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# Optimal Design of GFRP-Plywood Variable Stiffness Plate

Janis Sliseris[a,]*, Girts Frolovs[b], Karlis Rocens[c], Vadims Goremikins[d]

[a,b,c,d]*Department of Structural Engineering, Faculty of Building and Civil Engineering, Riga Technical University, Kalku st. 1, LV-1001 Riga, Latvia*

**Abstract**

The new optimization methods of outer layer fibre directions and concentrations of plywood plate with glass fibre-epoxy outer layers are proposed. The first method minimizes structural compliance. It consists of two phases. The fibre directions are optimized in the first phase and concentrations in the second phase. The second method maximizes buckling load for the first buckling mode. The increase of stiffness is about 31% of plate with optimized fibre direction and concentration comparing to similar non-optimized plate. The buckling load of single span rectangular plate could be increased about 34% when optimized GFRP-plywood plate by proposed method.


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*Keywords*: Plywood; Glass fibre; fibre orientation; fibre concentration; minimal compliance; maximal buckling load.

## 1. Introduction

Laminated plates and shells with variable stiffness have been intensively investigated during past two decades.

These kinds of structures are becoming more popular due to ability to achieve increased strength-to-mass and stiffness-to-mass ratios by tailoring material properties. The fibre steering machines are becoming more popular in manufacturing variable stiffness glass or carbon fibre plates.

An optimal variable stiffness plate could be obtained by optimization of fibre orientation angle [1], [2], [3] or thickness optimization [4-5]. A lamina with the variable stiffness and curved fibres provide a great flexibility to achieve needed natural frequencies, mode shapes [6], vibration amplitudes [7] and buckling load [8]. It is necessary to design constant thickness plates in many cases. Optimal properties of constant thickness plate or shell are obtained by using Genetic Algorithm [9-10] or Ant Colony algorithm [11], [12], [13] in cases of complicated objective function or many design variables. It is necessary to take into account inter-laminar stress of variable stiffness lamina [14] in some cases.

Problem of optimal fibre orientation angle of multilayer lamina is successfully solved by using topology optimization approach [15-16], discrete material optimization method [17], [18], [19], Ant colony algorithm [20] or Genetic algorithm [21]. Optimization of structural elements are done by taking into account uncertainty and nonlinear effects [22-23].

Flexural plates, like GFRP-plywood, with variable stiffness have not been investigated enough by now. The optimization method for this type of structure should be specially created. Therefore this publication is proposing a new optimization method for GFRP-plywood lamina fibre direction and concentration optimization and providing some typical results.

* Corresponding author.
*E-mail address:* janis.sliseris@gmail.com


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## 2. Optimization for minimal compliance

The lamination parameters that define stacking sequence of lamina by 12 parameters are usually used in optimization due to the convex relationship between stiffness and lamination parameters. The lamination parameters are related to each other, therefore the problem with feasible region always appears in optimization procedure. As well as an extra procedure for stacking sequence rendering from lamination parameters is necessary. To simplify this optimization technique and make it more applicable to the flexural plates with symmetrical layup authors propose a new method. This method is based on structural compliance minimization:

$$\min_{\phi,k} U^T(\phi,\ k) K(\phi,\ k) U(\phi,\ k), \tag{1}$$

where $\phi$ fibre orientation angles, $U(\phi,k)$ – isplacement vector, $K(\phi,k)$ – lobal stiffness matrix.

$$\phi = \{\phi_1,\ \phi_2, \ldots, \phi_{Ne}\} \tag{2}$$

$$k = \{k_1,\ k_2, \ldots,\ k_{Ne}\} \tag{3}$$

This method directly optimizes fibre orientation angle and concentration of only outer layers of symmetrical lamina. The outer layers play the most significant role in stiffness of flexural plate.

The material flexural stiffness matrix $D_i$ of $i$-th finite element is modified by using coordinate transformation matrix $N$ and fibre concentration coefficients $k$ :

$$D_i = k_i N^T(\phi_i) D_i^0 N(\phi_i) \tag{4}$$

The proposed method is based on algorithm that is shown in Fig.1. The algorithm consists of three loops. The first loop runs until the convergence criteria are satisfied. The second loop goes through all finite elements from 1 to $Ne$ (number of finite elements). The third loop goes through all discrete values of fibre orientation angles from 1 to $N$.

The fibre orientation angles are changed by special procedure $R(x)=x_j$. This procedure changes orientation angle to $x_j$ in region with center in $i$-th finite element and radius $R_{inf}$.

The finite element analysis is done inside all loops. The value of compliance function $C(i,j)$ (index $i$ indicate $i$-th discrete angle and index $j$ indicate $j$-th finite element) is calculated by using the results of finite element analysis.

There is a special procedure that updates values of fibre orientation angles $x$ inside the first loop. The updated value of $x$ is obtained in each finite element according to minimal compliance.

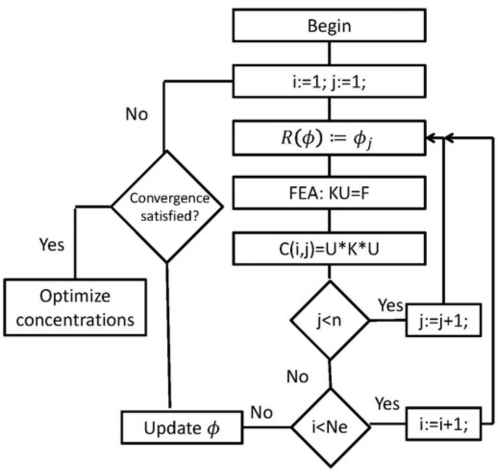


Fig. 1. The optimization algorithm of the fibre direction $\phi$

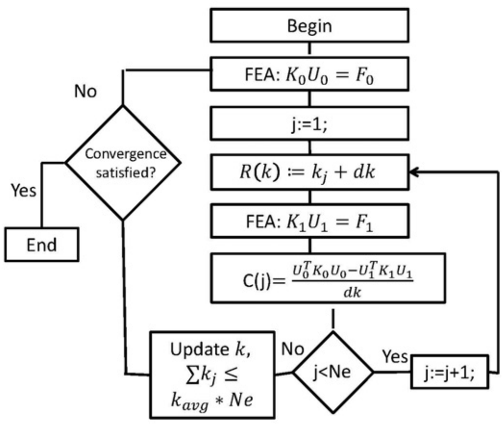


Fig. 2. The optimization algorithm of the fibre concentration $k$ optimization for maximal buckling strength

The fibre concentrations are updated by using following algorithm:

$$k_i = max\begin{cases} k_{min} \\ min\begin{cases} k_{max} \\ \left(k_i * \dfrac{C(i)}{L}\right)^{0.5} \end{cases} \end{cases} \tag{5}$$

where $k_{min}$, $k_{max}$ – minimal/maximal value of possible concentration, $L$ – rameter that is used to limit the sum of concentrations in all finite elements.

## 3. Optimization for maximal buckling strength

The optimization method maximizes the coefficient $\lambda y$ which the in-plane loads must be multiplied to cause buckling. The minimal buckling coefficient $\lambda_{min}$ s maximized by choosing appropriate fibre orientation angles and concentrations

$$(\phi, k) \max \lambda_{min} (\phi, k) \tag{6}$$

$$\left(K(\phi,\ k) - \lambda(\phi,\ k)K_G(\phi,\ k)\right]u(\phi,\ k)^i = 0 \tag{7}$$

Where $K$ s global stiffness matrix, $u^i$ s the $i$-th buckling mode and $r$ s the total number of degrees of freedom, $K_G$ s geometrical matrix, which is obtained using Mindlin plate theory and neglecting terms with third and higher powers in displacement gradients.

The optimization algorithms are similar to those shown in Fig. 1 and Fig. 2. The difference is in finite element analysis procedures. When buckling analysis is done than Eigen-value problem is solved, which is defined by Eq. 7.

## 4. Results

The optimal fibre orientation angles and concentration ratio were obtained for 19 layer symmetrical birch plywood sheet. The plywood sheet has the following lay-up [$\phi_i$, 0, 90, … 90,0, $\phi_i$]. The total thickness of the sheet is 26 mm. The outer layer of sheet was made of glass fibre- epoxy. The birch plywood were used with following elastic properties [24]: $E_1 = 16\,400$ MPa, $E_2 = 500$ MPa, $G_{12} = 890$ MPa, $v_{12} = 0.3$. The glass fibre of grade $E$ was analysed using the following elastic properties: $E_1 = E_2 = 85\,000$ MPa, $G_{12} = 35\,420$ MPa, $v_{12} = v_{21} = 0.2$ [25]. The epoxy glue was assumed to have the following elastic properties: $E_1 = E_2 = 2400$ MPa, $v_{12} = v_{21} = 0.3$, $G_{12} = 1308$ MPa [26].

Four discrete values of fiber orientation angle were used: 0/45/90/135.

The angle is between $x$-axis (horizontal axis) and fiber central axis. In all cases the influence radius was constant $R_{inf}$=0.15(m).

The plates are loaded by 1 KPa uniformly distributed transversal load when searching for minimal compliance. The in-plane load in horizontal axis direction was applied in optimization of maximal buckling load.

### *4.1. Minimal compliance optimization*

The optimization was done for single span and three span plywood plates with outer layer made of glass fibre-epoxy composite. The single span plate with 2.1 m spans in both directions and simply supported on all edges was optimized. Due to symmetry, one quarter of plate was optimized. The numerical results show that optimal direction of glass fibres are 45/135 degrees, see Fig. 3.

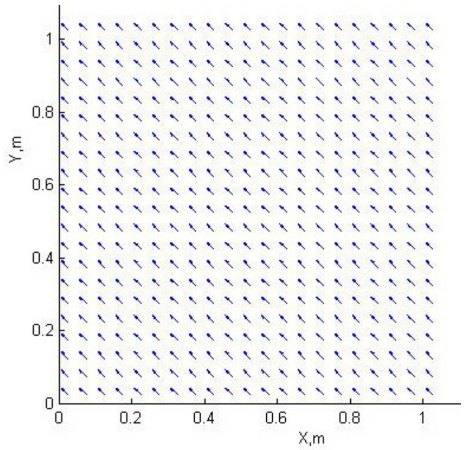


Fig. 3. Fibre direction plot of a single span rectangular plate with dimensions 2.1 m × 2.1 m (due to symmetry is shown one quarter of plate)

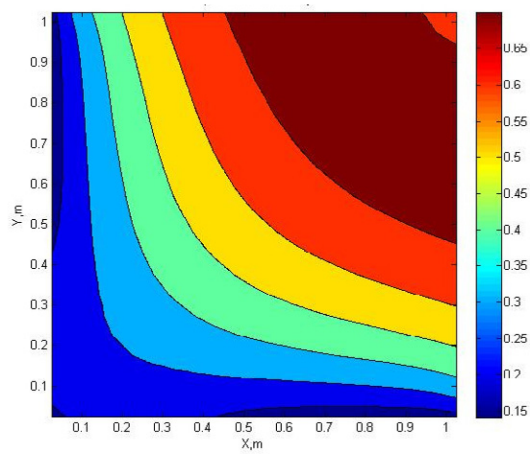


Fig. 4. Fibre relative concentration plot of a single span rectangular plate with dimensions 2.1 m × 2.1 m (due to symmetry is shown one quarter of plate)

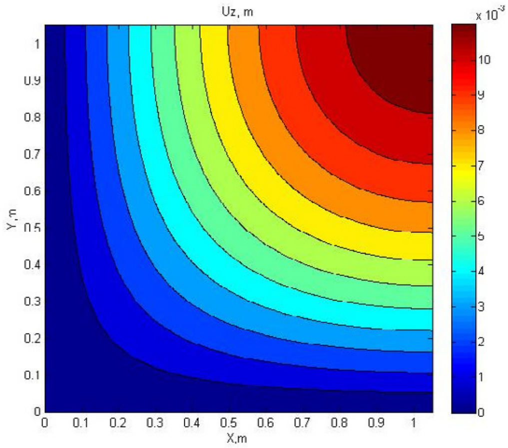


Fig. 5. Deflection plot of a single span rectangular non-optimized plate with dimensions 2.1 m × 2.1 m(due to symmetry is shown one quarter of plate).

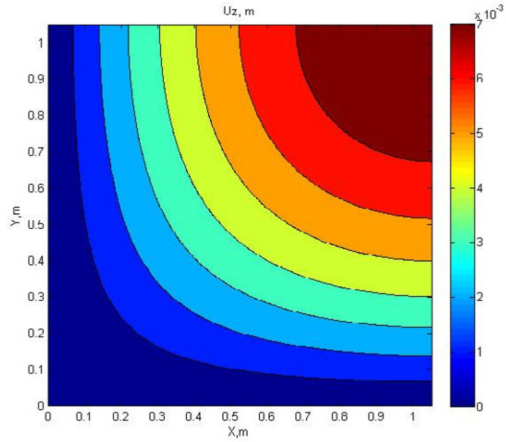


Fig. 6. Deflection plot of a single span rectangular optimized plate with dimensions 2.1 m × 2.1 m (due to symmetry is shown one quarter of plate)

The optimal fibre concentration plot is shown in Fig. 4. It can be seen that maximal amount of fibre should be put in the central part of plate.

The maximal deflection of non-optimized plate is 0.0117 m (see Fig. 5). The plate's with optimized fibre directions and constant concentration the maximal deflection is 0.0101 m. The maximal deflection of the plate with optimal fibre directions and concentrations is 0.008 m (see Fig. 6). The difference between rotation angles of non-optimized and optimized plates are the same as for maximal deflection.

The increase of stiffness of the optimized plate is not significant when single span plate has one dimension significantly bigger than other.

The optimization was done also for three span plates. The plots of optimal fibre directions and concentration of plate with three equal spans 0.7 m × 0.7 m × 0.7 m in both directions are shown in Fig. 7 and Fig. 8. The fibre direction is orthogonal with support lines. Maximal fibre concentrations are necessary on support lines. The fibre relative concentrations in middle of spans are about 25% less than on support lines.

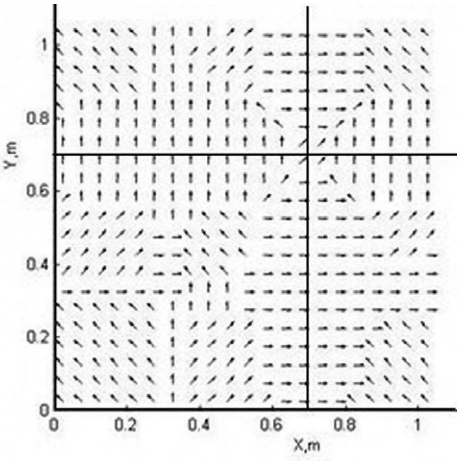


Fig. 7. Fibre direction plot of three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions (due to symmetry is shown one quarter of plate)

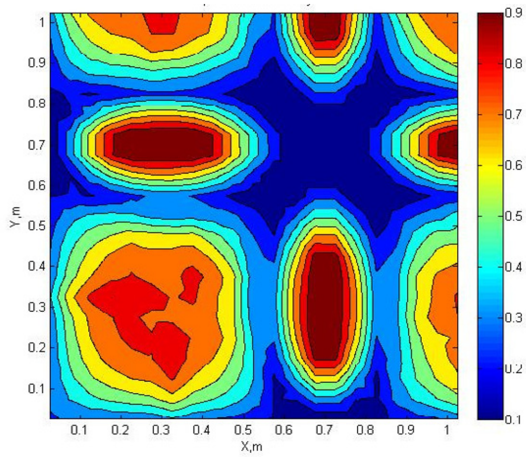


Fig. 8. Fibre relative concentration plot of three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions (due to symmetry is shown one quarter of plate)

The obtained optimal fibre orientation results show that single simply supported plate could be effectively manufactured by making four different regions of plate, see Fig. 3. However, the concentration that is close to necessary one could be achieved when additional four discrete regions are made (by strengthening the places where the fibre concentration is over 0.6), see Fig. 4.

The three span plates could be divided into rectangular discrete domains to obtain fibre orientation and concentrations that are close to the necessary ones, see Fig. 7 and Fig. 8.

### *4.2. Maximal buckling load optimization*

The local buckling problem of skin sheets of sandwich plates could be significantly reduced with variable stiffness GFRP-plywood shins. The fibre directions and concentrations are optimized to obtain maximal buckling load for the first buckling mode.

The optimal fibre directions and concentrations are obtained for single span rectangular plate which is clamped on all edges. The plate is loaded with uniformly distributed in-plane load on the right edge in horizontal direction.

The plot of optimal fibre direction is shown in Fig. 9. The fibre concentrations are shown in Fig. 10. The fibre direction in central part of plate is coincident with load direction. The fibres should be orthogonally to edges in the central part of edges. The fibres should be oriented in 45/135 degrees in the corners of plate.

The maximal fibre concentrations are in the central part of the edges and in the centre of plate, see Fig. 10.

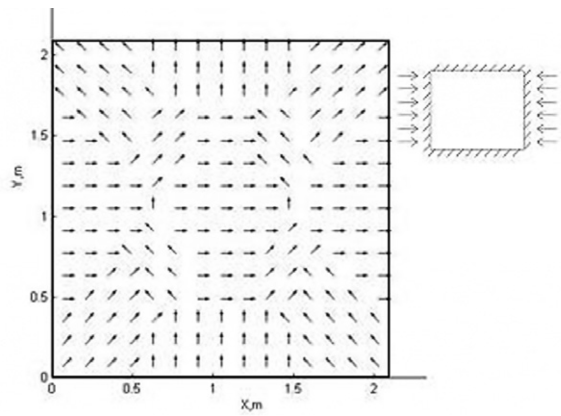


Fig. 9. Fibre direction plot of a single span rectangular plate with dimensions 2.1 m × 2.1 m

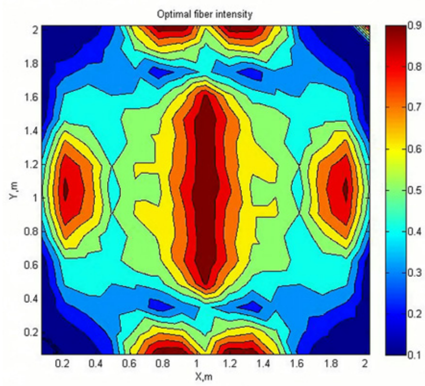


Fig. 10. Fibre relative concentration plot of a single span rectangular plate with dimensions 2.1 m × 2.1 m (the perfect symmetry is not achieved because of numerical computation errors (like round-up) when solving eigenvalue problem).

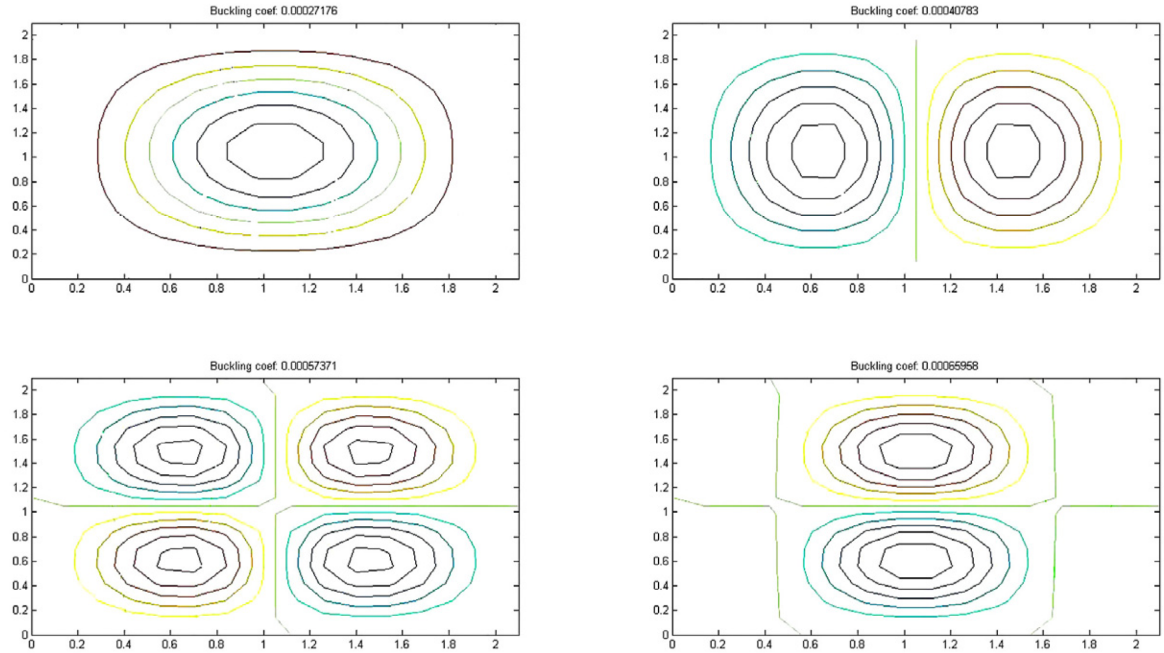


Fig. 11. Plot of buckling modes and appropriate first four buckling coefficients of a single span non-optimized rectangular plate with dimensions 2.1 m × 2.1 m (the scales of horizontal and vertical axes are different to provide better visualization)

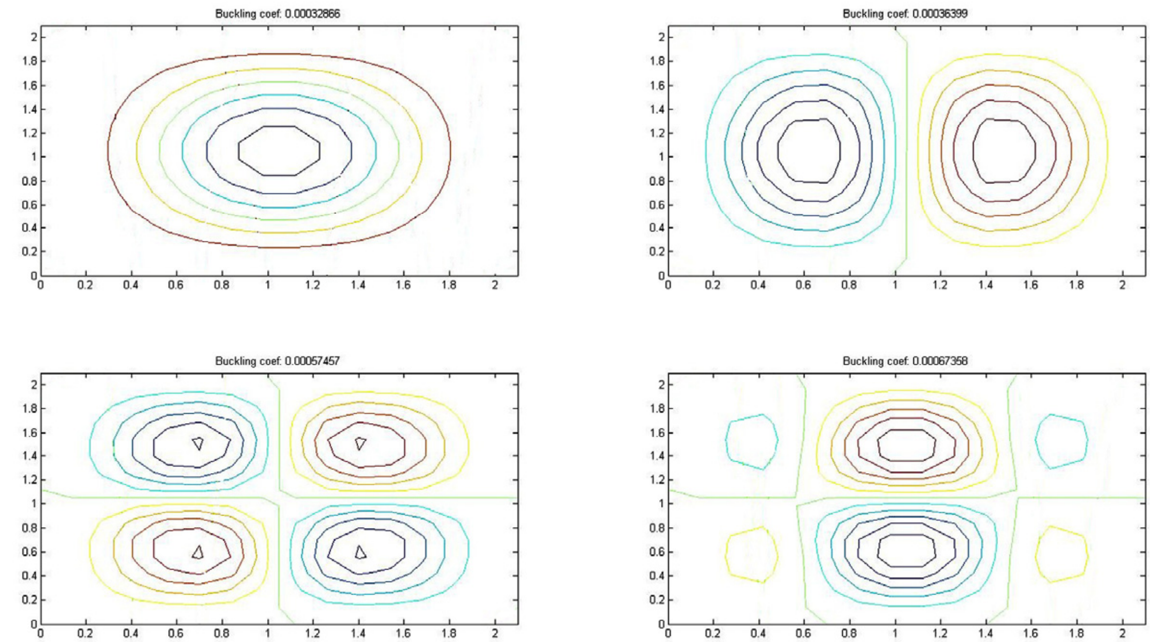


Fig. 12. Plot of buckling modes and appropriate first four buckling coefficients of a single span fibre optimized rectangular plate with dimensions 2.1 m × 2.1 m (the scales of horizontal and vertical axes are different to provide better visualization)

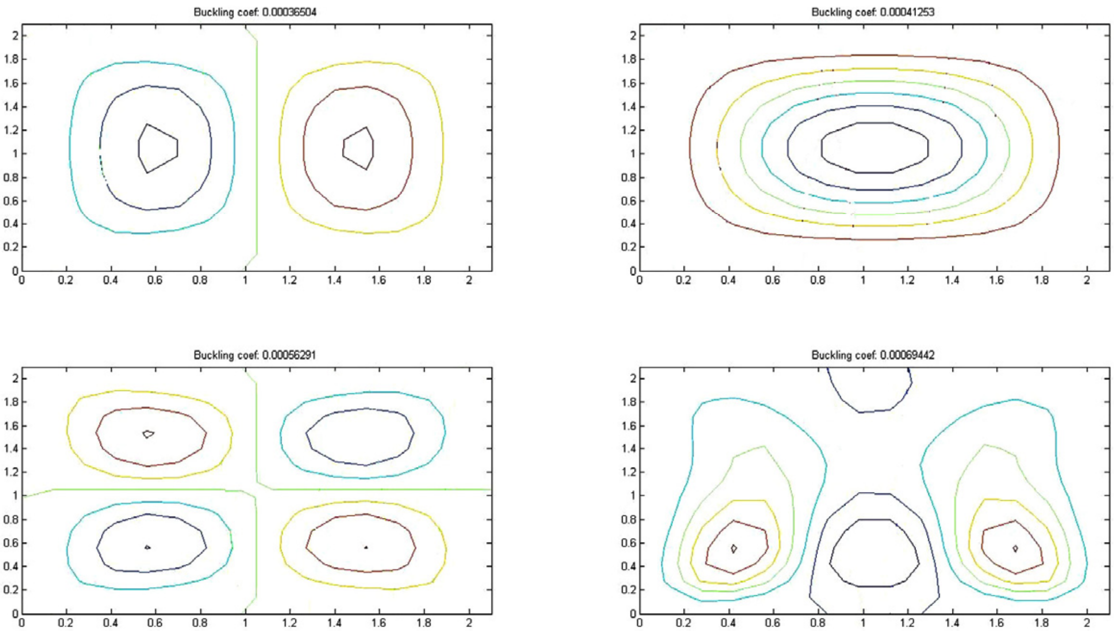


Fig. 13. Plot of buckling modes and appropriate first four buckling coefficients of a single span fibre and concentration optimized rectangular plate with dimensions 2.1 m × 2.1 m (the scales of horizontal and vertical axes are different to provide better visualization)

The rectangular plate's first four buckling modes and appropriate buckling coefficients are shown in Fig. 11 (non-optimized plate), Fig. 12 (optimized only fibre directions) and Fig. 13 (optimized fibre directions and concentrations).

The increase of buckling load is about 21% when optimized only fibre direction, but when optimized fibre directions and concentrations then buckling load is increased for about 34% for the first buckling mode. However, the buckling load for second, third and fourth modes do not increase significantly, sometimes even decrease.

The results show that the first two buckling modes are transposed for the case when fibre directions and concentrations are optimized.

### *4.2.1. Lamina with discrete variable stiffness*

The manufacturing of continuously varying fibre placement may cause difficulties in manufacturing of glass GFRP-plywood composite. Therefore we propose to make composite with discrete variable stiffness. It means that in discrete domains- areas of plate there are different fibre orientation and concentrations. We use only discrete fibre orientation angles (0/45/90/135 degrees) and relative concentrations (0.155/0.5/0.65/0.85).

For each area direction of glass fibres (0/45/90/135 degrees) and fibre relative concentration *I*- fiber volume fraction (0.155/0.5/0.65/0.85) is chosen. For example, if $I=0.5$, than equal volume of fibres and epoxy matrix are used in that layer. Three types of plates were analysed – non-optimized directions (0 degrees) and non-optimized relative concentration (0.5); optimized directions (0/45/90/135 degrees) and non-optimized relative concentration (0.5); optimized directions (0/45/90/135 degrees) and optimized relative concentrations (.155/0.5/0.65/0.85);

Reduced modulus of elasticity (for relative concentration $I=0.5$) for glass fibre layer are calculated [25] $E_1$=58000 MPa; $E_2$=$E_3$=16400 MPa;$G_{12}$=$G_{13}$=6000 MPa;$G_{23}$=15000 MPa; $\nu_{12}$= $\nu_{23}$=0,233; $\nu_{13}$=0.056.

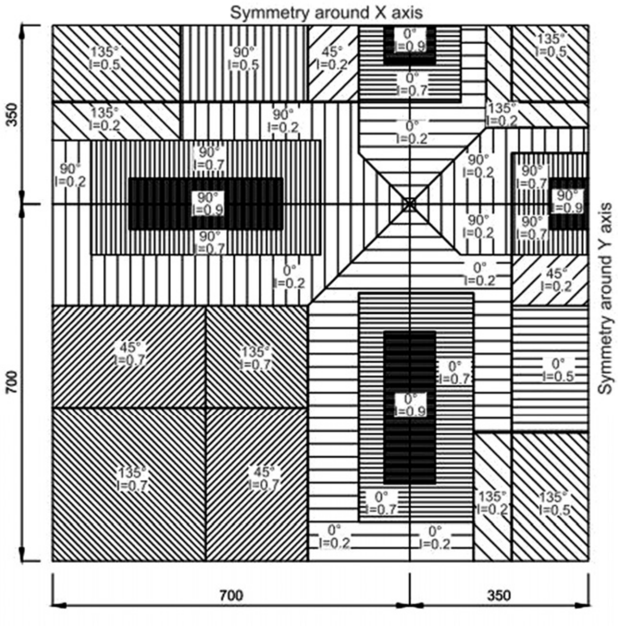


Fig. 14. The plot of discrete domains including fibre orientations and concentrations of a three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions (due to symmetry is shown one quarter of plate)

The chosen discrete domains with intensities are shown in Fig. 14.

The plates are loaded by 1 KPa uniformly distributed transversal load the same as for one span plate example. Due to symmetry, one quarter of plate was analysed.

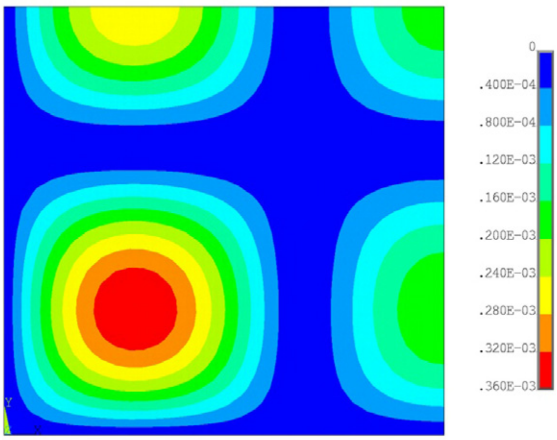


Fig. 15. Deflection plot of a three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions for non-optimized plate (due to symmetry is shown one quarter of plate).

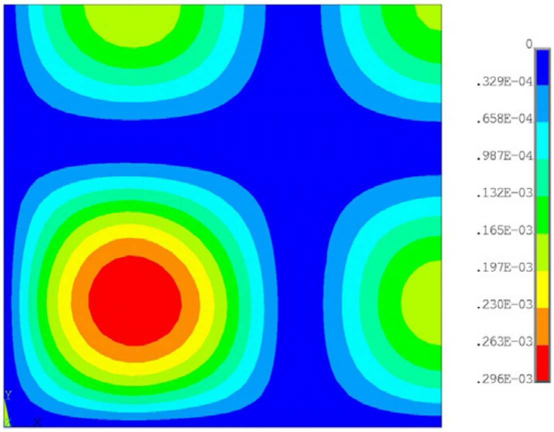


Fig. 16. Deflection plot of a three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions for plate with optimized fibre directions (due to symmetry is shown one quarter of plate)

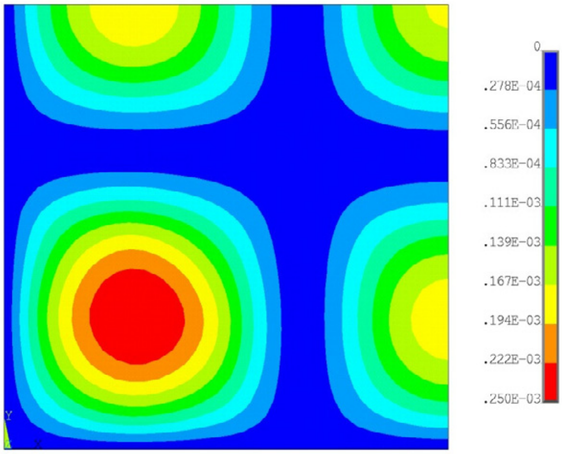


Fig. 17. Deflection plot of a three span rectangular plate with spans 0.7 m × 0.7 m × 0.7 m in both directions for plate with optimized fibre directions and concentrations (due to symmetry is shown one quarter of plate).

The thickness of GFRP layer is 1.75 mm, but the thickness of plywood layers is 1.21 mm.

The maximal deflection of a non-optimized plate is 0.360 mm, see Fig. 15. For the plate with optimized fibre directions and constant concentration the maximal deflection is 0.295 mm, see Fig. 16. The maximal deflection of the plate with optimal fibre directions and concentrations is 0.250 mm, see Fig. 17. The difference between rotation angles of non-optimized and optimized plates is the same as for maximal deflection.

The maximum displacements were in the middle of first and last span of the plate for the three span plate. Comparing all three plates it is found that the decrease of displacements is 18% when optimized only fibre direction. When optimized fibre directions and concentrations then displacements are decreased for about 31% comparing to non-optimized plate.

Stress in direction of fibres increase with maximal values in direction of fibres. They increase for about 21% (from 7,70 MPa to 9,35 MPa). It means that stress in plywood reduces.

## 5. Conclusions

The new fibre direction and concentration optimization methods, which minimize structural compliance and maximize first buckling load, are proposed.

The GFRP-plywood composite plates provide a grate possibility to increase stiffness and buckling load more than 30%.

The plate with optimal fibre directions and concentrations has a different buckling modes comparing to a non-optimized plate.

The optimized plate with discrete variable stiffness provides a reduction of displacements about 31%. However normal stress in direction of glass fibres incereases about 21% comparing to non-optimized plate.

In the future there should be created a method that automatically calculates optimal dimensions of discrete domains and fibre directions, concentrations in each discrete domain of the plate. The experimental investigations of GFRP-plywood plate should be made in the future.

## Acknowledgement

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