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Minimum pilot power for service coverage in WCDMA networks

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Abstract Pilot power management is an important issue for efficient resource utilization in WCDMA networks. In this paper, we consider the problem of minimizing pilot power subject to a coverage constraint. The constraint can be used to model various levels of coverage requirement, among which full coverage is a special case. The pilot power minimization problem is $\mathcal{NP}$-hard, as it generalizes the set covering problem. Our solution approach for this problem consists of mathematical programming models and methods. We present a linear-integer mathematical formulation for the problem. To solve the problem for large-scale networks, we propose a column generation method embedded into an iterative rounding procedure. We apply the proposed method to a range of test networks originated from realistic network planning scenarios, and compare the results to those obtained by two ad hoc approaches. The numerical experiments show that our algorithm is able to find near-optimal solutions with a reasonable amount of computing effort for large networks. Moreover, optimized pilot power considerably outperforms the ad hoc approaches, demonstrating that efficient pilot power management is an important component of radio resource optimization. As another part of our numerical study, we examine the trade-off between service coverage and pilot power consumption.

Keywords WCDMA · Pilot power · Coverage · Optimization

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1 Introduction

In a WCDMA network, a cell announces its presence through a Common Pilot Channel (CPICH), a fixed rate downlink physical channel that carries a pre-defined bit/symbol sequence [2]. Normally, each cell has only one CPICH, called Primary CPICH or P-CPICH, for broadcasting the pilot signal over the entire cell. The CPICH signals, or *pilot signals*, are used for channel quality estimation, cell selection/reselection, and handover [1]. The CPICH Received Signal Code Power (RSCP) and CPICH E_c/N_0 (the ratio of the received energy per chip to the power density in the band) are among the most important user equipment measurements related to the link performance. The former is used for handover evaluation, downlink open loop power control, uplink open loop power control, and for the calculation of pathloss which becomes possible because the CPICH transmit power can be read from the System Information broadcasted by radio network controllers [3]. The CPICH E_c/N_0 measurement is used by mobile terminals for cell selection/re-selection and may also be used for handover evaluation.

A mobile terminal in a WCDMA network continuously monitors pilot signals and is typically attached to the cell from which the strongest pilot signal is received. A number of factors, such as transmit power, attenuation, total interference, and thermal noise, affect the strength of the received pilot signal. The CPICH transmit power, or *pilot power* in short, plays an important role in system engineering of WCDMA networks since it strongly affects network coverage.

From the radio network planning point of view, the pilot power should be minimized to leave as much power as possible for traffic channels in order to increase cell capacity. This is especially important if the operator uses an OTSR (Omni Transmit Sectorial Receive) configuration which allows to save on power amplifiers and/or if the power levels of other downlink common channels, e.g., SCH (Synchronization Channel), PICH (Paging Indicator Channel), CCPCH (Common Control Physical Channel), and AICH (Acquisition Indicator Channel), are set relative to CPICH. The latter is a common practice [7]. Excessive pilot power adds also to the total downlink interference, increases cell overlap areas causing higher cell loads, and may lead to larger areas suffering from pilot pollution. On the other hand, if the pilot power is too low, the decreased cell dominance area may cause overload of some neighboring cells and/or coverage holes in the network.

Typically, 5–10% of the maximum downlink cell power is used for CPICH [9], but there is no standard approach for finding an optimal pilot power configuration. Among the most popular approaches is the *uniform pilot power* setting where all cells use the same pilot power [6, 9, 12]. Uniform pilot power performs poorly from the power consumption point of view. Moreover, it causes a high level of total interference in the network, large cell overlapping areas, and high pilot power pollution. Finding manually an optimal setting of pilot power levels in the network is a tedious task, especially for large networks. As a manual approach for assigning the pilot power is slow and prone to error, there is a need of pilot power management techniques that can be implemented and performed automatically. In [10], the authors demonstrated that a rule-based optimization technique for setting pilot power levels significantly outperforms a manually-designed solution in terms of the network cost.

The authors of [8] studied the problem of minimizing the pilot power subject to coverage constraints (similar to the problem studied here) and presented a heuristic algorithm that adjusts the pilot power of one cell in each iteration. In [13], a cost-minimization method is used in network simulations. Based on some target values for coverage and traffic load, the method attempts to minimize the deviation from the target values by adjusting pilot power levels using a gradient descent procedure. The authors of [12, 15, 16] considered power management for load balancing, and showed that network performance can be enhanced by proper adjustments of the CPICH transmit power.

In this paper, we study the problem of providing service coverage using a minimum amount of pilot power and thus, resolving a trade-off between power consumption for pilot signalling and coverage. Our solution approach to the problem consists of mathematical programming techniques. Our solution method comprises a column generation method and an iterative rounding procedure. The method is suitable for finding near-optimal solutions for large-scale networks and is able to find solutions within a few percent from optimality in a reasonable amount of time. In addition to an efficient solution approach, our second contribution is the modeling work itself. To the best of our knowledge, our work of mathematical modeling of pilot power optimization is original and novel. Specifically, the model in Section 3.1 provides a systematic description of the task of pilot power optimization. The model allows finding optimal solutions for small networks. Then, the enhanced model in Section 3.2 is significant not only in the sense that it enables an efficient method for finding near-optimal solutions to large-scale networks, but also because the model yields a sharp bound (derived from the Linear Programming or LP, relaxation). This is important since otherwise it is very difficult (if not impossible) to find out the solution quality. Moreover, our numerical study on large-scale, real-life networks provides valuable insights into the significant amount of power gain that can be achieved by means of mathematical optimization. As the second part of our numerical study, we examine the trade-off between coverage and power consumption.

The remainder of the paper is organized as follows. In Section 2 we formalize the optimization problem, and present two ad hoc solution approaches. The mathematical formulations are discussed in Section 3, and our optimization method is described in Section 4. Numerical results are presented in Section 5. Finally, in Section 6 we draw some conclusions and discuss forthcoming research.

2 The optimization problem

2.1 System model

Consider a WCDMA network consisting of m cells. Let $\mathcal{I}$ denote the set of cells, i.e., $\mathcal{I} = \{1, \dots, m\}$. The service area is represented by a grid of bins, for which the signal propagation predictions (or measurements) are known. We denote the total number of bins by n, and define $\mathcal{J} = \{1, \dots, n\}$. The size of a bin determines the resolution a set of the signal propagation data. We assume that the signal propagation conditions are the same across a bin, and denote the power gain between the antenna of cell i and bin j by g_{ij}. We assume that $0 < g_{ij} < 1, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$. (Throughout the paper, linear scale will be used for all parameters and variables.)

Let P_i^T denote the total transmission power available in cell i. This amount of power is shared among the CPICH channel, other common channels, and dedicated traffic channels. Let y_i denote the amount of power allocated to the pilot signal in cell i. The amount of power left for other purposes in cell i is at most $P_i^T - y_i$. A higher value of y_i results in less power available to serve user traffic.

In bin j, the received pilot power of cell i is $g_{ij} y_i$. In addition to the pilot signal, some interfering signals, including the signals for user traffic in cell i and signals from other base stations, are received in the bin. We consider network scenarios with high traffic load and assume that all base stations operate at full power. The assumption represents the worst-case interference scenario. Dealing with this scenario is reasonable when planning service coverage, because if the CPICH coverage is achieved in some area under this assumption, then the downlink coverage of the area is guaranteed for any traffic scenario. (The same assumption has been also used, for example, in [12].) Under this assumption, the total amount of interference in bin j with respect to cell i reads

$$I_{ij} = \alpha_j\big(P_i^T - y_i\big)g_{ij} + \sum_{k\in\mathcal{I}:k\neq i} P_k^T g_{kj} + \nu_j, \tag{1}$$

where $\alpha_j \in (0, 1)$ is the non-orthogonality factor in bin j, and ν_j is the thermal noise power in bin j.

The strength of a pilot signal is measured by its carrier-to-interference ratio (CIR), or CPICH E_{C/N_0}. We assume that a bin can be covered by a cell if and only if the corresponding CPICH CIR is no less than a threshold γ_0. For cell i and bin j, the CIR requirement is therefore

$$\gamma_{ij} = \frac{g_{ij}y_i}{I_{ij}} = \frac{g_{ij}y_i}{\alpha_j\big(P_i^T - y_i\big)g_{ij} + \sum_{k\in\mathcal{I}:k\neq i} P_k^T g_{kj} + \nu_j} \geq \gamma_0. \tag{2}$$

To access network service, a mobile terminal must be able to detect at least one pilot signal that satisfies the CIR requirement. Thus, for providing service in bin j, a necessary condition is that at least one pilot signal satisfies (2). From (2), it can be easily derived that, if cell i provides coverage in bin j, then pilot power y_i must be at least P_{ij}, defined as

$$P_{ij} = \gamma_0 \cdot \frac{\alpha_j P_i^T g_{ij} + \sum_{k\in\mathcal{I}:k\neq i} P_k^T g_{kj} + \nu_j}{(1+\gamma_0\alpha_j)g_{ij}}. \tag{3}$$

Pilot power is a configuration parameter that should not be changed very often. A pilot power setting, optimized for a particular traffic scenario, may perform poorly when the demand pattern changes. For this reason, cells are considered to be equally important in our system model which justifies minimization of the total pilot power in the network.

The requirement of coverage is represented by a parameter $d \leq n$, where d is the number of bins that are required to be covered by at least one pilot signal. The case of full coverage corresponds to $d = n$. Our optimization problem amounts to choosing the pilot power levels of the cells to cover at least d bins, such that the amount of total pilot power is minimized. We use MPP to denote this optimization problem.

2.2 Problem complexity

We have the following result concerning the complexity of problem MPP.

Proposition 1. *Problem MPP is $\mathcal{NP}$-hard.*

Proof: It is sufficient to show that a special case of the problem is $\mathcal{NP}$-hard. For the case of full coverage ($d = n$), an $\mathcal{NP}$-hardness proof, in which the well-known set covering problem is polynomially reduced to MPP, is provided in [14]. □

Given the $\mathcal{NP}$-hardness result, it is unlikely that there exists any exact and polynomial-time algorithm for MPP. However, using a mathematical programming method tailored for the problem, near-optimal solutions can be obtained with reasonable computing effort even for large networks.

2.3 Two ad hoc solutions

It is worth mentioning two ad hoc strategies for setting the pilot power. We will describe these two strategies for the case of full coverage and present analytical solutions. The solutions can, however, be easily adapted to the case when $d < n$. The first strategy is uniform pilot power, by which the pilot power is the same in all cells. In the second strategy, referred to as the *gain-based pilot power*, a bin is always covered by the cell with the highest power gain. For the system model in Section 2.1, the formulas of uniform power and gain-based power were, to the best of our knowledge, originally derived and presented in [14] and [11], respectively.

Uniform pilot power. We use y^U to denote the power level used by a cell in the solution of uniform pilot power. A necessary condition for covering bin j is that y^U is at least as large as the minimum of P_{ij} among all cells, i.e., $y^U \geq \min_{i\in\mathcal{I}} P_{ij}$. Moreover, this condition must hold for every bin, leading to the following inequality,

$$y^U \geq \max_{j\in\mathcal{J}} \min_{i\in\mathcal{I}} P_{ij}. \tag{4}$$

We further observe that after setting all pilot power levels to $\max_{j\in\mathcal{J}} \min_{i\in\mathcal{I}} P_{ij}$, every bin is covered by at least one pilot signal. It follows immediately that we can change (4) to equality, that is, $y^U = \max_{j\in\mathcal{J}} \min_{i\in\mathcal{I}} P_{ij}$. The total amount of power in the solution of uniform pilot power is therefore

$$P^U = m \cdot y^U. \tag{5}$$

It is straightforward to adapt the solution of uniform pilot power to the case of partial coverage (i.e., $d < n$), by sorting the bins in ascending order with respect to $\min_{i\in\mathcal{I}} P_{ij}$. The dth bin in the sorted sequence yields the minimum uniform pilot power for which the coverage requirement is satisfied.

The approach of using uniform pilot power is efficient in simple propagation scenarios, where the signal attenuation is essentially determined by distance. In such scenarios, cell sizes will be roughly the same for fairly uniformly distributed traffic and equally-spread base stations. However, for an in-homogeneous planning situation, using the same pilot power for all cells results in an unnecessarily large power consumption for pilot signals.

Gain-based pilot power. A second ad hoc solution for setting pilot power is to assign cells to bins based on the power gain values. In this solution, a bin is always covered by the cell with the maximum power gain. For bin j, we use $c(j)$ to denote the cell that maximizes the power gain among all cells, i.e.,

$$c(j) = \arg\max_{i \in \mathcal{I}} g_{ij}. \tag{6}$$

Note that, if the power limit P_i^T is the same for all cells, then for any bin, the cell with the maximum power gain is also the cell with the minimum required power level. In this case, $c(j)$ can be equivalently defined as $c(j) = \arg\min_{i \in \mathcal{I}} P_{ij}$.

By choosing the cell with the maximum gain for every bin, and setting the power levels accordingly, we obtain a solution in which all bins are covered. In this solution, the pilot power of cell i equals

$$y_i^G = \max_{j \in \mathcal{J}: c(j)=i} P_{ij}. \tag{7}$$

The total power in the gain-based pilot power solution is therefore

$$P^G = \sum_{i \in \mathcal{I}} y_i^G. \tag{8}$$

There are several ways to obtain a power-minimization heuristic for the case of partial coverage, by adapting the above procedure for computing gain-based pilot power. One such heuristic is as follows. For every bin, we find the maximum power gain among all cells, i.e., $g_{c(j),j}$, which is used to sort the bins in descending order. The pilot power solution is then determined by the first d bins in the sorted sequence. The gain-based pilot power is quite intuitive for a network planner. In fact, it significantly outperforms the solution of uniform pilot power. However, our results also show that this solution can still be quite far from the optimum, especially for large networks.

3 Mathematical formulations

3.1 A cell-bin formulation

Problem MPP can be mathematically represented by a cell-bin formulation. The formulation contains the following three sets of variables.

$$y_i = \text{The pilot power of cell } i,$$

$$x_{ij} = \begin{cases} 1 & \text{if the pilot signal of cell } i \text{ covers bin } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$s_j = \begin{cases} 1 & \text{if bin } j \text{ is covered by the pilot signal} \\ & \text{of at least one cell,} \\ 0 & \text{otherwise.} \end{cases}$$

It should be noted that only some of the combinations between cells and bins are relevant for defining the set of x-variables, because usually a bin can only be covered by a small number of cells. For convenience, we define a set $C(j)$, which consists of cells that can cover bin j using a feasible pilot power, i.e., $C(j) = \{i \in \mathcal{I} : P_{ij} \leq P_i^T\}$.

The cell-bin formulation of MPP is as follows.

$$[\text{MPP-CBF}]\, P^* = \min \sum_{i \in \mathcal{I}} y_i \tag{9}$$

$$\text{s. t.} \sum_{i \in C(j)} x_{ij} \geq s_j, \quad \forall j \in \mathcal{J}, \tag{10}$$

$$P_{ij} x_{ij} \leq y_i, \quad \forall j \in \mathcal{J}, \quad \forall i \in C(j), \tag{11}$$

$$\sum_{j \in \mathcal{J}} s_j \geq d, \tag{12}$$

$$x_{ij} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, \quad \forall i \in C(j), \tag{13}$$

$$0 \leq s_j \leq 1, \quad \forall j \in \mathcal{J}. \tag{14}$$

In MPP-CBF, a constraint of (10) ensures that variable s_j is one only if bin j is covered by one or more cells. By constraints (11), the pilot power y_i must be at least P_{ij}, if cell i covers bin j. Finally, the coverage requirement is formulated in constraint (12). Note that the non-negativity restrictions on the y-variables are implicitly handled by (11). Also, we do not have to restrict the s-variables to be integral, because for any integer solution of x, there is at least one optimal integral solution (which can be computed easily) of the s-variables. MPP-CBF is a quite straightforward linear-integer formulation for MPP. From a computational point of view, however, this formulation is not efficient. In particular, the LP relaxation of MPP-CBF is very weak (i.e., the LP optimum is often far away from the integer optimum). Even for small networks, solving MPP-CBF to optimality is out of reach of a standard branch-and-bound solution technique.

3.2 An enhanced formulation

To avoid the aforementioned weakness of formulation MPP-CBF, we derive a second, enhanced formulation to the problem. The enhancement is based on the observation that the optimal pilot power of any cell belongs to a discrete set. More specifically, the optimal value of y_i is one member of the set $\{P_{ij}, j = 1, \ldots, n : i \in C(j)\}$, because otherwise the total power can be reduced while maintaining the same level of coverage. Assume that y_i equals P_{ij^*} in an optimal solution, then j^* is the bin with the highest pilot power requirement among all bins covered by i in this solution.

To simplify the presentation of the enhanced formulation, we define one set $B(i)$ for every cell i. The set $B(i)$ contains all bins that may be covered by the cell, i.e.,

$$B(i) = \left\{ j \in \mathcal{J} : P_{ij} \leq P_i^T \right\} \tag{15}$$

Instead of using continuous variables to represent pilot power, in the enhanced formulation we use binary variables to enumerate all possible power levels. The following are the variable definitions of the enhanced formulation.

$$z_{ik} = \begin{cases} 1 & \text{if the pilot power of cell } i \text{ equals } P_{ik}, \\ 0 & \text{otherwise.} \end{cases}$$

$$s_j = \begin{cases} 1 & \text{if bin } j \text{ is covered by the pilot signal of at least one cell,} \\ 0 & \text{otherwise.} \end{cases}$$

If cell i covers bin k, then it also covers bin j if $P_{ij} \leq P_{ik}$. This information is represented by the following set of indication parameters.

$$a_{ijk} = \begin{cases} 1 & \text{if bin } j \text{ is covered by the pilot signal of cell } i, \\ & \text{provided that the pilot power of cell } i \text{ equals } P_{ik}, \\ 0 & \text{otherwise.} \end{cases}$$

The enhanced formulation for MPP is as follows.

$$[\text{MPP-EF}]\, P^* = \min \sum_{i \in \mathcal{I}} \sum_{k \in B(i)} P_{ik} z_{ik} \tag{16}$$

$$\text{s. t.} \sum_{k \in B(i)} z_{ik} \leq 1, \quad \forall i \in \mathcal{I}, \tag{17}$$

$$\sum_{i \in C(j)} \sum_{k \in B(i)} a_{ijk} z_{ik} \geq s_j, \quad \forall j \in \mathcal{J}, \tag{18}$$

$$\sum_{j \in \mathcal{J}} s_j \geq d, \tag{19}$$

$$z_{ik} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, \quad \forall k \in B(i), \tag{20}$$

$$0 \leq s_j \leq 1, \quad \forall j \in \mathcal{J}. \tag{21}$$

In MPP-EF, a constraint of (17), further referred to as a *cell pilot constraint*, states that if cell i covers any bin, then its pilot power equals P_{ik} for some $k \in B(i)$. The constraint also allows a pilot power to be zero, in case a cell does not cover any bin. Consequently, for any cell, at most one term in the objective function can be positive. Constraints (18), or *bin coverage constraints*, link the power-selection variables to the coverage variables. The coverage requirement is stated in (19). As in the cell-bin formulation, it is not necessary to impose integrality constraints on the s-variables in the enhanced formulation.

The LP relaxation of the enhanced formulation provides a sharper bound to P^* than that of the cell-bin formulation. We formalize this result in the following proposition.

Proposition 2. *The lower bound provided by the LP relaxation of MPP-EF is at least as strong as that of MPP-CBF. In addition, there are instances for which the former is strictly better than the latter.*

Proof: The proof is similar to the proof for the full coverage case presented in [14]. □

4 The solution approach

4.1 Overview

We propose a solution approach in which a column generation method is embedded into an iterative rounding procedure. For large networks, an obvious difficulty of solving MPP-EF is its size. We observe that the number of variables (columns) exceeds far more the number of constraints in this formulation. To be able to efficiently solve the LP relaxation of MPP-EF, we apply a column generation method. Column generation decomposes a linear program (typically with a large number of variables) into a master problem and a subproblem. The former contains only a subset of the columns. The latter is a separation problem for the dual LP, and is used to check optimality, i.e., whether additional columns need to be added to the master problem or not.

The optimal LP solution to MPP-EF usually contains fractional-valued variables. To find an integer solution, we embed the column generation method into an iterative rounding procedure. The rounding procedure selects the largest fractional-valued z-variable in the LP optimum, and rounds the value of this variable up to one. The column generation method is then invoked again to solve the (modified) LP problem with an additional constraint stating that the value of the selected z-variable equals one. These steps are then repeated, until the column generation method produces an integer solution. The column generation method and the iterative rounding procedure generate both lower and upper bounds to the optimum. These two bounds define an interval within which the minimum total pilot power lies.

4.2 Column generation

Let us consider the LP relaxation of MPP-EF. In the LP relaxation, the integrality constraints of the z-variables are relaxed. We denote by MPP-LP the LP problem obtained by replacing (20) with $z_{ik} \geq 0, \forall i \in \mathcal{I}, \forall k \in B(i)$. In a column generation context, this LP problem is also referred to as the *master problem*.

We start the column generation method by considering a restricted version of MPP-LP (*restricted master problem*), where the sets of bins $B(i), i \in \mathcal{I}$ are replaced by some subsets $B'(i) \subseteq B(i), i \in \mathcal{I}$. Usually, the size of $B'(i)$ is much smaller than that of $B(i)$. Let us denote the restricted master problem by MPP-MAS.

We assume that the sets $B'(i), i \in \mathcal{I}$, contain sufficiently many members such that MPP-MAS is feasible. (As will be clear later on, this condition can be easily satisfied.) The optimal solution to MPP-MAS is clearly feasible to MPP-LP. To examine whether this solution is also optimal to MPP-LP, we need either to identify a cell i and a bin $k \in B(i) \setminus B'(i)$ for which the variable z_{ik} should be added to MPP-MAS so the current solution can be improved, or to show that no such variable exists. In LP terms, this amounts to examining whether there exists any z-variable for which the reduced cost is strictly negative. If such a variable is found, it is added to MPP-MAS, which is then re-optimized. If, on the other hand, all reduced costs are nonnegative, then the optimal solution to MPP-MAS is also optimal to MPP-LP.

The reduced cost of a z-variable depends on the optimal dual solution to MPP-MAS. Let us denote by (π, μ) an optimal dual solution to MPP-MAS, where $\mu = \{\mu_i : i \in \mathcal{I}\}$ are the dual variables associated with the cell pilot constraints, and $\pi = \{\pi_j : j \in \mathcal{J}\}$ are the dual variables associated with the bin coverage constraints. By LP duality, the reduced cost of z_{ik} is then given by

$$\begin{aligned}\bar{c}_{ik} &= P_{ik} - \sum_{j \in \mathcal{J}: i \in C(j)} \pi_j a_{ijk} - \mu_i \\ &= P_{ik} - \sum_{j \in B(i)} \pi_j a_{ijk} - \mu_i. \end{aligned} \quad (22)$$

Note that the second equality of (22) is due to the fact that, for cell i, the sets $\{j \in \mathcal{J} : i \in C(j)\}$ and $B(i)$ coincide.

For cell i, there exists some z-variable with a negative reduced cost if and only if the minimum of $\bar{c}_{ik}, k \in B(i)$, is negative. We are therefore interested in the solution to the optimization problem of $\min_{i \in \mathcal{I}, k \in B(i)} \bar{c}_{ik}$, which is referred to as the column generation subproblem. Observe that the subproblem decomposes by cell, and, for cell i, the minimum reduced cost of the z-variables is found by computing (22) for all $k \in B(i)$:

$$\bar{c}_i^* = \min_{k \in B(i)} \bar{c}_{ik}. \quad (23)$$

If $\bar{c}_i^* < 0$, then the corresponding variable, z_{ik^*}, where $k^* = \arg\min_{k \in B(i)} \bar{c}_{ik}$, is added to the restricted master problem MPP-MAS by setting $B'(i) = B'(i) \cup \{k^*\}$. Examining all the cells, at most m variables will be added to MPP-MAS in the same iteration. The restricted master problem is then re-optimized, and we proceed to the next iteration. If $\bar{c}_{ik} \geq 0, \forall i \in \mathcal{I}, \forall k \in B(i)$, then the optimal solution to MPP-MAS is also optimal to MPP-LP.

The column generation method solves MPP-LP correctly because in the worst case, all the z-variables are added, (generated) to MPP-MAS, which then becomes identical to MPP-LP. In addition, this will occur after a finite number of iterations. Typically, only a few of the z-variables are generated before MPP-LP is solved to optimality. For large-scale problems, this greatly reduces the computational effort for solving MPP-LP.

In the first iteration of the method, we need to initialize the sets $B'(i), i \in \mathcal{I}$. These sets should be defined to ensure the feasibility of the restricted master problem. One possibility for this purpose is to use the solution of gain-based pilot power discussed in Section 2.3 and to assign to set $B'(i)$ the single bin which defines the pilot power of cell i in the solution.

4.3 The iterative rounding procedure

The column generation method is efficient for solving the LP relaxation of MPP-EF. To ensure integer optimality, a branch-and-bound scheme, which embeds column generation into its enumeration tree, is necessary. This requires, however, very long computing time for large networks. We therefore consider an iterative rounding procedure, which is aimed to generate a near-optimal solution.

In one iteration, the procedure rounds one fractional-valued variable in the optimal solution of the LP relaxation up to one. Let $\bar{z} = \{\bar{z}_{ik}, i \in \mathcal{I}, k \in B'(i)\}$ be an optimal solution to the LP relaxation. The rounding procedure chooses the variable whose value is largest among all fractional-valued variables in this solution. We denote such a variable by $\bar{z}_{i^*k^*}$, i.e.,

$$\bar{z}_{i^*k^*} = \max_{i \in \mathcal{I}, k \in B^F(i)} \bar{z}_{ik}. \quad (24)$$

In (24), $B^F(i) = \{k \in B'(i) : 0 < \bar{z}_{ik} < 1\}$. The following constraint is then added to MPP-MAS,

$$z_{i^*k^*} = 1. \quad (25)$$

Adding constraint (25) makes the current solution $\bar{z}$ infeasible to MPP-MAS, because $\bar{z}_{i^*k^*} < 1$. We need therefore re-optimize MPP-MAS. In addition, re-optimization of MPP-MAS alters the optimal values of the dual variables, which, in turn, may result in negative reduced costs for some z-variables that are currently not present in the restricted master problem. If this occurs, the re-optimization process must add new elements to some of the sets $B'(i)$, $i \in \mathcal{I}$, in order to solve MPP-LP with the new constraint (25) to optimality. In other words, we apply again the column generation method to solve the modified version of MPP-LP.

The iterative rounding procedure, which repeatedly applies column generation to a sequence of LPs, can be summarized as follows,

1. Solve MPP-LP using the column generation method.
2. If all z-variables are integral, terminate.
3. Find a variable $z_{i^*k^*}$ that solves (24).
4. Add constraint (25) to MPP-LP, go to Step 1.

Because adding (25) will always increase the value of a z-variable, feasibility is maintained throughout the iterative rounding procedure. Moreover, the procedure generates an integer solution within a finite number of steps.

5 Numerical results

5.1 Test networks

We present computational results obtained for six test networks of various sizes. Among them, networks N1 and N6 are real-life planning scenarios. In particular, N1 is provided by Ericsson Research, Sweden, and N6 is a planning scenario for the city of Berlin, provided by the MOMENTUM project group [5]. Tables 1 and 2 show the network statistics and the parameter setting used in our experiments, respectively.

For each of the test networks, we are given predicted attenuation values for a specific network configuration for each cell. These values, beside the pathloss component, include also the shadowing (or slow fading) component modelled statistically as a zero-mean log-normal distribution with a standard deviation of 8 dB. Figure 1 shows the cumulative distribution function of the best-server attenuation values (for a bin, this is the smallest attenuation among those between this bin and the antennas of potentially covering cells) of all the test networks. Power gains used in our system model are derived from the attenuation values by changing the sign and applying the scale transformation.

Table 1 Test network statistics

Network	Sites	Cells (m)	Bins (n)	Area[1], [m^2]	Bin size, [m^2]
N1	22	60	1375	1280 × 1800	40 × 40
N2	15	42	2708	2400 × 2000	40 × 40
N3	25	70	5029	2880 × 2800	40 × 40
N4	50	140	9409	4000 × 4000	40 × 40
N5	65	188	15112	5200 × 5200	40 × 40
N6	50	148	22500	7500 × 7500	50 × 50

[1] For some instances, a small portion of the area is not subject to service coverage.

Table 2 Parameter setting

Parameter	Networks N1-N5	Network N6
P_i^T	15 W, for all cells	19.95 W, for all cells
γ_0	0.015	0.01
α_j	0.4, for all bins	{0.327, 0.633, 0.938}, depending on bin type (urban, rural, or mixed)
ν_j	10^{-13} W, for all bins	$1.5488 \cdot 10^{-14}$ W, for all bins

5.2 Results and analysis

In the first part of our computational study, we focus on the case of pilot power minimization subject to full service coverage (i.e., $d = n$). For each of the test networks, we experimented with the following solution approaches: the two ad hoc strategies discussed in Section 2.3, a standard linear integer solver [4], and, finally, the column generation method and the iterative rounding procedure. All the computational experiments have been conducted on a Sun UltraSPARC station with a 400 MHz CPU and 1 GB physical memory.

We present the results of uniform pilot power and the gain-based pilot power in Table 3. The table displays the total power consumption as well as the average power per cell, in Watt, for the two solutions. We note that uniform pilot power is significantly outperformed by the gain-based pilot power approach. Specifically, the pilot power of a cell ranges between 1.0 W and 2.3 W in the former approach, whereas the latter approach leads to an average power of less than 1.0 W for all the test networks.

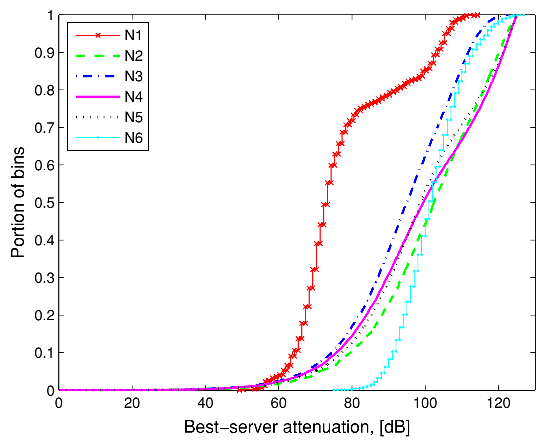

Fig. 1 Cumulative distribution functions of the best-server attenuation values

Table 3 Two ad hoc solutions

	Uniform power approach		Gain-based approach	
Network	Total (P^U)	Average	Total (P^G)	Average
N1	65.2384	1.0873	31.6053	0.5268
N2	50.5748	1.2042	30.9283	0.7364
N3	82.2443	1.1892	52.8006	0.7543
N4	175.9084	1.2565	106.0703	0.7576
N5	270.2355	1.4374	150.1760	0.7988
N6	345.0963	2.3317	147.0142	0.9933

An attempt was made to solve the formulation MPP-EF of our test networks using CPLEX [4]. We then applied the column generation method and the iterative rounding procedure. These results are summarized in Table 4. For networks N1-N4, the left part of the table shows the optimal total and average pilot power, as well as the computing time used by CPLEX. For the other two networks, CPLEX did not manage to find the optimal solution (or any near-optimal integer solution) because of lack of memory. The results of the column generation method and the iterative rounding procedure, including the total and average pilot power of the integer solution, the LP bound, the relative gap between the upper and lower bounds, and the computing time, are displayed in the right part of Table 4.

We observe that the LP relaxation of MPP-EF yields a good bound to the integer optimum, and that the iterative rounding procedure finds near-optimal solutions. The relative gap between the integer solution and the LP bound is less than 2% for the first five test networks, and less than 5% for the last test network. Examining the results obtained by our method, we observe up to 25% improvement over the solutions of gain-based pilot power.

The main advantage of our solution method is its capability of finding a feasible solution of high quality for large networks, for which a standard solver fails. From a practical point of view, the quality of the obtained solutions is sufficiently high for the purpose of network planning, because of the uncertainty in the network data (the power gain values in particular).

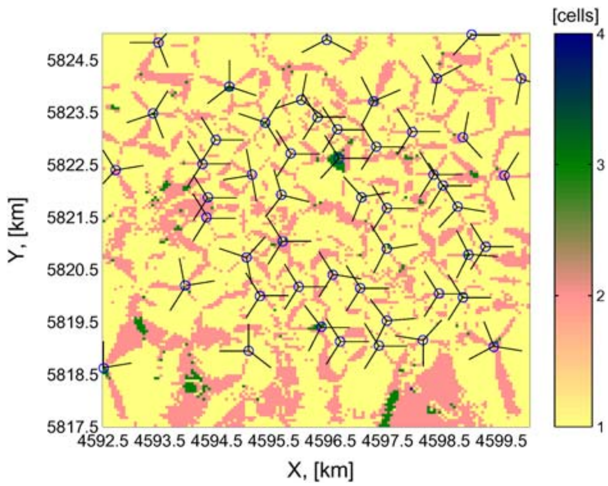

Fig. 2 Coverage statistics, network N6

For network N6 (the city of Berlin), the solution found by our method is further illustrated in Figs. 2 and 3. Figure 2 depicts the number of cells with a pilot signal satisfying the CIR requirement in each bin of the service area. Dark pixels represent bins covered by several cells. (Since full coverage is a requirement for this solution, the minimum number of covering cells in any bin is one.) Observe that most parts of

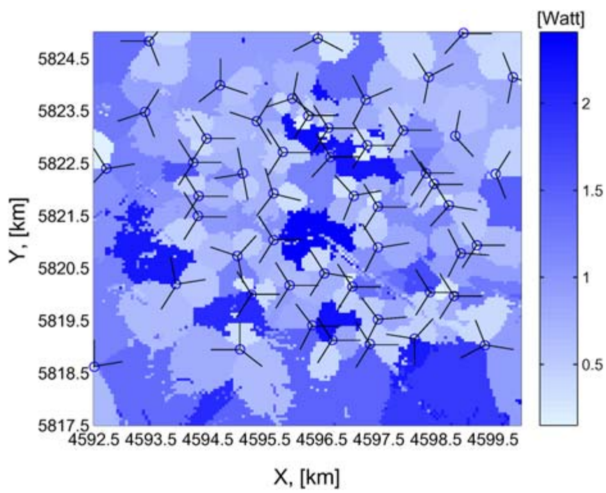

Fig. 3 Pilot power, network N6

Table 4 Optimal and near-optimal pilot power solutions

				Column generation and iterative rounding				
	CPLEX solution			Integer solution		LP bound	Gap (%)	Time
Network	P^*	Average	Time	Total	Average			
N1	27.8689	0.4645	0.3s	27.8884	0.4648	27.8387	0.18	0.4s
N2	26.3226	0.6267	5m50s	26.5643	0.6325	26.2803	1.09	6s
N3	46.6949	0.6671	55m	46.9224	0.6703	46.4233	1.08	34s
N4	92.4697	0.6605	2h47m	93.1238	0.6652	92.0477	1.17	4m46s
N5	—	—	—	127.9024	0.6803	125.4856	1.93	13m50s
N6	—	—	—	115.1785	0.7782	109.7558	4.94	3h2m

the area are covered by one or two pilot signals and only a few percent are covered by three or four cells. This is a significant improvement over the uniform solution where it turned out that more than 70% of the area are covered by more than one pilot signal. Figure 3 presents the optimized pilot power setting. For each bin, the color represents the power level of the strongest pilot signal. We observe that in the optimized solution most cells use a pilot power of less than 1.5 W. The number of cells that need the power level of the uniform solution (2.33 W) is very small.

In the next part of our computational study, we examine the impact of the level of coverage on the total power consumption of the pilot signals. There are several reasons for considering partial coverage. First, in many real-life planning scenarios, guaranteeing full coverage can be very expensive from both economic and resource management points of view. Therefore it is rather common (especially in rural areas) to have a coverage requirement of less than 100%. Second, reducing the coverage requirement will most probably lead to significant savings in power consumption. Because in our system model the pilot power is planned for the worst-case interference scenario, a solution obtained for a coverage requirement of slightly less than 100% is likely to be sufficiently good for the average interference scenario. Thus adopting a pilot power solution derived from partial coverage is of interest for a network operator, provided that the resulting saving in power is large. Third, solving our optimization model under partial coverage reveals bins that are most expensive in terms of radio resource management. This provides an operator with useful information when measures other than pilot power are considered for improving coverage. Examples of such measures include adjusting radio base station antenna azimuth to yield a better span, and antenna downtilt for reducing interference.

We chose two of the test networks, N1 and N6, for which we let the number of bins in the coverage constraint, d, take 21 different (uniformly spaced) values in the range of $[0.8 \cdot n, n]$. For each of these values, we applied the method of column generation and iterative rounding to generate upper and lower bounds to the optimal total power. The results, together with those obtained from the two ad hoc approaches, are shown in Figs. 4 and 5, where the upper and lower bounds (UB and LB) are plotted using solid and dotted lines, respectively.

From the two figures, it is evident that the total pilot power grows rapidly with respect to the degree of coverage. For network N6, for example, the amount of pilot power for covering 90% of the service area is only about 55% of that for full coverage, and is approximately 60% of the total amount of uniform pilot power needed to provide 90% coverage. Reducing pilot power leads to higher network capacity (in those areas covered by pilot signals). We also observe that, except for the case of full coverage of network N6, the integer solution found by the method is extremely close to optimum in the two figures. Another observation is that although in both figures the difference in the total amount of pilot power between the solutions decreases when decreasing the coverage degree, the gap between the ad hoc solutions and the optimized pilot power is significant for a large network (N6).

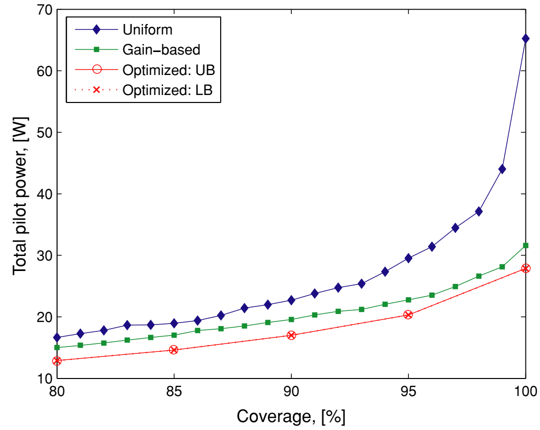

Fig. 4 Total pilot power vs coverage, N1

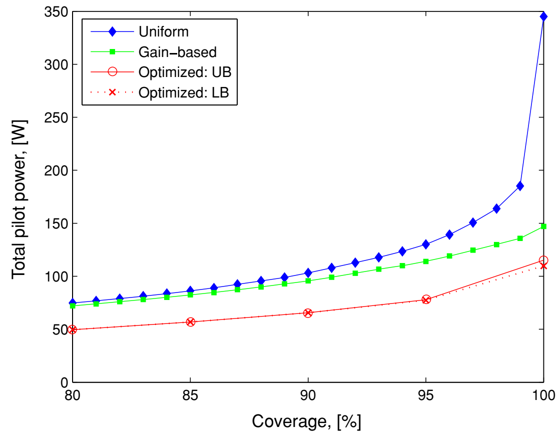

Fig. 5 Total pilot power vs coverage, N6

6 Conclusions

Engineering WCDMA networks gives rise to many optimization problems. In this paper, we have addressed the problem of providing service coverage using a minimum amount of pilot power. We presented two mathematical models and a solution approach based on column generation and iterative rounding. The approach is aimed to find a near-optimal solution within a reasonable amount of computing time.

Several conclusions can be drawn from our computational study. First, even for the scenario of worst-case interference, full coverage by pilot signals needs less than five percent of the total downlink power in a network. Second, a slight decrease in the degree of coverage enables considerable reductions in the pilot power. Coverage versus power consumption is thus an important trade-off in WCDMA network design. Moreover, optimized pilot power considerably outperforms ad hoc approaches, and, therefore, mathematical models can be very helpful for maximizing power efficiency in WCDMA networks.

An extension of the current research is pilot power optimization for the purpose of load balancing. The power of a pilot signal influences the cell size, and thereby the load of the cell. Taking into account the variation of traffic intensity over the service area, pilot power levels can be adjusted to equalize cell load. Another possible extension is joint optimization of pilot power and radio base station antenna configurations, e.g., antenna azimuth and antenna tilt. An interesting topic to be addressed in our forthcoming work is to develop an algorithm that can rapidly come up with a rather good (but not necessarily very close to optimal) pilot power setting. This would be very useful when pilot power is to be optimized for a large number of alternative network designs (i.e., various combinations of antenna tilt and azimuth).

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