_{i=0}^{l-1} 2^i.\gamma_i$$
$$ls^0 = [x_1^0, x_2^0, x_3^0, \ldots, x_M^0]$$
$$x_i^{(0)} = [ls_{i,1}^{(0)}, ls_{i,2}^{(0)}, \ldots, ls_{i,NLS}^{(0)}]^T, \quad i = 1,2,\ldots,M \qquad (A-8)$$

(iv) Calculate the fitness function for each satisfied decoded value of a binary string.

$$f(ls) = \frac{1}{1 + J_{calculted}} \qquad (A-9)$$

A3.4. Selection strategy

The selection of parents to produce successive generations plays an important role in the GA. The goal is to allow the "fittest" individuals to be selected more often to reproduce. There are a number of selection methods proposed in the literature; fitness proportionate selection, ranking and tournament selection. Tournament selection is used in this work. In tournament selection, '*M*' individuals are selected randomly from the population, and the best of the '*M*' is inserted into the new population for further

Table 14
Comparison of DE and its variants with GA, CAPSO, PSO and DFP techniques based on class interval and proportionate frequency for 25-bus test system.

No. of class	DE			EPSDE			SaDE			GA			CAPSO			PSO			DFP		
	Class interval of objective function		Proff	Class interval of objective function		Proff	Class interval of objective function		Proff	Class interval of objective function		Proff	Class interval of objective function		Proff	Class interval of objective function		Proff	Class interval of objective function		Proff
	$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$		$J_{(k+1)min}$	$J_{(k+1)max}$	
1	1.9248	1.9345	0.35	1.8330	1.8440	0.50	1.8648	1.8774	0.45	2.0295	2.04516	0.20	2.0640	2.0802	0.15	2.0993	2.1157	0.20	2.1167	2.1314	0.15
2	1.9345	1.9542	0.25	1.8440	1.8550	0.20	1.8774	1.8900	0.25	2.04516	2.0608	0.10	2.0802	2.0963	0.00	2.1157	2.1322	0.00	2.1314	2.1461	0.10
3	1.9542	1.9689	0.20	1.8550	1.8661	0.15	1.8900	1.9027	0.15	2.0608	2.0765	0.20	2.0963	2.1124	0.15	2.1322	2.1486	0.10	2.1461	2.1609	0.00
4	1.9689	1.9836	0.10	1.8661	1.8771	0.10	1.9027	1.9154	0.10	2.0765	2.0921	0.30	2.1124	2.1286	0.25	2.1486	2.1651	0.60	2.1609	2.1756	0.35
5	1.9836	1.9983	0.10	1.8771	1.8881	0.05	1.9154	1.9280	0.05	2.0921	2.1078	0.20	2.1286	2.1447	0.45	2.1651	2.1815	0.10	2.1756	2.1903	0.40

Proff—proportionate frequency.

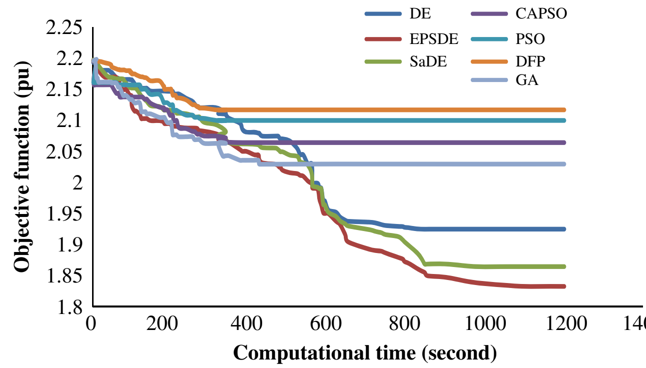

Fig. 2. Plot of convergence of objective function with respect to computational time using DE, SaDE, EPSDE, GA, CAPSO, PSO and DFP algorithm for 25-bus system.

genetic processing. Tournaments are often held between pairs of individuals, although larger tournaments can be used. This procedure is repeated until the mating pool is filled.

Calculate probability for selection (p_s) of each string:

$$p_{si} = \frac{f_i}{\sum_{j=1}^{M} f_j} \quad (A-10)$$

A3.5. Crossover operation

The crossover operator is mainly responsible for the global search property of the GA. The operator basically combines substructures of two parent chromosomes to produce new structures, with the chosen crossover probability (p_c). For binary-coded GA, there exist a number of crossover operators. Crossover can occur at a single position (single crossover), or at number of different positions (multiple crossover). In this work two point crossovers is employed in which two crossover sites are chosen and offspring are created by swapping the bits between the chosen crossover sites.

A3.6. Mutation

The final operator in the genetic algorithm is mutation. The mutation operator is used to inject new genetic material into the population. Mutation changes randomly the new offspring. For binary encoding bit-wise mutation is preferred which switches a few randomly chosen bits from 1 to 0 or from 0 to 1 with a small mutation probability (p_m). After mutation, the new generation is complete and the procedure begins again with the fitness evaluation of the population.

Appendix B

B1. Class interval and proportionate frequency [11]

Obtain class interval and proportionate frequency using following steps:

Step-1: number of classes

$$(k) = 1 + 3.3 \log_{10} n \quad (B-1)$$

where, 'k' is the number of classes and 'n' is the number of sample

Step-2:

$$\Delta w = (J_{max} - J_{min})/k \quad (B-2)$$

where, Δw is the class width of frequency distribution and J_{max} is the maximum value of objective function, J_{min} is the minimum value of objective function

Step-3:

$$J_{k+1} = (k \times \Delta w + J_{min}) \quad k = 1,2,\ldots,5 \qquad (B-3)$$

where, J_{k+1} is the class interval of objective function

Step-4:

$$proff = f/n \qquad (B-4)$$

where, *proff* is the proportionate frequency and *f* is the frequency of class interval.

B2. Confidence interval and length of confidence interval [11]

Obtain confidence interval and length of confidence interval using following steps:

Step-1: arithmetic means value of the objective function:

$$\bar{J} = \left[1/n \sum_{j=1}^{n} (J_j)\right] \qquad (B-5)$$

Step-2: standard deviation of objective function:

$$\sigma = \left[\{1/(n-1)\} \sum_{j=1}^{n} (J_j - \bar{J})^2\right]^{1/2} \qquad (B-6)$$

Step-3: confidence level $(\gamma) = 0.95$; $c = 2.0452$.

Step-4: standard error of the mean objective function

$$(\varepsilon) = c\sigma/\sqrt{n} \qquad (B-7)$$

Step-5: confidence interval of the objective function

$$[(\bar{J} - \varepsilon) \leq \mu \leq (\bar{J} + \varepsilon)] \qquad (B-8)$$

Step-6: length of confidence interval

$$(L) = [(\bar{J} + \varepsilon) - (\bar{J} - \varepsilon)]c = 2\varepsilon c \qquad (B-9)$$

References

[1] C.W. Taylor, Concept of under voltage load shedding for voltage stability, IEEE Transactions on Power Delivery 7 (2) (1992) 480–487.
[2] T.Q. Tuan, J. Fandino, N. Hadjsaid, J.C. Sabonnadiere, H. Vu, Emergency load shedding to avoid risk of voltage instability using indicators, IEEE Transactions on Power Systems 19 (1) (1994) 341–351.
[3] R. Balanathan, N.C. Pahalawaththa, U.D. Annakkage, P.W. Sharp, Under voltage load shedding to avoid voltage instability, IEE Proceedings—Generation, Transmission and Distribution 145 (2) (1998) 175–181.
[4] G.J. Anders, Probability concepts in electric power systems, Wiley, New York, 1990.
[5] R. Billinton, L. Wenyuan, Reliability assement of electric power systems using Monte-Carlo method, Plenum, New York, 1994.
[6] A. Wiszniewski, New criteria of voltage stability margin for the purpose of load shedding, IEEE Transactions on Power Delivery 22 (3) (2007) 1367–1371.
[7] A. Girgis, S. Mathure, Application of active power sensitivity to frequency and voltage variations on load shedding, International Journal of Electrical Power Systems Research 80 (3) (2010) 306–310.
[8] X. Fu, X. Wang, Determination of load shedding to provide voltage stability, International Journal of Electrical Power and Energy Systems 33 (3) (2011) 515–521.
[9] T. Amraee, A.M. Ranjbar, R. Feuillet, Adaptive under-voltage load shedding scheme using model predictive control, International Journal of Electrical Power Systems Research 81 (7) (2011) 1507–1513.
[10] H. Song, S.D. Baik, B. Lee, Determination of load shedding for power-flow solvability using outage-continuation power flow (OCPF), IEE Proceedings—Generation, Transmission and Distribution 153 (3) (2006) 321–325.
[11] E.W. Kreyszig, Advance engineering mathematics, John Wiley and Sons, Inc., 2001.
[12] L.D. Arya, D.K. Sakravadia, D.P. Kothari, Corrective rescheduling for static voltage stability control, International Journal of Electrical Power and Energy Systems 27 (2005) 3–12.
[13] K. Price, R. Storn, J. Lampinen, Differential evolution: a practical approach to global optimization, Springer, 2005.
[14] J. Tvrdik, Adoption in differential evolution: a numerical comparison, Applied Soft Computing Journal 9 (3) (2009) 1149–1155.
[15] J. Lampinen, A constraint handling approach for differential evolution algorithm, in: Proceedings of the 2002 Congress on Evolutionary Computation (CEC-2002), vol. 2, pp. 1468–1473.
[16] V.L. Huang, A.K. Qin, P.N. Suganthan, Self-adaptive differential evolution algorithm for constrained real-parameter optimization, IEEE Congress on Evolutionary Computation (CEC) (2006) 17–24.
[17] R. Mallipeddi, P.N. Suganthan, Ensemble differential evolution algorithm for CEC2011 problems, IEEE Congress on Evolutionary Computation (CEC) (2011) 1557–1564.
[18] P.R. Bijwe, D.P. Kothari, L.D. Arya, Alleviation of line over loads and voltage variations by corrective rescheduling, Proceedings of the IEE—Part C 140 (4) (1993) 249–255.
[19] L.D. Arya, S.C. Choube, M. Shrivastava, D.P. Kothari, Loadability margin enhancement using co-ordinated aggregation based particle swarm optimization (CAPSO), International Journal of Electrical Power and Energy Systems 32 (9) (2010) 975–984.
[20] L.D. Arya, S.C. Choube, D.P. Kothari, Emission constrained secure economic dispatch, International Journal of Electrical Power and Energy Systems 19 (5) (1997) 279–285.
[21] S. He, E. Prempain, Q.H. Wu, An improved particle swarm optimization for mechanical design optimization problems, Journal of Engineering Optimization 36 (5) (2004) 585–605.
[22] J.B. Park, K.S. Lee, J.R. Shin, K.Y. Lee, A particle swarm optimization for economic dispatch with non-smooth cost functions, IEEE Transactions on Power System 20 (1) (2005) 34–42.
[23] M. Varadarajan, K.S. Swarup, Differential evolutionary algorithm for optimal reactive power dispatch, International Journal of Electrical Power and Energy Systems 30 (8) (2008) 435–441.
[24] J. Vlachogiannis Gand, K.Y. Lee, A comparative study on particle swarm optimization for optimal steady state performance of power systems, IEEE Transactions on Power Systems 21 (4) (2006) 1318–1328.
[25] K. Deb, Optimization for engineering design algorithms, PHI, Inc., 2010.
[26] S. Das, P.N. Suganthan, Differential evolution: a survey of the state-of-the-art, IEEE Transactions on Evolutionary Computation 15 (1) (2011) 4–31.
[27] A. Zhou, B.Y. Qu, H. Li, S.Z. Zhao, P.N. Suganthan, Multiobjective evolutionary algorithm: a survey of the state of the art, Swarm and Evolutionary Computation 1 (1) (2011) 32–49.
[28] J. Derrac, S. García, D. Molina, F. Herrera, A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms, Swarm and Evolutionary Computation 1 (1) (2011) 3–18.
[29] A.E. Eiben, S.K. Smit, Parameter tuning for configuring and analyzing evolutionary algorithms, Swarm and Evolutionary Computation 1 (1) (2011) 19–31.