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Hotelling's T^2 charts with variable sample size and control limit

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Abstract

The ideas of variable sampling interval (VSI), variable sample size (VSS), variable sample size and sampling interval (VSSI), and variable parameters (VP) in the univariate case have been successfully applied to the multivariate case to improve the efficiency of Hotelling's T^2 chart with fixed sampling rate (FSR) in detecting small process shifts. However, the main disadvantage in using most of these control schemes is an increasing in the complexity due to the adaptive changes in sampling intervals. In this paper, retaining the lengths of sampling intervals constant, a variable sample size and control limit (VSSC) T^2 chart is proposed and described. The statistical efficiency of the VSSC T^2 chart in terms of the average time to signal a shift in process mean vector is compared with that of the VP, VSSI, VSS, VSI, and FSR T^2 charts. From the results of comparison, it shows that the VSSC T^2 chart for a (very) small shift in the process mean vector gives a better performance than the VSSI, VSS, VSI, and FSR T^2 charts; meanwhile, it presents a similar performance to the VP T^2 chart. Furthermore, from the viewpoint of practicability, it is more convenient for administrating the control chart than the VSI, VSSI, and VP T^2 chart. Thus, it may provide a good option for quick response to small shifts in a multivariate process.

Keywords: Statistical quality control; Multivariate control chart; Genetic algorithms

1. Introduction

The rapid growth of modern data-acquisition technology uses during production has made it common to monitor several correlated quality characteristics simultaneously. As a result, various types of multivariate control charts have been proposed for statistical process control works. A natural multivariate extension to the univariate Shewhart chart is the Hotelling's multivariate control procedure (1947). For this procedure, it is assumed that a sequence of $(p \times 1)$ random vectors $X_1, X_2, X_3, \ldots$, each representing individual observation or sample mean vector of p related quality characteristics, are observed over time. The p related quality characteristics are assumed jointly distributed as p-variate normal with mean vector μ_0 and covariance matrix Σ_0. When X_i's represent the independent

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sample mean vectors with sample size of n, the Hotelling's multivariate control chart signals that a process mean shift has occurred as soon as

$$T_i^2 = n(X_i - \overline{X})'\overline{S}^{-1}(X_i - \overline{X}) > k, \quad (1)$$

where $\overline{X}$ and $\overline{S}$ are the averaged sample mean vector and sample covariance matrix, respectively, from m initial $(p \times 1)$ random vectors prior to on-line process monitoring; k is a specified action limit that is given by Alt (1984) as

$$k = C(m,n,p)F_{p,v,\alpha}, \quad (2)$$

where $C(m,n,p) = \frac{p(m+1)(n-1)}{(mn-m-p+1)}$, $v = mn - m - p + 1$, and $F_{p,v,\alpha}$ is the upper α percentage point of F distribution with p and v degrees of freedom if sample size $n > 1$. Moreover, $C(m,n,p) = \frac{p(m+1)(m-1)}{m^2-mp}$ and $v = m - p$ if sample size $n = 1$.

Recently multivariate CUSUM and multivariate EWMA schemes (Crosier, 1988; Lowry et al., 1992) have been proposed and shown better powerful than T^2 control chart particularly for small or moderate process shifts. In order to increase the power of the original T^2 control chart, He and Grigoryan (2005) proposed the multivariate multiple sampling (MMS) control chart scheme, which is a multivariate extension of a double sampling (DS) $\overline{X}$ chart with least two sampling stages. The assumption of MMS or DS charts is that the minimum time between successive samples is negligible. The DS $\overline{X}$ chart was proposed by Daudin (1992) to improve the statistical efficiency of the $\overline{X}$ chart without increased sampling. Daudin's work has also been successfully extended to monitoring of process variability (Grigoryan and He, 2005) as well as joint monitoring of process mean and variability (He and Grigoryan, 2006).

On the other hand, Aparisi extended the ideas of adaptive sample size, sampling interval, and control limits in the univariate case (e.g., Reynolds et al., 1988; Tagaras, 1998; Prabhu et al., 1994; Costa, 1997) to the multivariate case, and proposed three types of modified T^2 charts with variable sample size (VSS), variable sampling interval (VSI), and variable sample size and sampling interval (VSSI) features, respectively (see Aparisi, 1996; Aparisi and Haro, 2001; Aparisi and Haro, 2003), given that the μ_0 and Σ_0 were known. In contrast to the MMS or DS charts, the assumption of VSS, VSI, and VSSI charts is that the minimum time between successive samples is positive (He et al., 2002). The study results indicate that the VSI T^2 charts obtain a great improvement in the value of average time to signal (ATS) for moderate process shifts while the VSSI T^2 charts obtain a great improvement for small process shifts. As compared with the MMS chart with two sampling stages, the VSSI T^2 chart is better in detecting small shifts in the process mean vector. Although the MMS chart with multiple sampling stages ($\geqslant 3$) begins to give a better performance than the VSSI T^2 chart, it might become more difficult to work with the MMS chart in industrial practice.

Chen and Chiou (2005) considered that μ_0 and Σ_0 are unknown and extended Aparisi's works to the fully variable parameters (VP) T^2 charts, which allow the sample size, the sampling interval and the control limit changeable simultaneously. From numerical comparisons, it indicates that for small process shifts the VP T^2 charts provide better ATS values than the VSSI charts, remaining the ATS values almost the same for moderate process shifts.

Although the adaptive T^2 charts have been shown to be quicker than Hotelling's T^2 charts with fixed sampling rate (FSR) in detecting small or moderate shifts for a process, the adaptive sampling schemes for the VSI, VSSI, and VP T^2 charts use two sampling intervals (the short and long intervals) in which the sampling times in a given time period will be unpredictable and cause administrative inconvenience. Consequently, this paper aims to propose the variable sample size and control limit (VSSC) T^2 chart in which the waiting time between successive samples are fixed. In the next section, a description and an example of the VSSC T^2 charts are presented. Like the way in He and Grigoryan (2005), the statistical design of the VSSC T^2 charts is formulated as a design optimization problem in Section 3. Applying the genetic algorithms, the VSSC T^2 chart is statistically designed to compare with the FSR T^2 chart and other types of adaptive T^2 charts in terms of their speed in detecting off-target conditions. Finally, concluding remarks are presented in the last section.

2. The VSSC T^2 control chart

When a FSR T^2 chart is used to monitor a multivariate process, a sample of size n_0 is drawn every h_0 hours, and the value of the T^2 statistic (i.e. sample point) is plotted on a control chart with $k_0 = C(m,n_0,p)F_{p,v,\alpha_0}$ as the control limit or action limit.

The VSSC T^2 chart is a modification of the FSR T^2 chart. Let n_1 and n_2 be the minimum and maximum sample sizes, respectively, such that $n_1 < n_0 < n_2$ while keeping the sampling interval fixed at h_0 for the administration consideration. The decision to switch between the maximum and minimum sample size depends on position of the prior sample point on the control chart. If the prior sample point $(i - 1)$ falls in the safe region, the minimum sample size n_1 will be used for the current sample point (i); if the prior sample point $(i - 1)$ falls in the warning region, the maximum sample size n_2 will be used for the current sample point (i). Finally, if the prior sample point falls in the action region, then the process is considered out-of-control. Here the safe, warning, and action regions are given by the warning limit w_j and the action limit $k_j = C(m, n_j, p)F_{p,v_j,\alpha_j}$ (safe region is given by $[0, w_j]$, warning region is given by $(w_j, k_j]$, and action region is given by (k_j, ∞)), respectively, where $j = 1$ if the prior sample point comes from the small sample, and $j = 2$ if the prior sample point comes from the large sample. It is assumed that $w_1 > w_2$ and $k_1 > k_0 > k_2$. Moreover, for the sake of simplicity we set w_j as follows:

$$p_0 = \Pr\{T_i^2 < w_1 | T_i^2 < k_1\} = \Pr\{T_i^2 < w_2 | T_i^2 < k_2\}, \quad (3)$$

where p_0 is the conditional probability of a sample point falling in the safe region, given that it did not fall in the action region. Eq. (3) implies that p_0 is independent of the sample size.

The following function summarizes the control scheme of the VSSC T^2 chart:

$$(n(i), w(i), k(i)) = \begin{cases} (n_2, w_2, k_2) & \text{if } w(i-1) < T_{i-1}^2 \leqslant k(i-1), \\ (n_1, w_1, k_1) & \text{if } 0 \leqslant T_{i-1}^2 \leqslant w(i-1). \end{cases} \quad (4)$$

During the in-control period, it is assumed that the sizes of samples are chosen at random between two values when the process is starting or after a false alarm. Small size is selected with probability of p_0 , whereas large size is selected with probability of $(1 - p_0)$.

As the warning limit and action limit are varied, depending on the sample size, it is possible for the practitioner to employ one chart for a small sample and another chart for a large sample. However, this is a tedious process. To avoid it, one may construct a control chart with two scales, one on left hand side and the other on the right hand side. The observation from small sample can be plotted according to the left scale, and the one from large sample can be plotted according to the right scale. However, it is still difficult for a practitioner to plot the point because the left scale is not proportional to the right scale. Costa (1999) recommended breaking the left scale and plotting these sample points anywhere inside the right region, regardless of the right position. In this way, the effort to monitor a process with the VSSC control chart or with the FSR control chart is almost the same. Fig. 1 illustrates an example of the VSSC T^2 chart with one hour fixed sampling interval length. The first sample is taken at 8:00 with small sample size. The second sample is taken at 9:00 with large sample size because the first sample point falls in the warning region. Since the second sample point still falls in the warning region, the third sample is taken at 10:00 with large sample size. Once more, the fourth sample is taken at 11:00 with small sample size because the third sample point falls in the safe region. The sampling procedure continues until a sample point falls in the action region such as the seventh sample in Fig. 1.

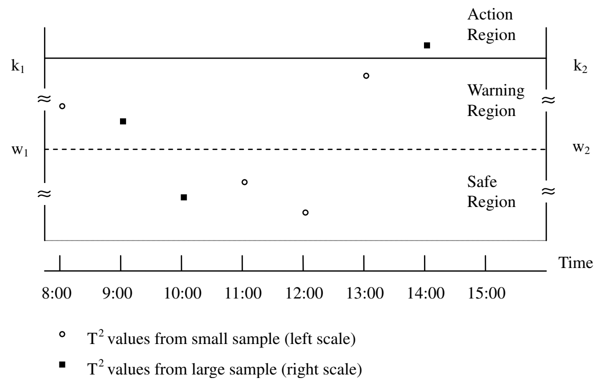

Fig. 1. An example of the VSSC T^2 chart.

3. Statistical design of the VSSC T^2 chart

In evaluating the statistical efficacy of the VSSC T^2 chart, it is reasonable to compare the performance of the VSSC T^2 chart with the FSR or other available type's T^2 charts under equal conditions. The performance of the VSSC T^2 chart is relative to a decision of the following chart design parameters: the sample size (n_1 and n_2), the action limit (k_1 and k_2), and the warning limit (w_1 and w_2). In this paper, the design for the above parameters is formulated as a design optimization problem, which can be mathematically written as follows:

$$\min_{n_1,n_2,k_1,k_2,w_1,w_2} \mathrm{ATS}_1, \tag{5}$$

$$\text{s.t. } n_1p_0 + n_2(1-p_0) = n_0, \tag{6}$$

$$\alpha_1p_0 + \alpha_2(1-p_0) = \alpha_0. \tag{7}$$

The objective function (5) is to minimize ATS_1, i.e. the out-of-control average time length it takes the VSSC T^2 chart to signal a process mean shift. When the sampling interval remains constant, this value is given as a constant multiple (h_0) of the out-of-control average run length (ARL_1), which is defined as the average number of samples before the chart produce a signal.

Since the adaptive control limit is considered in the VCCS T^2 chart, the ARL_1 value for this chart depends on the first sample. When the first sample size is small, the ARL_1 is obtained based on the number of sample points, U_1, in the safe region taken from the start of the process to the time the chart signals. Thus, U_1 is a geometric random variable with parameter $(1-q)$, where q is the conditional probability of obtaining another point in the safe region, given that the current sample point belongs to the safe region. Thus,

$$q = p_{11} + p_{12}\sum_{i=1}^{\infty} p_{22}^{i-1}p_{21}, \tag{8}$$

where

$$p_{11} = \Pr\{T_i^2 < w_1 | T_i^2 \sim C(m,n_1,p)F_{p,v_1,\lambda_1}\},$$
$$p_{12} = \Pr\{w_1 < T_i^2 < k_1 | T_i^2 \sim C(m,n_1,p)F_{p,v_1,\lambda_1}\},$$
$$p_{21} = \Pr\{T_i^2 < w_2 | T_i^2 \sim C(m,n_2,p)F_{p,v_2,\lambda_2}\},$$
$$p_{22} = \Pr\{w_2 < T_i^2 < k_2 | T_i^2 \sim C(m,n_2,p)F_{p,v_2,\lambda_2}\},$$

where the F_{p,v_j,λ_j} represents the non-central F distribution with p and v_j degrees of freedom and non-centrality parameter λ_j for $j=1$ and 2. The non-centrality parameter λ_j is given by $\lambda_j = n_j(\mu_1-\mu_0)'\Sigma_0{}^{-1}(\mu_1-\mu_0)$. If we let $\delta = \sqrt{(\mu_1-\mu_0)'\Sigma_0^{-1}(\mu_1-\mu_0)}$, then λ_j can be rewritten by $\lambda_j = n_j\delta^2$, where δ is the Mahalanobis distance used to measure a change in the process mean vector.

Let N_i^1 be the number of subsequent sample points from the current sample point (i) until another sample point outside the warning region, given that the current sample point belongs to the safe region. The N_i^1's are independent and identically distributed with $\Pr\{N_i^1 = 1\} = 1-p_{12}$ and $\Pr\{N_i^1 = 1+j\} = p_{12}p_{22}^{j-1}(1-p_{22})$ for $j=1,2,\ldots,\infty$. Moreover, the expected value of N_i^1 are

$$E(N_i^1) = 1 + \frac{p_{12}}{1-p_{22}}. \tag{9}$$

As a result, the total number of sample points from the start of the process to the time the chart signals is

$$\mathrm{N}_1 = \sum_{i=1}^{U_1} N_i^1. \tag{10}$$

Then, using the Wald's identity, the ARL_1 can be written as

$$E(N_1) = E(U_1)E(N_i^1) = \frac{1-p_{22}+p_{12}}{D}, \tag{11}$$

where $D = 1 - p_{11} - p_{22} + p_{11}p_{22} - p_{12}p_{21}$.

Similarly, when the first sample is large, let U_2 be the number of sample points in the warning region taken from the start of the process to the time the chart signals. Also let N_i^2 be the number of subsequent sample points (i) from the current sample point until another sample point outside the safe region, given that the current sample point belongs to the warning region. Then, the ARL_1 is determined as the expected value of N_2 – the total number of sample points from the start of the process to the time the chart signals, and it is expressed as

$$E(N_2) = E(U_2)E(N_i^2) = \frac{1-p_{11}+p_{21}}{D}. \tag{12}$$

Since the first sample is chosen at random with probability of p_0 for being small and $(1-p_0)$ for being large, the ARL_1 is given by

$$\mathrm{ARL}_1 = p_0E(N_1) + (1-p_0)E(N_2). \tag{13}$$

Constraints (6) and (7) ensure that the VSSC T^2 chart and the FSR T^2 chart or other type's T^2 charts have the "matched" in-control performances, in terms of the same false alarm rate and identical number of items per unit time during the in-control period. Accordingly, the optimal values of the design parameters: n_1, n_2, w_1, w_2, α_1 (or k_1), and α_2 (or k_2) for the operation of VSSC T^2 would be drawn from the optimization model.

For a particular application, the procedure to solve the optimization model may be programmed as follows.

Input variables: n_0, h_0, α_0, m, p, and δ

Computation:

1. Randomly generate the values for one pair of (n_1, n_2) or (α_1, α_2) as well as one element from each remaining pair.
2. Determine the remainders by the constraints (6) and (7).
3. Determine the values of w_j by the following formula.

$$w_j = C(m, n_j, p)F^{-1}_{p,mn_j-m-p+1}((1-\alpha_j)p_0), \quad j = 1, 2, \tag{14}$$

where $F^{-1}_{p,mn_j-m-p+1}(\cdot)$ is the inverse of the F distribution function with p and $(mn_j - m - p + 1)$ degrees of freedom.

4. Repeat the above three steps until an optimal solution with optimal objective value is obtained.

Output variables: n_1^*, n_2^*, w_1^*, w_2^*, α_1^*, α_2^*, and ARL_1^*.

4. Using genetic algorithms to solve the model

As aforementioned above, the design optimization problem is formulated as a decision problem with the mixed continuous-discrete decision variables and a discontinuous and non-convex solution space. If typical non-linear programming techniques are used to solve this optimization problem, they may perhaps be inefficient and time-consuming. Genetic Algorithms (GAs) are well suitable for solving such a problem because they have a less chance of converging to local optima in a multimodal space as compared with the typical techniques.

GAs are search algorithms that were developed based on an analogy with natural selection and population genetics in biological system (Goldberg, 1989). They have been commonly used or modified for solving many kinds of optimization problems. Recently, several extensive applications to the design optimization problem of quality control charts have been presented, e.g., Aparisi and García-daíz (2003), He et al. (2002), He and Grigoryan (2002), He and Grigoryan (2005), He and Grigoryan (2006), Grigoryan and He (2005), Chen (2004). When applying GAs to the model (5)–(7), the operations of GAs include the four steps: (1) randomly generate an initial solution population where each candidate solution $(n_1, n_2, w_1, w_2, \alpha_1, \alpha_2)$ in the population is represented as a string of bits; (2) assign each bit string a value according to a fitness function (i.e., the objective function that minimizes the ATS_1) and select strings from the old population randomly a but biased by their fitness; (3) recombine these strings by using the crossover and mutation operators; (4) produce a new generation of strings that are more fit than the previous one. The termination condition is achieved when the number of generations is large enough or a satisfied fitness value is obtained.

In the statistical design of the VSSC T^2 chart, the tool used for carrying out the GAs was a commercial software package called EVOLVER (http://www.palisade.com.au/evolver) that can function as an add-in to Microsoft Excel.

The quality of the solution generated by the GAs might depend on the setting of their control parameters: the population size (PS), the crossover probability (CP), and the mutation rate (MR). In order to find the optimal setting of these parameters that minimizes the fitness function (i.e., ATS_1), an orthogonal array experiment is developed in this study. The com-

Table 1
Parameters and levels in the GAs

Parameters	Level 1	Level 2	Level 3
PS	50	75	100
CP	0.10	0.30	0.50
MR	0.05	0.10	0.25

Table 2
Experiment layout of L9 (3^4) orthogonal array and results

Assay	PS	CP	MR	4	y_1	y_2	y_3	SN
1	1	1	1	1	9.5007	9.5018	9.4999	−19.55520355
2	1	2	2	2	9.5013	9.5010	9.5046	−19.55657488
3	1	3	3	3	9.5023	9.4999	9.5001	−19.55514263
4	2	1	2	3	9.5280	9.5047	9.5005	−19.56459154
5	2	2	3	1	9.5044	9.5246	9.5000	−19.56331140
6	2	3	1	2	9.5067	9.4999	9.5006	−19.55666661
7	3	1	3	2	9.4999	9.5041	9.5002	−19.55578268
8	3	2	1	3	9.5534	9.5324	9.5000	−19.58060500
9	3	3	2	1	9.5011	9.4999	9.5037	−19.55590452

putation experiment was carried out for the case of two correlated quality characteristics with $n_0 = 2$, $m = 600$, $\delta = 0.75$, $h_0 = 1$, and $\alpha_0 = 0.005$. In the orthogonal array experiment, three levels of each parameter are planned as shown in Table 1. The L9(3^4) orthogonal array is used to assign the three parameters in its first three columns. In the experiment of L9(3^4) orthogonal array, there are totally nine assays (different level combinations of the three parameters). For each assay, three replicates of ATS_1 values from the GAs (denoted by y_1, y_2, and y_3) are recorded in Table 2. Because the ATS_1 is a characteristic of smaller-the-better characteristic, the appropriate signal-to-noise ratio (SN) for evaluating the experiment results (Taguchi, 1987) is

$$SN = -10 \times \log\left(\frac{1}{r}\sum_{i=1}^{r} y_i^2\right), \quad (15)$$

where r is the total number of replicates of ATS_1 per assay. The values of SN ratio for each assay are listed in Table 2 (Note that SN ratio is a larger-the-better index). Accordingly, the sum of SN ratio at each level for each parameter can be obtained and shown in Table 3. Based on the information in Table 3, it is observed that diverse levels for each parameter make no significant difference to the sum of SN ratio, i.e., diverse levels for each parameter did not significantly affect the solution generated by the GAs. The outcome might be caused by that

Table 3
Sum of SN ratio at each level for each parameter in the GAs

	PS	CP	MR
Level 1	−58.6669*	−58.6756	−58.6925
Level 2	−58.6846	−58.7005	−58.6771
Level 3	−58.6923	−58.6677*	−58.6742*

Table 4
Comparison between out-of-control ATS values for the schemes VSSC and FSR

n_0	m	δ	n_1/n_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS_{VSSC}	ATS_{FSR}	%
2	600	0.25	1/43	0.000/0.207	19.78/3.15	7.54/2.98	65.94	145.15	54.57
		0.50	1/20	0.000/0.091	17.15/4.80	5.93/3.96	22.04	76.20	71.08
		0.75	1/11	0.000/0.050	23.01/6.01	4.64/3.87	9.50	37.35	74.56
		1.00	1/8	0.000/0.035	19.65/6.75	3.92/3.52	5.20	19.13	72.82
		1.25	1/6	0.000/0.024	17.50/7.46	3.24/3.04	3.46	10.48	66.98
		1.50	1/5	0.000/0.019	15.49/8.01	2.79/2.67	2.63	6.16	57.31
3	300	0.25	1/58	0.000/0.138	18.01/3.98	681/3.58	44.64	127.92	65.10
		0.50	1/22	0.000/0.052	21.18/5.93	4.77/3.91	13.34	55.21	75.84
		0.75	1/13	0.000/0.029	18.68/7.09	3.63/3.32	5.88	23.62	75.11
		1.00	1/9	0.000/0.020	22.09/7.88	2.80/2.67	3.54	11.10	68.11
		1.25	1/7	0.000/0.014	16.98/8.53	2.22/2.15	2.57	5.79	55.61
		1.50	2/7	0.000/0.013	11.89/8.77	3.23/3.13	2.06	3.37	38.87
4	200	0.25	1/66	0.000/0.108	22.81/4.48	6.31/3.83	34.20	113.75	69.93
		0.50	1/24	0.000/0.038	22.93/6.57	4.16/3.64	9.69	42.13	77.00
		0.75	1/14	0.000/0.020	16.71/7.83	2.98/2.82	4.51	16.43	72.55
		1.00	2/10	0.000/0.019	17.76/7.95	2.82/2.68	2.87	7.33	60.85
		1.25	2/9	0.001/0.014	13.61/8.64	2.54/2.45	2.15	3.77	42.97
		1.50	3/8	0.004/0.008	11.20/9.65	3.22/3.17	1.74	2.25	22.67
5	150	0.25	1/72	0.000/0.087	20.51/4.91	5.94/3.98	27.94	101.94	72.59
		0.50	1/26	0.000/0.029	17.13/7.10	3.76/3.40	7.74	33.38	76.81
		0.75	2/16	0.001/0.023	21.68/7.60	3.15/2.94	3.74	12.15	69.22
		1.00	3/12	0.001/0.019	14.02/8.05	3.05/2.91	2.47	6.26	60.54
		1.25	4/10	0.004/0.012	11.52/8.90	3.59/3.50	1.89	2.73	30.77
		1.50	4/9	0.005/0.005	10.82/10.72	3.22/3.21	1.52	1.71	11.11
10	80	0.25	1/87	0.000/0.045	18.00/6.30	4.76/3.92	14.74	64.28	77.07
		0.50	3/33	0.000/0.021	20.82/7.82	2.99/2.81	4.11	14.25	71.16
		0.75	5/22	0.001/0.015	14.33/8.59	2.49/2.41	2.28	4.40	48.18
		1.00	9/18	0.004/0.010	11.10/9.41	4.40/4.31	1.63	1.96	16.84
		1.25	9/13	0.005/0.005	10.84/10.80	2.79/2.78	1.24	1.25	0.80
		1.50	9/11	0.005/0.005	10.84/10.81	1.40/1.40	1.05	1.05	0.00

Two correlated quality characteristics, fixed sampling interval length $h_0 = 1$, and $\alpha_0 = 0.005$.
$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

a large number of 100,000 generations that spent about 3 minutes was employed in each run through the study. Even so, the optimal level combination of the three parameters in the GAs should be appropriately chosen: PS = 50; CP = 0.5; MR = 0.25.

5. Performances of the VSSC T^2 charts

In this section we compare the VSSC T^2 charts with the FSR T^2 charts and other type's adaptive T^2 charts in order to evaluate the statistical performances of the VSSC charts.

5.1. Comparing the VSSC and FSR T^2 charts

The comparisons between the VSSC and FSR charts were conducted for the situations in which two or four related quality characteristics were simultaneous control. Tables 4 and 5 illustrate the values of n_1, n_2, w_1, w_2, $\alpha_1(k_1)$, and $\alpha_2(k_2)$ that minimize ATS_1 by fixing h_0 at 1.00 and α_0 at 0.005 when different degrees of the process mean shifts (δ): 0.25, 0.50, 0.75, 1.00, 1.25, and 1.50 are present. To make a rational choice of m, we began with picking a value from the range of $800p/3(n_0 - 1)$ to $400p/(n_0 - 1)$ in Nedumaran and Pignatiello (1999) for different combinations of p and n_0. The range was speculated so that the T^2 charts based on the estimated parameters can perform in a similar manner to the charts based on true parameters during the on-line process monitoring stage. After that, a sensitivity analysis is conducted to investigate the effects of m values on the optimal design parameters and ATS.

As compared with the FSR T^2 chart, the improvements (%) in the ATS_1:

Table 5
Comparison between out-of-control ATS values for the schemes VSSC and FSR

n_0	m	δ	n_1/n_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS_{VSSC}	ATS_{FSR}	%
2	1400	0.25	1/51	0.000/0.229	20.53/5.63	11.69/5.45	78.77	160.30	50.86
		0.50	1/24	0.000/0.110	22.40/7.54	9.87/6.77	28.72	99.31	71.08
		0.75	1/14	0.001/0.051	18.29/9.45	8.44/7.24	13.08	53.89	75.73
		1.00	1/9	0.000/0.039	22.53/10.11	7.25/6.60	6.86	28.68	76.08
		1.25	1/7	0.001/0.025	18.44/11.20	6.48/6.17	4.38	15.76	72.21
		1.50	1/6	0.000/0.023	20.52/11.33	6.01/5.76	3.18	9.12	65.13
3	700	0.25	1/69	0.000/0.169	26.70/6.44	10.89/6.09	53.98	146.82	63.23
		0.50	1/28	0.000/0.065	22.61/8.86	8.61/7.04	17.45	76.05	77.05
		0.75	1/15	0.000/0.034	22.80/10.44	6.93/6.40	7.66	35.16	78.21
		1.00	1/10	0.000/0.021	20.63/11.59	5.75/5.53	4.40	16.71	73.67
		1.25	1/8	0.000/0.017	21.70/12.11	5.05/4.91	3.04	8.56	64.49
		1.50	2/7	0.002/0.017	17.33/12.03	6.02/5.82	2.39	4.79	50.10
4	500	0.25	1/83	0.000/0.128	21.45/7.17	10.38/6.59	42.12	134.93	68.78
		0.50	1/30	0.000/0.048	26.64/9.61	7.80/6.82	12.59	60.05	79.03
		0.75	1/16	0.000/0.024	22.52/11.26	6.06/5.76	5.66	24.69	77.08
		1.00	1/11	0.000/0.016	22.88/12.20	4.93/4.79	3.45	10.93	68.44
		1.25	2/9	0.000/0.016	20.71/12.19	5.07/4.92	2.48	5.41	54.16
		1.50	3/8	0.003/0.013	16.13/12.80	6.90/5.88	1.99	3.05	34.75
5	400	0.25	1/89	0.000/0.109	26.05/7.58	9.91/6.77	34.41	124.40	72.34
		0.50	1/31	0.000/0.037	24.64/10.25	7.16/6.51	9.93	48.58	79.56
		0.75	1/18	0.001/0.021	24.84/11.60	5.63/5.39	4.61	18.28	74.78
		1.00	2/13	0.000/0.017	21.16/12.06	5.21/5.04	2.90	7.72	62.44
		1.25	3/11	0.002/0.014	17.10/12.60	5.41/5.29	2.16	3.79	43.01
		1.50	4/10	0.004/0.008	15.29/13.92	6.46/6.40	1.73	2.20	21.36
10	150	0.25	1/110	0.000/0.056	22.58/9.26	8.66/7.08	18.57	86.53	78.54
		0.50	2/38	0.000/0.021	22.30/11.64	5.93/5.55	5.14	21.51	76.10
		0.75	4/24	0.000/0.016	22.56/12.28	4.96/4.81	2.66	6.41	58.50
		1.00	7/20	0.004/0.009	15.80/13.58	5.64/5.57	1.86	2.60	28.46
		1.25	9/16	0.005/0.005	15.10/14.92	6.87/6.85	1.38	1.47	6.12
		1.50	9/11	0.005/0.005	15.08/15.05	3.37/3.37	1.11	1.11	0.00

Four correlated quality characteristics, fixed sampling interval length $h_0 = 1$, and $\alpha_0 = 0.005$.
$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

$$\frac{\mathrm{ATS_{FSR}} - \mathrm{ATS_{VSSC}}}{\mathrm{ATS_{FSR}}} \times 100,$$

using the VSSC control schemes are great especially for small ($\delta < 1.00$) process mean shifts. This means a shorter time to detect an out-of-control condition. Sometimes, when the average sample size is large with large process shift ($\delta = 1.50$), the improvement are minor. It may be observed that the space between the maximum and minimum control limits tends to be narrow as the magnitude of shift in the process increases. Identical outcomes are found for the variable warning limits the sample sizes.

A sensitivity analysis, which measures the effect of m values on the optimal design parameters and ATS_1 values, has been studied. It is found that, from the result of sensitivity analysis, only the maximum action and warning limits are sensitive to m values. Fig. 2 shows their values against different m values

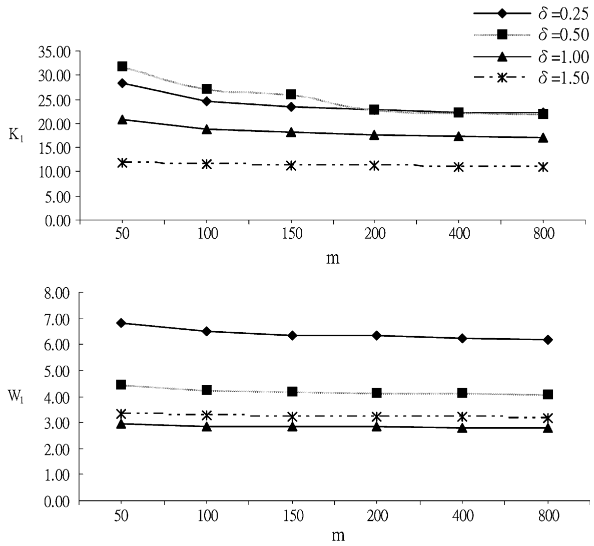

Fig. 2. k_1 and w_1 versus different m values for case: $p = 2$, $n_0 = 4$, and $\delta = 0.25$, 0.50, 1.00, and 1.50.

Table 6
Comparison between out-of-control ATS for the schemes VSSC, VP, VSSI, VSS, VSI and FSR ($n_0 = 2$, $m = 600$, $p = 2$, $h_0 = 1$, $\alpha_0 = 0.005$)

	δ	n_1/n_2	h_1/h_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS	$\mathrm{ATS_{FSR}}$
VSSC	0.25	1/43	1.00/1.00	0.000/0.207	19.78/3.15	7.54/2.98	65.94	145.15
	0.50	1/20	1.00/1.00	0.000/0.091	17.15/4.80	5.93/3.96	22.04	76.20
	0.75	1/11	1.00/1.00	0.000/0.050	23.01/6.01	4.64/3.87	9.50	37.35
	1.00	1/8	1.00/1.00	0.000/0.035	19.65/6.75	3.92/3.52	5.20	19.13
	1.25	1/6	1.00/1.00	0.000/0.024	17.50/7.46	3.24/3.04	3.46	10.48
	1.50	1/5	1.00/1.00	0.000/0.019	15.49/8.01	2.79/2.67	2.63	6.16
VP	0.25	1/43	1.02/0.01	0.000/0.205	18.50/3.17	7.54/3.00	65.80	145.15
	0.50	1/19	1.06/0.01	0.000/0.083	15.86/4.99	5.81/4.03	21.62	76.20
	0.75	1/10	1.12/0.03	0.000/0.045	20.09/6.24	4.42/3.79	8.65	37.35
	1.00	1/6	1.24/0.03	0.000/0.025	20.09/7.42	3.24/3.04	4.10	19.13
	1.25	1/3	2.00/0.01	0.000/0.010	17.87/9.30	1.39/1.37	2.17	10.48
	1.50	1/3	2.00/0.01	0.000/0.010	18.51/9.29	1.39/1.37	1.42	6.16
VSSI	0.25	1/182	1.01/0.01	0.005/0.005	10.73/10.61	9.21/9.13	100.47	145.15
	0.50	1/40	1.03/0.02	0.005/0.005	10.73/10.62	7.04/6.99	31.28	76.20
	0.75	1/16	1.07/0.01	0.005/0.005	10.73/10.62	5.32/5.29	11.08	37.35
	1.00	1/8	1.17/0.01	0.005/0.005	10.73/10.63	3.86/3.84	4.79	19.13
	1.25	1/3	2.00/0.01	0.005/0.005	10.73/10.67	1.38/1.38	2.41	10.48
	1.50	1/3	2.00/0.01	0.005/0.005	10.73/10.67	1.38/1.38	1.46	6.16
VSS	0.25	1/185	1.00/1.00	0.005/0.005	10.73/10.62	9.23/9.15	100.58	145.15
	0.50	1/41	1.00/1.00	0.005/0.005	10.73/10.62	7.09/7.03	31.88	76.20
	0.75	1/18	1.00/1.00	0.005/0.005	10.73/10.63	5.56/5.52	12.16	37.35
	1.00	1/10	1.00/1.00	0.005/0.005	10.73/10.63	4.35/4.33	6.16	19.13
	1.25	1/8	1.00/1.00	0.005/0.005	10.73/10.64	3.86/3.84	3.91	10.48
	1.50	1/6	1.00/1.00	0.005/0.005	10.73/10.64	3.20/3.19	2.84	6.16
VSI	0.25	2/2	3.51/0.01	0.005/0.005	10.73/10.73	0.67/0.67	135.63	145.15
	0.50	2/2	4.62/0.01	0.005/0.005	10.73/10.73	0.47/0.47	60.11	76.20
	0.75	2/2	7.02/0.01	0.005/0.005	10.73/10.73	0.31/0.31	21.29	37.35
	1.00	2/2	11.52/0.01	0.005/0.005	10.73/10.73	0.18/0.18	6.75	19.13
	1.25	2/2	19.12/0.01	0.005/0.005	10.73/10.73	0.11/0.11	2.32	10.48
	1.50	2/2	31.49/0.01	0.005/0.005	10.73/10.73	0.06/0.06	1.24	6.16

$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

for the cases of $p = 2$, $n_0 = 4$, and $\delta = 0.25$, 0.50, 1.00, or 1.50. As shown in Fig. 2, the maximum control or warning limit is decreasing as the m value initially increases. But, it remains almost unvarying as long as m goes beyond Nedumaran and Pignatiello's lower bound: $800p/3(n_0 - 1)$.

5.2. Comparisons with the VSI, VSS, VSSI and VP T^2 charts

As one can see in the introduction, there are other useful control schemes such as the VP, VSI, VSS, and VSSI one for the FSR T^2 charts in decreasing the average time to signal a small process shift. As a result, in evaluating the usefulness of the VSSC scheme it seems reasonable to make comparisons between them.

Before the comparisons, it is necessary to decide the optimal design parameters in these control schemes for producing the minimal ATS_1, given the values of n_0, h_0, α_0, p, m and δ. In doing so, an extension of the model (5)–(7) as follows will be used to decide these optimal design parameters.

$$\underset{n_1,n_2,h_1,h_2,k_1,k_2,w_1,w_2}{\text{Min}} \text{ATS}_1, \tag{16}$$

$$\text{s.t. } n_1 p_0 + n_2(1 - p_0) = n_0, \tag{17}$$

$$h_1 p_0 + h_2(1 - p_0) = h_0, \tag{18}$$

$$\alpha_1 p_0 + \alpha_2(1 - p_0) = \alpha_0, \tag{19}$$

where the objective function ATS_1 in (16) is expressed mathematically by

$$\text{ATS}_1 = p_0 E(\text{TS}_1) + (1 - p_0) E(\text{TS}_2),$$

where $E(\text{TS}_i)$, $i = 1, 2$ represent the average time to signal an out-of-control condition when the first sample size is small or large, respectively. As shown in Chen and Chiou (2005),

Table 7
Comparison between out-of-control ATS for the schemes VSSC, VP, VSSI, VSS, VSI and FSR ($n_0 = 4$, $m = 200$, $p = 2$, $h_0 = 1$, $\alpha_0 = 0.005$)

	δ	n_1/n_2	h_1/h_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS	ATS_{FSR}
VSSC	0.25	1/66	1.00/1.00	0.000/0.108	22.81/4.48	6.31/3.83	34.20	113.75
	0.50	1/24	1.00/1.00	0.000/0.038	22.93/6.57	4.16/3.64	9.69	42.13
	0.75	1/14	1.00/1.00	0.000/0.020	16.71/7.83	2.98/2.82	4.51	16.43
	1.00	2/10	1.00/1.00	0.000/0.019	17.76/7.95	2.82/2.68	2.87	7.33
	1.25	2/9	1.00/1.00	0.000/0.014	13.61/8.64	2.54/2.45	2.15	3.77
	1.50	3/8	1.00/1.00	0.000/0.008	11.20/9.65	3.22/3.17	1.74	2.25
VP	0.25	1/66	1.05/0.01	0.000/0.107	19.89/4.50	6.31/3.84	34.04	113.75
	0.50	1/21	1.17/0.01	0.000/0.032	18.09/6.91	3.87/3.48	8.93	42.13
	0.75	1/10	1.50/0.01	0.003/0.008	11.89/9.63	2.22/2.18	3.49	16.43
	1.00	1/5	3.97/0.01	0.000/0.007	16.16/10.19	0.58/0.57	1.53	7.33
	1.25	2/5	2.99/0.01	0.000/0.007	17.61/9.94	0.82/0.81	1.12	3.77
	1.50	3/5	1.99/0.01	0.002/0.008	12.54/9.82	1.39/1.38	1.04	2.25
VSSI	0.25	1/164	1.02/0.02	0.005/0.005	10.99/10.65	7.74/7.56	52.80	113.75
	0.50	1/33	1.10/0.01	0.005/0.005	10.99/10.66	4.74/4.67	11.43	42.13
	0.75	1/11	1.42/0.02	0.005/0.005	10.99/10.68	2.42/2.40	3.77	16.43
	1.00	1/5	3.98/0.01	0.005/0.005	10.99/10.73	0.58/0.58	1.55	7.33
	1.25	2/5	2.99/0.01	0.005/0.005	10.82/10.73	0.82/0.82	1.13	3.77
	1.50	3/5	2.00/0.01	0.005/0.005	10.81/10.73	1.39/1.39	1.03	2.25
VSS	0.25	1/165	1.00/1.00	0.005/0.005	10.99/10.65	7.57/7.57	53.12	113.75
	0.50	1/37	1.00/1.00	0.005/0.005	10.99/10.66	4.97/4.89	12.53	42.13
	0.75	1/18	1.00/1.00	0.005/0.005	10.99/10.67	3.49/3.44	5.29	16.43
	1.00	2/13	1.00/1.00	0.005/0.005	10.99/10.68	3.43/3.39	3.14	7.33
	1.25	2/10	1.00/1.00	0.005/0.005	10.99/10.69	2.79/2.76	2.24	3.77
	1.50	3/8	1.00/1.00	0.005/0.005	10.82/10.70	3.22/3.20	1.75	2.25
VSI	0.25	4/4	3.84/0.01	0.005/0.005	10.76/10.76	0.60/0.60	100.72	113.75
	0.50	4/4	6.48/0.01	0.005/0.005	10.76/10.76	0.34/0.34	25.71	42.13
	0.75	4/4	12.98/0.01	0.005/0.005	10.76/10.76	0.16/0.16	5.11	16.43
	1.00	4/4	26.65/0.01	0.005/0.005	10.76/10.76	0.08/0.08	1.44	7.33
	1.25	4/4	49.99/0.01	0.005/0.005	10.76/10.76	0.04/0.04	1.03	3.77
	1.50	4/4	49.99/0.01	0.005/0.005	10.76/10.76	0.04/0.04	1.00	2.25

$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

$$E(\mathrm{TS}_1) = \frac{h_1(1-p_{22}) + h_2 p_{12}}{D}, \tag{20}$$

$$E(\mathrm{TS}_2) = \frac{h_2(1-p_{11}) + h_1 p_{21}}{D}. \tag{21}$$

For the purpose of comparison, the minimal ATS_1 for the VP chart may be produced by adjusting the values of n_1, n_2, h_2, and α_1, which can result in the remainders h_1 and α_2 by Eqs. (17)–(19) and w_j ($j = 1, 2$) by Eq. (14). Similarly, we seek out the optimal values of n_1, n_2, and h_2 as well as their remainders for fixed α_j's at constant in designing the VSSI chart; the optimal values of n_1 and n_2 as well as their remainders for fixed α_j's and h_j's at constant in designing the VSS chart; and finally the optimal values of h_1 and h_2 as well as their remainders for fixed α_j's and n_j's at constant in designing the VSI chart.

Tables 6–9 give the results of comparisons among the VSSC, VP, VSSI, VSS, VSI, and FSR T^2 charts for two and four related quality characteristics in the cases of $n_0 = 2$ or 4, and the values of δ increased from 0.25 to 1.50 by 0.25. From these tables, it is observed that for very small δ values the VSSC chart is superior to the VSSI, VSS, VSI and FSR charts. Moreover, the VSSC and VP chart result in similar performances of out-of-control ATS values. However, from the viewpoint of administration, the VSSC chart which always takes a sample at fixed times are more convenient than the VP chart. As a result, it can be concluded that the VSSC chart is a good option to quickly detect a small magnitude of process shift. For large process shifts ($\delta > 1$), the VSSC scheme is a trivial improvement on the FSR scheme in comparison with the VSSI, VSI, and VP schemes. But, this must not

Table 8
Comparison between out-of-control ATS for the schemes VSSC, VP, VSSI, VSS, VSI and FSR ($n_0 = 2$, $m = 1400$, $p = 4$, $h_0 = 1$, $\alpha_0 = 0.005$)

	δ	n_1/n_2	h_1/h_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS	$\mathrm{ATS}_{\mathrm{FSR}}$
VSSC	0.25	1/51	1.00/1.00	0.000/0.229	20.53/5.63	11.69/5.45	78.77	160.30
	0.50	1/24	1.00/1.00	0.000/0.110	22.4/7.54	9.87/6.77	28.72	99.31
	0.75	1/14	1.00/1.00	0.001/0.051	18.29/9.45	8.44/7.24	13.08	53.89
	1.00	1/9	1.00/1.00	0.000/0.039	22.53/10.11	7.25/6.60	6.86	28.68
	1.25	1/7	1.00/1.00	0.001/0.025	18.44/11.20	6.48/6.17	4.38	15.76
	1.50	1/6	1.00/1.00	0.000/0.023	20.52/11.33	6.01/5.76	3.18	9.12
VP	0.25	1/49	1.02/0.02	0.000/0.234	23.12/5.57	11.63/5.39	78.42	160.30
	0.50	1/23	1.05/0.01	0.000/0.106	22.34/7.64	9.76/6.81	28.31	99.31
	0.75	1/12	1.10/0.01	0.000/0.053	22.40/9.35	8.05/6.94	12.13	53.89
	1.00	1/8	1.17/0.01	0.001/0.030	19.01/10.73	6.89/6.45	5.95	28.68
	1.25	1/5	1.33/0.01	0.000/0.020	22.93/11.74	5.41/5.24	3.19	15.76
	1.50	1/3	2.00/0.01	0.000/0.010	22.93/13.36	3.37/3.33	1.92	9.12
VSSI	0.25	1/235	1.00/0.01	0.005/0.005	14.97/14.87	13.55/13.47	113.22	160.30
	0.50	1/52	1.02/0.01	0.005/0.005	14.97/14.87	11.26/11.20	41.15	99.31
	0.75	1/21	1.05/0.01	0.005/0.005	14.97/14.88	9.32/9.28	15.80	53.89
	1.00	1/11	1.11/0.01	0.005/0.005	14.97/14.88	7.71/7.68	7.06	28.68
	1.25	1/6	1.25/0.01	0.005/0.005	14.97/14.89	5.96/5.94	3.63	15.76
	1.50	1/3	2.00/0.01	0.005/0.005	14.97/14.92	3.35/3.35	2.07	9.12
VSS	0.25	1/235	1.00/1.00	0.005/0.005	14.97/14.87	13.55/13.47	113.46	1160.30
	0.50	1/53	1.00/1.00	0.005/0.005	14.97/14.87	11.29/11.26	41.59	99.31
	0.75	1/22	1.00/1.00	0.005/0.005	14.97/14.88	9.43/9.38	16.68	53.89
	1.00	1/13	1.00/1.00	0.005/0.005	14.97/14.88	8.14/8.11	8.26	28.68
	1.25	1/9	1.00/1.00	0.005/0.005	14.97/14.88	7.16/7.13	4.98	15.76
	1.50	1/7	1.00/1.00	0.005/0.005	14.97/14.89	6.43/6.41	3.49	9.12
VSI	0.25	2/2	5.11/0.01	0.005/0.005	14.97/14.97	1.62/1.62	151.58	160.30
	0.50	2/2	7.58/0.01	0.005/0.005	14.97/14.97	1.26/1.26	81.28	99.31
	0.75	2/2	12.39/0.01	0.005/0.005	14.97/14.97	0.94/0.94	33.80	53.89
	1.00	2/2	22.40/0.01	0.005/0.005	14.97/14.97	0.67/0.67	12.03	28.68
	1.25	2/2	43.07/0.01	0.005/0.005	14.97/14.97	0.46/0.46	4.23	15.76
	1.50	2/2	49.98/0.01	0.005/0.005	14.97/14.97	0.43/0.43	1.81	9.12

$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

Table 9
Comparison between out-of-control ATS for the schemes VSSC, VP, VSSI, VSS, VSI and FSR ($n_0 = 4$, $m = 500$, $p = 4$, $h_0 = 1$, $\alpha_0 = 0.005$)

	δ	n_1/n_2	h_1/h_2	α_1/α_2	k_1/k_2	w_1/w_2	ATS	ATS_{FSR}
VSSC	0.25	1/83	1.00/1.00	0.000/0.128	21.45/7.17	10.38/6.59	42.12	134.93
	0.50	1/30	1.00/1.00	0.000/0.048	26.64/9.61	7.80/6.82	12.59	60.05
	0.75	1/16	1.00/1.00	0.000/0.024	22.52/11.26	6.06/5.76	5.66	24.69
	1.00	1/11	1.00/1.00	0.000/0.016	22.88/12.20	4.93/4.79	3.45	10.93
	1.25	2/9	1.00/1.00	0.000/0.016	20.71/12.19	5.07/4.92	2.48	5.41
	1.50	3/8	1.00/1.00	0.003/0.013	16.13/12.80	6.90/5.88	1.99	3.05
VP	0.25	1/82	1.04/0.01	0.000/0.129	22.19/7.15	10.36/6.58	41.89	134.93
	0.50	1/28	1.12/0.04	0.000/0.044	24.67/9.80	7.61/6.75	12.05	60.05
	0.75	1/14	1.29/0.03	0.003/0.012	16.49/12.87	5.64/5.51	4.93	24.69
	1.00	1/7	1.98/0.02	0.001/0.009	19.43/13.54	3.39/3.34	2.23	10.93
	1.25	2/5	2.97/0.01	0.000/0.007	20.69/14.10	2.40/2.37	1.32	5.41
	1.50	3/5	2.99/0.01	0.000/0.007	20.70/14.10	2.40/2.37	1.10	3.05
VSSI	0.25	1/214	1.01/0.01	0.005/0.005	15.18/14.89	11.99/11.81	63.20	134.93
	0.50	1/44	1.07/0.01	0.005/0.005	15.18/14.90	8.64/8.54	15.55	60.05
	0.75	1/16	1.25/0.01	0.005/0.005	15.18/14.91	6.01/5.95	5.39	24.69
	1.00	1/7	2.00/0.01	0.005/0.005	15.18/14.94	3.37/3.35	2.35	10.93
	1.25	1/5	4.00/0.01	0.005/0.005	15.18/14.96	1.93/1.92	1.31	5.41
	1.50	2/5	2.99/0.01	0.005/0.005	15.18/14.96	2.39/2.38	1.09	3.05
VSS	0.25	1/215	1.00/1.00	0.005/0.005	15.18/14.89	12.01/11.82	63.44	134.93
	0.50	1/47	1.00/1.00	0.005/0.005	15.18/14.90	8.80/6.76	16.41	60.05
	0.75	1/21	1.00/1.00	0.005/0.005	15.18/14.90	6.69/8.69	6.72	24.69
	1.00	1/13	1.00/1.00	0.005/0.005	15.17/14.91	5.41/5.36	3.88	10.93
	1.25	2/11	1.00/1.00	0.005/0.005	15.18/14.92	5.73/5.68	2.64	5.41
	1.50	3/9	1.00/1.00	0.005/0.005	15.18/14.93	6.45/6.43	2.02	3.05
VSI	0.25	4/4	5.95/0.01	0.005/0.005	14.98/14.98	1.47/1.47	121.78	134.93
	0.50	4/4	11.29/0.01	0.005/0.005	14.98/14.98	0.99/0.99	39.80	60.05
	0.75	4/4	26.27/0.01	0.005/0.005	14.98/14.98	0.61/0.61	9.23	24.69
	1.00	4/4	49.96/0.01	0.005/0.005	14.98/14.98	0.43/0.43	2.30	10.93
	1.25	4/4	49.99/0.01	0.005/0.005	14.98/14.98	0.43/0.43	1.17	5.41
	1.50	4/4	49.99/0.01	0.005/0.005	14.98/14.98	0.43/0.43	1.03	3.05

$\alpha_1 = 0.000$ means that the optimal value of k_1 make the risk of false alarm nearly zero.

cause a fuss because for large shifts the values of ATS by themselves are already fairly small.

6. Conclusions

In this paper, a VSSC T^2 chart is proposed for increasing the detection speed of the Hotelling's T^2 chart for small process shifts. The statistical design of the VSSC chart is formulated as an optimization problem which minimizes the average time to signal an out-of-control condition (ATS_1), given the average sample size and false alarm rate constraints when the process is in control. By means of applying the genetic algorithms to the optimization problem, the optimal design parameters and ATS_1 corresponding to different numbers of quality characteristics were tabled for specific average sample size, sampling interval, false alarm rate, and magnitude of process mean shift. The statistical performance of VSSC chart measured by ATS_1 was compared with the FSR, VSS, VSI, VSSI, VP T^2 charts. The results of comparisons show that the VSSC chart obtain a great and consistent improvement on the FSR T^2 charts. As compared with the VSS, VSSI, and VSSI T^2 charts the VSSC T^2 chart performs excellent for very small shifts in process mean. Moreover, the VSSC T^2 chart and the VP T^2 chart have a similar performance, but from the viewpoint of administration, the VSSC T^2 chat provides an easier tool to achieve a comparable efficiency.

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