International Journal of Rock Mechanics & Mining Sciences 58 (2013) 118–126

Contents lists available at SciVerse ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Ranking the sawability of ornamental stone using Fuzzy Delphi and multi-criteria decision-making techniques

Reza Mikaeil [a,*], Yilmaz Ozcelik [b], Reza Yousefi [c], Mohammad Ataei [d], Seyed Mehdi Hosseini [e]

[a] Department of Mining and Metallurgical Engineering, Urmia University of Technology, Urmia 3619995161, Iran
[b] Hacettepe University, Department of Mining Engineering, Ankara, Turkey
[c] School of mechanical engineering, Sharif University of Technology, Tehran, Iran
[d] Faculty of Mining, Petroleum and Geophysics, Shahrood University of Technology, Shahrood, Iran
[e] Department of mining, Shahrood Branch, Islamic Azad University, Shahrood, Iran

A R T I C L E I N F O

Article history:
Received 22 May 2011
Received in revised form
7 May 2012
Accepted 4 September 2012
Available online 23 November 2012

Keywords:
Rock sawability
Production rate
Analytic hierarchy process
Fuzzy Delphi
TOPSIS

A B S T R A C T

The main purpose of this study is to compare the many different rock properties in the rock sawability. The comparison was realized with the combination of the analytic hierarchy process (AHP) and Fuzzy Delphi method and also TOPSIS method. The analysis is one of the multi-criteria techniques providing useful support in selecting among several alternatives with different objectives and criteria. FDAHP method was used in determining the weights of the criteria by decision makers and then ranking the sawability of the rocks was determined by TOPSIS method. The study was supported by the results obtained from a questionnaire carried out to know the opinions of the experts in this subject. During the research process, the rock sawability was evaluated in terms of production rate of sawn rock. Prediction of production rate is important in the cost estimation and the planning of the stone plants. The new developed ranking method may be used for evaluating production rate of ornamental stone at any stone factory with different stone. Some factors such as uniaxial compressive strength, Schmiazek F-abrasivity, mohs hardness and Young's modulus must be obtained for the best production rate ranking.

1. Introduction

Prediction of rock sawability is important in the cost estimation and the planning of the stone plants. A correct estimation of rock sawability helps to make the planning of the rock sawing projects more efficiently. Rock sawability depends on non-controlled parameters related to rock characteristics and controlled parameters related to properties of cutting tools and equipment. In the same working conditions, the sawing process and its results are strongly affected by mineralogical and mechanical properties of rock. Up to now, many studies have been done on the relationship between sawability and rock characteristics in stone processing. Burgess proposed a regression model for sawability, which was based on mineralogical composition, hardness, grain size and abrasion resistance [1]. Ertingshausen investigated the power requirements during cutting of Colombo Red granite in up and down cutting modes. He found out that the required power was less for the up-cutting mode when the cutting depth was below 20–25 mm. For deeper cuts, however, the power consumption was less for the down-cutting mode [2]. Wright and Cassapi tried to correlate the petrographic analysis and physical properties with sawing results. The research indicated cutting forces to have the closest correlation [3]. Birle et al. [4] presented similar work in 1986, but again considered only blade life as the criterion on which a ranking system should be based. Hausberger concluded that an actual sawing test was the most reliable method for determining the machinability of a rock type. He observed that the higher proportion of minerals with well defined cleavage planes helps the cutting to be easier [5]. Jennings and Wright gave an overall assessment of the major factors which affect saw blade performance. They found out that hard materials usually require a smaller size diamond than do softer stones because the load per particle is not sufficiently high and greater clearance is required for swarf. Conversely, if large diamond grits are used on hard materials, the penetration of the diamond is limited, and normally either excessive grit pull-out will occur or large wear flats will appear on the diamond particles [6]. Clausen et al. carried out a study on the acoustic emission during single diamond scratching of granite and suggested that acoustic emission could be used in sawability classification of natural stones. They also concluded that the cutting process is affected by the properties and frequency of minerals,

* Corresponding author. Tel.: +98 9113278634; fax: +98 273 333 55 09.
E-mail address: Reza.mikaeil@gmail.com (R. Mikaeil).

1365-1609/$ - see front matter Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.ijrmms.2012.09.002

grain size and degree of interlocking [7]. Tonshoff and Asche [8] discussed the macroscopic and microscopic methods of investigating saw blade segment wear. Luo [9] investigated the worn surfaces of diamond segments in circular saws for the sawing of hard and relatively soft granites. He found out that for the sawing of hard granite, the worn particles were mainly of the macro-fractured crystal and/or pull-out hole type. Webb and Jackson [10] showed that a good correlation could be obtained between saw blade wear performance and the ratio of normal to tangential cutting forces during the cutting of granite. Özçelik [11] investigated the working conditions of diamond wire cutting machines in marble industry. Xu investigated the friction characteristics of the sawing process of granites with diamond segmented saw blade. The results of the experimental studies indicated that most of the sawing energy is expended by friction of sliding between diamonds and granites [12]. Xu et al. found that about 30 percent of the sawing energy might be due to the interaction of the swarf with the applied fluid and bond matrix. Most of the energy for sawing and grinding is attributed to ductile ploughing [13]. Özçelik et al. [14] reviewed the wear on diamond beads in the cutting of different rock types by the ridge regression. Brook developed a new index test, called Brook hardness, which has been specifically developed for sliding diamond indenters. The consumed energy is predictable from this new index test [15]. Konstanty presented a theoretical model of natural stone sawing by means of diamond-impregnated tools. In the model, the chip formation and removal process are quantified with the intention of assisting both the toolmaker and the stonemason in optimising the tool composition and sawing process parameters, respectively [16]. Eyuboglu et al. investigated the relationship between blade wear and the sawability of andesitic rocks. They also showed that the wear rate could be predicted from the statistical model by using a number of andesite properties. The model indicated the Shore scleroscope hardness as the most important rock property affecting wear rate [17]. Xu et al. carried out an experimental study to investigate the characteristics of the force ratio in the sawing of granites with a diamond segmented blade. They measured consumed power for determining the tangential and the normal force components, horizontal and vertical force components. It was found out that the force components and their ratios did not differ much for different granites, in spite of the big differences in sawing difficulty [18]. Özçelik et al. investigated the effects of textural properties on marble cutting with diamond wire. They concluded that there are significant relationships between texture coefficient of limestone with unit wear and cutting rate [19]. Gunaydin et al. investigated the correlations between sawability and different brittleness using regression analysis. They concluded that sawability of carbonate rocks can be predicted from the rock brittleness [20]. Ersoy et al. experimentally studied the performance and wear characteristics of circular diamond saws in cutting of different types of rocks. They derived a statistical predictive model for the saw blade wear where specific cutting energy, silica content, bending strength, and Schmidt rebound hardness were the input parameters of the model [21,22]. Delgado et al. experimentally studied the relationship between the sawability of granite and its micro-hardness. In their study, sawing rate was chosen as the sawability criterion, and the micro-hardness of granite was calculated from mineral Vickers micro-hardness. Experimental results indicated that the use of Vickers hardness microindentor could provide more precise information in sawability studies [23]. Özçelik [24] experimentally studied the effect of mineralogical and petrographical properties of marble on cutting by diamond wire. Özçelik [25] obtained the optimum working conditions of diamond wire cutting machines in the marble industry. Özçelik [26] investigated the effect of marble textural characteristics on the sawing efficiency of diamond segmented frame saws. Mikaeil et al. developed a new statistical model to predicting the production rate of carbonate rocks based on uniaxial compressive strength and equal quartz content. Additional, they investigated the sawability of some important Iranian stone [27]. Yousefi et al. [28] studied the factors affecting on the sawability of the ornamental stone.

Especially, among the previous studies some researchers have developed a number of classification systems for ranking the sawability of rocks. Wei et al. evaluated and classified the sawability of granites by means of the fuzzy ranking system. In their study, wear performance of the blade and the cutting force were used as the sawability criteria. They concluded that with the fuzzy ranking system, by using only the tested petrographic and mechanical properties, a convenient selection of a suitable saw blade could be made for a new granite type [29]. Similarly, Tutmez et al. developed a new fuzzy classification of carbonate rocks based on rock characteristics such as uniaxial compressive strength, tensile strength, Schmidt hammer value, point load strength, impact strength, Los Angeles abrasion loss and P-wave velocity. By this fuzzy approach, marbles used by factories were ranked three linguistic qualitative categories: excellent, good and poor [30]. Kahraman et al. developed a quality classification of building stones from P-wave velocity and its application to stone cutting with gang saws. They concluded that the quality classification and estimation of slab production efficiency of the building stones can be made by ultrasonic measurements [31]. Mikaeil et al. developed a new classification system for assessing the carbonate rock sawability. In the new classification system, each rock was assigned a rating from 10 to 100, with a higher rating corresponding greater ease of sawing. Based on the CRSi (carbonate rock sawability index) rating, the sawing rate was classified into five categories: Excellent, Good, Fair, Poor, and Very poor. Moreover, they presented a new estimation model for predicting the production rate with respect to machine characteristics (rotational speed of saw and saw diameter) and CRSi by using multiple curvilinear regression analysis [32].

The performance of any stone factory is affected by the complex interaction of numerous factors. These factors that affect the production cost can be classified as production rate, energy, labor, water, diamond saw blade, polishing pads, filling material and packing. Among the above factors, production rate is one of the most important factors. In this study, it is aimed to develop a new hierarchy model for ranking the production rate of ornamental stone in sawing process with circular saw. By this model, ornamental stone were ranked with respect to its production rate. This model can be used for cost analysis and project planning as a decision making index. To make a right decision on production rate of ornamental stone, all known criteria related to the problem should be analyzed. Although an increasing in the number of related criteria makes the problem more complicated and more difficult to reach a solution, this may also increase the correctness of the decision made because of those criteria. Due to the arising complexity in the decision process, many conventional methods are able to consider limited criteria and may be generally deficient. Therefore, it is clearly seen that assessing all of the known criteria connected to the production rate by combining the decision making process is extremely significant.

The main purpose of this study is to compare the many different factors in the production rate of the ornamental stone. The comparison is performed with the combination of the analytic hierarchy process (AHP) and Fuzzy Delphi method and also the use of TOPSIS method. The analysis is one of the multi-criteria techniques that provide useful support in the choice among several alternatives with different objectives and criteria. FDAHP method is used in determining the weights of the criteria by decision makers and then ranking the production rate of the rocks is determined by TOPSIS method. The study is supported by results obtained from a questionnaire carried out to know the opinions of the experts in this subject.

2. Fuzzy sets and fuzzy numbers

To deal with the vagueness of human thought, Zadeh [33] first introduced the fuzzy set theory, which was oriented to the rationality of uncertainty due to imprecision or vagueness. A major contribution of fuzzy set theory is its capability of representing vague data. The theory also allows mathematical operators and programming to apply to the fuzzy domain. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one. With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision making is not taken into account [34]. Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world [35]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for common-sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution [36]. Fuzzy set theory is a better means for modeling imprecision arising from mental phenomena which are neither random nor stochastic. Human beings are heavily involved in the process of decision analysis. A rational approach toward decision making should take into account human subjectivity, rather than employing only objective probability measures. This attitude, towards imprecision of human behavior led to study of a new decision analysis filed fuzzy decision making [37]. A tilde '$\sim$' will be placed above a symbol if the symbol represents a fuzzy set. A triangular fuzzy number (TFN), $\tilde{M}$ is shown in Fig. 1. A TFN is denoted simply as $(l|m,m|u)$ or (l,m,u).

The parameters l, m and u, respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. Each TFN has linear representations on its left and right side such that its membership function can be defined as

$$\mu\left(x|\tilde{M}\right)=\begin{cases}0, & x<l,\\ (x-l)/(m-l), & l\leq x\leq m,\\ (u-x)/(u-m), & m\leq x\leq u,\\ 0, & x>u.\end{cases} \tag{1}$$

A fuzzy number can always be given by its corresponding left and right representation of each degree of membership:

$$\tilde{M}=\left(M^{l(y)},M^{r(y)}\right)=(l+(m-l)y,u+(m-u)y),\quad y\in[0,1], \tag{2}$$

where $l(y)$ and $r(y)$ denote the left side representation and the right side representation of a fuzzy number, respectively.

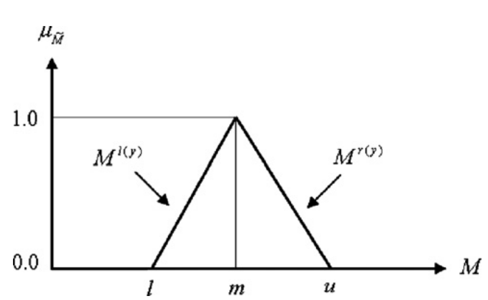

Fig. 1. A triangular fuzzy number, $\tilde{M}$.

3. Fuzzy Delphi analytic hierarchy process

The analytic hierarchy process (AHP) is an approach that is suitable for dealing with complex systems related to making a choice from among several alternatives and which provides a comparison of the considered options, firstly proposed by Saaty [38]. The AHP is based on the subdivision of the problem in a hierarchical form. In fact, the AHP helps organize the rational analysis of the problem by dividing it into its single parts; the analysis then supplies an aid to the decision makers who, making several pair-wise comparisons, can appreciate the influence of the considered elements in the hierarchical structure; the AHP can also give a preference list of the considered alternative solutions [38,39].

The AHP is a tool that can be used for analyzing different kinds of social, political, economic and technological problems, and it uses both qualitative and quantitative variables. The fundamental principle of the analysis is the possibility of connecting information, based on knowledge, to make decisions or previsions; the knowledge can be taken from experience or derived from the application of other tools. Among the different contexts in which the AHP can be applied, mention can be made of the creation of a list of priorities, the choice of the best policy, the optimal allocation of resources, the prevision of results and temporal dependencies, the assessment of risks and planning [39]. Although the AHP is to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style [40]. The traditional AHP method is problematic in that it uses an exact value to express the decision maker's opinion in a comparison of alternatives [41]. In addition, AHP method is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process [42]. To overcome all these shortcomings, FDAHP was developed for solving the hierarchical problems. Decision makers usually find that it is more confident to give interval judgments than fixed value judgments. This is because usually he/she is unable to explicit his/her preference to explicit about the fuzzy nature of the comparison process. Delphi method is a technique for structuring an effective group communication process by providing feedback of contributions of information and assessment of group judgments to enable individuals to re-evaluate their judgments. Since its development in the 1960s at Rand Corporation, Delphi method has been widely used in various fields [43–46]. On the other hand, Delphi method use crisp number and mean to become the evaluation criteria, these shortcomings might distort the experts' opinion. In order to deal with the fuzziness of human participants' judgments in traditional Delphi method, Ishikawa et al. [47] posited fuzzy set theory proposed by Zadeh [34] into the Delphi method to improve time-consuming problems such as the convergence of experts' options presented by Hwang and Lin [48]. The FDM is a methodology in which subjective data of experts are transformed into quasi-objective data using the statistical analysis and fuzzy operations. The main advantages of FDM [49] are that it can reduce the numbers of surveys to save time and cost and it also includes the individual attributes of all experts. This paper proposes the use of FDAHP for determining the weights of the main criteria which reflect the main properties of the rock affecting on rock sawability.

3.1. Methodology of FDAHP

The relative fuzzy weights of the decision elements are calculated using the following three steps based on the FDM and aggregate the relative fuzzy weights to obtain scores for the decision alternation.

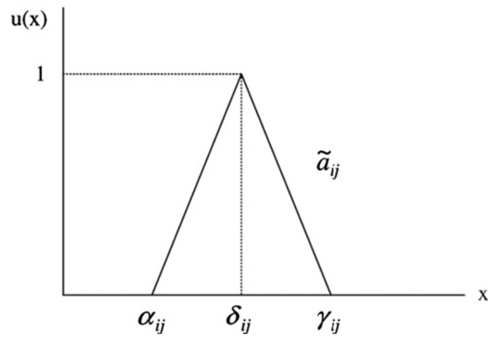

Fig.2. The membership function of the Fuzzy Delphi method.

First, compute the triangular fuzzy numbers (TFNs) $\tilde{a}_{ij}$ as defined in Eq. (3). In this work, the TFNs (shown as Fig. 2) representing the pessimistic, moderate and optimistic estimate is used to represent the opinions of experts for each activity time.

$$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}) \tag{3}$$

$$\alpha_{ij} = \mathrm{Min}(\beta_{ijk}), \quad k = 1, \ldots, n \tag{4}$$

$$\delta_{ij} = \left(\prod_{k=1}^{n} \beta_{ijk} \right)^{1/n}, \quad k = 1, \ldots, n \tag{5}$$

$$\gamma_{ij} = \mathrm{Max}\left(\beta_{ijk}\right), \quad k = 1, \ldots, n \tag{6}$$

where $\alpha_{ij} \le \delta_{ij} \le \gamma_{ij}$, $\alpha_{ij}, \delta_{ij}, \gamma_{ij} \in [1/9,1] \cup [1,9]$, and α_{ij}, δ_{ij}, γ_{ij} are obtained from Eqs. (4)–(6). α_{ij} indicates the lower bound and γ_{ij} indicates the upper bound. β_{ijk} indicates the relative intensity of importance of expert k between activities i and j. n is the number of experts in consisting of a group. Next, following the outline above, obtain a fuzzy positive reciprocal matrix, $\tilde{A} = [\tilde{a}_{ij}]$, $\tilde{a}_{ij} \times \tilde{a}_{ji} \approx 1$, $\forall i$, $j = 1,2,\ldots,n$, i.e.,

$$\tilde{A} = \begin{bmatrix} (1,1,1) & (\alpha_{12},\delta_{12},\gamma_{12}) & (\alpha_{13},\delta_{13},\gamma_{13}) \\ (1/\gamma_{12},1/\delta_{12},1/\alpha_{12}) & (1,1,1) & (\alpha_{23},\delta_{23},\gamma_{23}) \\ (1/\gamma_{13},1/\delta_{13},1/\alpha_{13}) & (1/\gamma_{23},1/\delta_{23},1/\alpha_{23}) & (1,1,1) \end{bmatrix} \tag{7}$$

Lastly, calculate the relative fuzzy weights of the evaluation factors:

$$\tilde{Z}_i = [\tilde{a}_{ij} \otimes \cdots \otimes \tilde{a}_{in}]^{1/n}, \quad \tilde{W}_i = \tilde{Z}_i \otimes \left(\tilde{Z}_i \oplus \cdots \oplus \tilde{Z}_n\right)^{-1} \tag{8}$$

where $\tilde{a}_1 \otimes \tilde{a}_2 \cong (\alpha_1 \times \alpha_2, \delta_1 \times \delta_2, \gamma_1 \times \gamma_2)$; the symbol $\otimes$ here denotes the multiplication of fuzzy numbers and the symbol $\oplus$ here denotes the addition of fuzzy numbers. $\tilde{W}_i$ is a row vector in consist of a fuzzy weight of the ith factor. $\tilde{W}_i = (\omega_1, \omega_2, \ldots, \omega_n)$ $i = 1, 2, \ldots n$, and W_i is a fuzzy weight of the ith factor.

4. TOPSIS method

TOPSIS (technique for order preference by similarity to ideal solution) is one of the useful multi-criteria decision making (MCDM) techniques to manage real-world problems [50]. TOPSIS method was firstly proposed by Hwang and Yoon [51]. According to this technique, the best alternative would be the one that is nearest to the positive ideal solution and farthest from the negative ideal solution [52]. The positive ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria [53]. In short, the positive ideal solution is composed of all the best values attainable of criteria, whereas the negative ideal solution consists of all the worst values attainable of criteria [54]. In this paper, TOPSIS method is used for determining the final ranking of the sawability of rocks. TOPSIS method is performed in the following steps.

Step 1. Decision matrix is normalized via

$$r_{ij} = \frac{w_{ij}}{\sqrt{\sum_{i=1}^{J} w_{ij}^2}} \qquad i = 1,2,3,\ldots,n, \quad j = 1,2,3,\ldots J \tag{9}$$

Step 2. Weighted normalized decision matrix is formed:

$$v_{ij} = w_i \times r_{ij}, \qquad i = 1,2,3,\ldots,n, \quad j = 1,2,3,\ldots J \tag{10}$$

Step 3. Positive ideal solution (PIS) and negative ideal solution (NIS) are determined:

$$A^* = \{v_1^*, v_2^*, \ldots, v_i^*, \ldots, v_n^*\} \quad \text{Maximum values} \tag{11}$$

$$A^- = \{v_1^-, v_2^-, \ldots, v_i^-, \ldots, v_n^-\} \quad \text{Minimum values} \tag{12}$$

Step 4. The distance of each alternative from PIS and NIS are calculated:

$$d_j^* = \sqrt{\sum_{i=1}^{n} \left(v_{ij} - v_j^*\right)^2} \tag{13}$$

$$d_j^- = \sqrt{\sum_{i=1}^{n} \left(v_{ij} - v_j^-\right)^2} \tag{14}$$

Step 5. The closeness coefficient of each alternative is calculated:

$$CC_j = \frac{d_j^-}{d_j^* + d_j^-} \tag{15}$$

Step 6. By comparing CC_j values, the ranking of alternatives are determined.

5. Application of FDAHP-TOPSIS method to multi-criteria comparison of sawability

5.1. Factors influencing rock sawability

Carbonate rock sawability depends on non-controlled parameters related to rock characteristics (such as textural characteristics, mechanical characteristics and environmental and structural characteristics) and controlled parameters related to properties of cutting tools and equipment (such as machine power, feed rate, depth of cut, peripheral speed, cutting mode, blade type, diameter of blade saw and Amount of water used). In the same working conditions, the sawing process and its results are strongly affected by physical and mechanical properties of rock. The important characteristics influencing the rock sawability are shown in Fig. 3. These characteristics are taken as criteria in this study. The texture used in Fig. 3. includes texture type and mineral composition of rock.

5.2. Determination of criteria's weights

Because different groups have varying objectives and expectations, they judge on rock sawability from different perspectives.

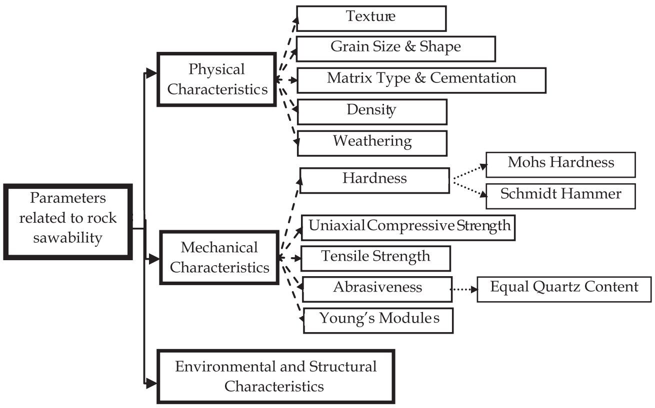

Fig. 3. Important characteristics influencing the rock sawability.

So, affecting criteria have different level of significance for different users. For this reason, five decision makers are selected from different areas and these decision makers evaluate the criteria. FDAHP is proposed to take the decision makers subjective judgments into consideration and to reduce the uncertainty and vagueness in the decision process. Decision makers from different backgrounds may define different weight vectors. They usually cause not only the imprecise evaluation but also serious persecution during decision process. For this reason, we proposed a group decision based on FDAHP to improve pair-wise comparison. First each decision maker (D_i), individually carry out pair-wise comparison by using Saaty's 1–9 scale [38]. An example of these pair-wise comparisons is shown as follow. "$C_{1\ldots12}$" is the criteria describing uniaxial compressive strength, mohs hardness, equal quartz content, abrasiveness, tensile strength, Young's modules, grain size and shape, texture, matrix type and cementation, Schmidt hammer rebound, density, weathering, respectively.

$$D_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \end{array} \\ \left[\begin{array}{cccccccccccc} 1 & 3 & 5 & 1 & 1 & 1 & 5 & 5 & 3 & 1/3 & 1 & 3 \\ 1/3 & 1 & 3 & 1/3 & 1/3 & 1/3 & 3 & 3 & 1 & 1/5 & 1/3 & 1 \\ 1/5 & 1/3 & 1 & 1/5 & 1/5 & 1/5 & 1 & 1 & 1/3 & 1/9 & 1/5 & 1/3 \\ 1 & 3 & 5 & 1 & 1 & 1 & 5 & 5 & 3 & 1/3 & 1 & 3 \\ 1 & 3 & 5 & 1 & 1 & 1 & 5 & 5 & 3 & 1/3 & 1 & 3 \\ 1 & 3 & 5 & 1 & 1 & 1 & 5 & 5 & 3 & 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 & 1/5 & 1/5 & 1/5 & 1 & 1 & 1/3 & 1/9 & 1/5 & 1/3 \\ 1/5 & 1/3 & 1 & 1/5 & 1/5 & 1/5 & 1 & 1 & 1/3 & 1/9 & 1/5 & 1/3 \\ 1/3 & 1 & 3 & 1/3 & 1/3 & 1/3 & 3 & 3 & 1 & 1/5 & 1/3 & 1 \\ 3 & 7 & 9 & 3 & 3 & 3 & 9 & 9 & 5 & 1 & 3 & 5 \\ 1 & 3 & 5 & 1 & 1 & 1 & 5 & 5 & 3 & 1/3 & 1 & 3 \\ 1/3 & 1 & 3 & 1/3 & 1/3 & 1/3 & 3 & 3 & 1 & 1/5 & 1/3 & 1 \end{array}\right] \end{array} \tag{16}$$

The weighting factors for each criterion were presented in the following steps:

1. Compute the triangular fuzzy numbers (TFNs):

$$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}) \tag{17}$$

According Eqs. (4)–(6),

$$\alpha_{ij} = \mathrm{Min}(\beta_{ijk}), \quad k = 1, \ldots, n \tag{18}$$

2. Create fuzzy pair-wise comparison matrix $\tilde{A}$. In this way, decision makers' pair-wise comparison values are transformed into triangular fuzzy numbers as in Table.

3. Calculate the relative fuzzy weights of the evaluation factors.

$$\tilde{Z}_1 = [\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \cdots \tilde{a}_{112}]^{1/12} = [0.6948, 0.968, 1.3738]$$
$$\tilde{Z}_2 = [\tilde{a}_{21} \otimes \tilde{a}_{22} \otimes \cdots \tilde{a}_{212}]^{1/12} = [0.7451, 0.968, 1.2989]$$
$$\tilde{Z}_3 = [\tilde{a}_{31} \otimes \tilde{a}_{32} \otimes \cdots \tilde{a}_{312}]^{1/12} = [0.5373, 0.7861, 1.2268]$$
$$\tilde{Z}_4 = [\tilde{a}_{41} \otimes \tilde{a}_{42} \otimes \cdots \tilde{a}_{412}]^{1/12} = [1.0284, 1.3098, 1.7181]$$
$$\tilde{Z}_5 = [\tilde{a}_{51} \otimes \tilde{a}_{52} \otimes \cdots \tilde{a}_{512}]^{1/12} = [1.0502, 1.3773, 1.8295]$$
$$\tilde{Z}_6 = [\tilde{a}_{61} \otimes \tilde{a}_{62} \otimes \cdots \tilde{a}_{612}]^{1/12} = [0.9, 1.1823, 1.5363]$$
$$\tilde{Z}_7 = [\tilde{a}_{71} \otimes \tilde{a}_{72} \otimes \cdots \tilde{a}_{712}]^{1/12} = [0.4665, 0.6661, 1.0885]$$
$$\tilde{Z}_8 = [\tilde{a}_{81} \otimes \tilde{a}_{82} \otimes \cdots \tilde{a}_{812}]^{1/12} = [0.508, 0.6897, 0.9733]$$
$$\tilde{Z}_9 = [\tilde{a}_{91} \otimes \tilde{a}_{92} \otimes \cdots \tilde{a}_{912}]^{1/12} = [0.4601, 0.7858, 1.2189]$$
$$\tilde{Z}_{10} = [\tilde{a}_{101} \otimes \tilde{a}_{102} \otimes \cdots \tilde{a}_{1012}]^{1/12} = [1.0284, 1.4483, 1.8295]$$
$$\tilde{Z}_{11} = [\tilde{a}_{111} \otimes \tilde{a}_{112} \otimes \cdots \tilde{a}_{1112}]^{1/12} = [0.7451, 1.1074, 1.4128]$$
$$\tilde{Z}_{12} = [\tilde{a}_{121} \otimes \tilde{a}_{122} \otimes \cdots \tilde{a}_{1212}]^{1/12} = [0.7717, 1.0353, 1.4128]$$
$$\sum \tilde{Z}_i = [8.9356, 12.327, 16.919]$$
$$\tilde{W}_1 = \tilde{Z}_1 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0411, 0.0785, 0.1537]$$
$$\tilde{W}_2 = \tilde{Z}_2 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.044, 0.0785, 0.1454]$$
$$\tilde{W}_3 = \tilde{Z}_3 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0318, 0.064, 0.1373]$$
$$\tilde{W}_4 = \tilde{Z}_4 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0608, 0.1063, 0.1923]$$
$$\tilde{W}_5 = \tilde{Z}_5 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0621, 0.1117, 0.2047]$$
$$\tilde{W}_6 = \tilde{Z}_6 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0532, 0.0959, 0.1719]$$
$$\tilde{W}_7 = \tilde{Z}_7 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0276, 0.054, 0.1218]$$
$$\tilde{W}_8 = \tilde{Z}_8 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.03, 0.056, 0.1089]$$
$$\tilde{W}_9 = \tilde{Z}_9 \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0272, 0.0637, 0.1364]$$
$$\tilde{W}_{10} = \tilde{Z}_{10} \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0608, 0.1175, 0.2047]$$
$$\tilde{W}_{11} = \tilde{Z}_{11} \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.044, 0.0898, 0.1581]$$
$$\tilde{W}_{12} = \tilde{Z}_{12} \otimes \left(\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3\right)^{-1} = [0.0456, 0.084, 0.1581] \tag{19}$$

The final weights of each parameter are calculated as follows: $W_1 = (\prod_{i=1}^{3} \omega_j)^{1/3} = 0.07928$, $W_2 = 0.0796$, $W_3 = 0.0655$, $W_4 = 0.1076$, $W_5 = 0.1125$, $W_6 = 0.0958$, $W_7 = 0.05675$, $W_8 = 0.0569$, $W_9 = 0.0619$, $W_{10} = 0.1136$, $W_{11} = 0.0857$, $W_{12} = 0.0847$. Mentioned priority weights are indicated for each criterion in Tables 1 and 2.

5.3. Selected parameters for assessing the rock sawability

In attempting to present a ranking system for assessing rock sawability, using all mentioned parameters is difficult from a practical point of view. In this ranking system three following rules have been considered: (a) the number of parameters used should be small, (b) equivalent parameters should be avoided, and (c) parameters should be considered within certain groups. Considering these rules, the parameters chosen for assessing the rock sawability are listed as follows: (a) uniaxial compressive strength (UCS); (b) Schmiazek F-abrasivity factor (SF-a); (c) mohs hardness (MH); (d) Young's modulus (YM). The reasons of the selections of these parameters are given below.

5.3.1. Uniaxial compressive strength (UCS)

Uniaxial compressive strength is one of the most important engineering properties of rocks. Rock material strength is used as an important parameter in many rock mass classification systems. Using this parameter in classification is necessary because

Table 1
Fuzzy pair-wise comparison matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1, 1, 1)	(0.7, 1, 1.4)	(0.7, 1.2, 2.3)	(0.6, 0.7, 1)	(0.6, 0.7, 1)	(0.6, 0.8, 1)
C_2	(0.7, 1, 1.4)	(1, 1, 1)	(0.7, 1.2, 1.7)	(0.7, 0.7, 0.8)	(0.6, 0.7, 1)	(0.7, 0.8, 1)
C_3	(0.4, 08, 1.4)	(0.6 0.8, 1.4)	(1, 1, 1)	(0.4, 0.6, 1)	(0.3, 0.6, 0.8)	(0.4, 0.6, 1)
C_4	(1, 1.4, 1.8)	(1.3, 1.4, 1.4)	(1, 1.7, 2.3)	(1, 1, 1)	(0.8, 1, 1.3)	(1, 1.1, 1.3)
C_5	(1, 1.4, 1.8)	(1, 1.4, 1.8)	(1.3, 1.8, 3)	(0.8, 1.1, 1.3)	(1, 1, 1)	(0.8, 1.1, 1.3)
C_6	(1, 1.3, 1.8)	(1, 1.3, 1.4)	(1, 1.6, 2.3)	(0.8, 1, 1)	(0.8, 0.9, 1.3)	(1, 1, 1)
C_7	(0.4, 0.7, 1.4)	(0.6, 0.7, 1)	(0.4, 0.8, 1)	(0.4, 0.5, 0.8)	(0.3, 0.5, 0.8)	(0.4, 0.5, 1)
C_8	(0.4, 0.7, 1)	(0.6, 0.7, 1)	(0.7, 0.9, 1)	(0.4, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.4, 0.6, 0.7)
C_9	(0.4, 0.8, 1.4)	(0.4, 0.8, 1.4)	(0.6, 1, 1.7)	(0.3, 0.6, 0.8)	(0.3, 0.6, 0.8)	(0.3, 0.6, 1)
C_{10}	(1.3, 1.5, 1.8)	(1, 1.5, 1.8)	(1, 1.8, 3)	(0.8, 1.1, 1. 3)	(0.8, 1.1, 1.3)	(1, 1.2, 1.3)
C_{11}	(1, 1.1, 1.4)	(0.7, 1.1, 1.4)	(0.7, 1.4, 2.3)	(0.6, 0.9 1)	(0.6, 0.8, 1)	(0.7, 0.9, 1)
C_{12}	(0.7, 1.1, 1.4)	(0.7, 1.1, 1.4)	(1, 1.3, 2.3)	(0.6, 0.8, 1)	(0.7, 0.8, 0.8)	(0.6, 0.8, 1)
	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
C_1	(0.7, 1.5, 2.3)	(1, 1.4, 2.3)	(0.7, 1.2, 2.3)	(0.6, 0.7, 0.8)	(0.7, 0.9, 1)	(0.7, 0.9, 1.4)
C_2	(1, 1.5, 1.7)	(1, 1.4, 1.7)	(0.7, 1.2, 2.3)	(0.6, 0.7, 1)	(0.7, 0.9, 1.4)	(0.7, 0.9, 1.4)
C_3	(1, 1.2, 2.3)	(1, 1.1, 1.4)	(0.6, 1, 1.7)	(0.3, 0.5, 1)	(0.4, 0.7, 1.4)	(0.4, 0.8, 1)
C_4	(1.3, 2, 2.3)	(1.4, 1.9, 2.3)	(1, 1.7, 3)	(0.8, 0.9, 1.3)	(1, 1.2, 1.8)	(1, 1.3, 1.8)
C_5	(1.3, 2.1, 3)	(1.4, 2.1, 3)	(1.3, 1.7, 3)	(0.8, 1, 1.3)	(1, 1.2, 1.8)	(1.3, 1.3, 1.4)
C_6	(1, 1.9, 2.3)	(1.4, 1.8, 2.3)	(1, 1.6, 3)	(0.8, 0.9, 1)	(1, 1.1, 1.4)	(0.4, 0.6, 1)
C_7	(1, 1, 1)	(0.6, 1, 1.4)	(0.4, 0.8, 1.7)	(0.3, 0.5, 1)	(0.4, 0.6, 1.4)	(0.4, 0.6, 1)
C_8	(0.7, 1, 1.7)	(1, 1, 1)	(0.6, 0.9, 1.7)	(0.3, 0.5, 0.7)	(0.4, 0.6, 1)	(0.4, 0.7, 1)
C_9	(0.6, 1.2, 2.3)	(0.6, 1.1, 1.7)	(1, 1, 1)	(0.3, 0.5, 1)	(0.4, 0.7, 1.4)	(0.4, 0.8, 1)
C_{10}	(1, 2.2, 3)	(1.4, 2.1, 3)	(1, 1.8, 3)	(1, 1, 1)	(1.3, 1.3, 1.4)	(1, 1.4, 1.8)
C_{11}	(0.7, 1.7, 2.3)	(1, 1.6, 2.3)	(0.7, 1.4, 2.3)	(0.7, 0.8, 0.8)	(1, 1, 1)	(0.7, 1.1, 1.4)
C_{12}	(1, 1.6, 2.3)	(1, 1.5, 2.3)	(1, 1.3, 2.3)	(0.6, 0.7, 1)	(0.7, 0.9, 1.4)	(1, 1, 1)

Table 2
Priority weights for criteria.

Criteria	Global weights
Uniaxial compressive strength (UCS)	0.1136
Mohs hardness (MH)	0.1125
Equal quartz content (EQC)	0.1076
Abrasiveness (SF-a)	0.0958
Tensile strength (TS)	0.0857
Young's modules (YM)	0.0847
Grain size and shape (GsS)	0.0796
Texture (T)	0.0793
Matrix type and cementation (Mt and C)	0.0655
Schmidt hammer rebound (SHR)	0.0619
Density (*D*)	0.0569
Weathering (W)	0.0568

strength of rock material constitutes the strength limit of rock mass [55]. Factors that influence the UCS of rocks are the constitutive minerals and their spatial positions, weathering or alteration rate, micro-cracks and internal fractures, density and porosity [44]. Therefore, uniaxial compressive strength test can be considered as representative of rock strength, density, weathering, texture and matrix type. Thus, the summation of the weights of five parameters (texture, weathering, density, matrix type and UCS) is considered as weight of UCS. In total, the weight of UCS is about 0.372.

5.3.2. *Schmiazek F-abrasivity factor (SF-a)*

Abrasiveness influences the tool wear and sawing rate substantially. Abrasiveness is mainly affected by various factors such as mineral composition, the hardness of mineral constituents and grain characteristics such as size, shape and angularity [56]. Schimazek's F-abrasiveness factor is depend on textural and mechanical properties and has good ability for evaluation of rock abrasivity. Therefore, this index is selected for using in ranking system. F-abrasivity factor is defined as

$$F = \frac{\mathrm{EQC} \times \mathrm{Gs} \times \mathrm{BTS}}{100} \quad (20)$$

where *F* is Schimazek's wear factor (N/mm), EQC is the equivalent quartz content percentage, Gs is the median grain size (mm), and BTS is the in direct Brazilian tensile strength. Regarding the rock parameters which are used in questionnaires, summation of the weights of abrasiveness, grain size, tensile strength and equivalent quartz content is considered as weight of Schimazek's F-abrasiveness factor. In total the weight of this factor is 0.3687.

5.3.3. *Mohs hardness (MH)*

Hardness can be interpreted as the rock's resistance to penetration. The factors that affect rock hardness are the hardness of the constitutive minerals, cohesion forces, homogeneity, and the water content of rock [44]. Thus, hardness is a good index of all above given parameters of rock material. Considering the importance of hardness in rock sawing, hardness, after Schmiazek F-abrasivity factor, is considered the most relevant property of rock material. Regarding the questionnaires, summation of the weights of mohs hardness and Schmidt hammer rebound value was considered as total weight of mean mohs hardness. In total, the weight of this factor is 0.1745.

5.3.4. *Young's modulus (YM)*

According to rock behavior during the fracture process, especially in sawing, the way that rocks reach the failure point has a great influence on sawability. The best scale for rock elasticity is Young's modulus. Based on ISRM suggested methods [57], the tangent Young's modulus at a stress level equal to 50% of the ultimate uniaxial compressive strength is used in this ranking system. Regarding the questionnaires, the weight of this factor is about 0.0847 in total.

5.4. *Field studies and laboratory tests*

Some stone factories in Shamsabad of Iran were selected and performances of diamond circular saw in term of the hourly production rates (P_h) were measured on 12 different granite and carbonate rocks. In Iranian factories, the usually very similar

Table 3
The result of laboratory and field studies.

	Rock samples	UCS MPa	BTS MPa	EQC %	Gs mm	SF-a N/mm	YM GPa	MH *n*	P_h m^2/h
1	Chayan (granite)	173	14.5	60.06	0.87	7.58	48.6	6.6	5
2	Ghermez Yazd (granite)	142	8.52	57.65	2.9	14.24	43.6	6.1	5.5
3	Sefid Nehbandan (granite)	145	9.2	64.3	4.1	24.25	35.5	5.75	5
4	Khoramdare (granite)	133	8.3	32.2	3.9	10.42	28.9	5.65	6
5	Morvarid Mashhad (granite)	125	7.4	30.3	3.8	8.52	31.2	5.6	6.5
6	Harsin (marble)	71.5	6.8	3.6	0.55	0.135	32.5	3.5	8.5
7	Anarak (marble)	74.5	7.1	3.4	0.45	0.109	33.6	3.2	9
8	Azarshahr (travertine)	53	4.3	2.8	1.01	0.122	20.7	2.9	11
9	Hajiabad (travertine)	61.5	5.6	2.6	0.85	0.124	21	2.9	10
10	Darebokhari (travertine)	63	5.4	2.7	0.87	0.127	23.5	2.95	10
11	Salsali (marble)	68	6.3	3.2	0.52	0.105	31.6	3.1	9
12	Haftoman (marble)	74.5	7.2	4	0.6	0.173	35.5	3.6	8

UCS: Uniaxial compressive strength, BTS: Brazilian tensile strength, EQC: Equivalent quartz content, Gs: Grain size, SF-a: Schmiazek F-abrasivity factor, YM: Young's Modules, MH: Mohs hardness, P_h: Production rate.

Table 4
Decision matrix.

	UCS C_1	SF-a C_2	YM C_3	MH C_4
1	173	7.58	48.6	6.6
2	142	14.24	43.6	6.1
3	145	24.25	35.5	5.75
4	133	10.42	28.9	5.65
5	125	8.52	31.2	5.6
6	71.5	0.135	32.5	3.5
7	74.5	0.109	33.6	3.2
8	53	0.122	20.7	2.9
9	61.5	0.124	21	2.9
10	63	0.127	23.5	2.95
11	68	0.105	31.6	3.1
12	74.5	0.173	35.5	3.6

Table 5
Normalized decision matrix.

	UCS C_1	SF-a C_2	YM C_3	MH C_4
1	0.4694	0.2362	0.4230	0.4194
2	0.3853	0.4438	0.3795	0.3876
3	0.3934	0.7558	0.3090	0.3653
4	0.3609	0.3247	0.2516	0.3590
5	0.3392	0.2655	0.2716	0.3558
6	0.1940	0.0042	0.2829	0.2224
7	0.2021	0.0034	0.2925	0.2033
8	0.1438	0.0038	0.1802	0.1843
9	0.1669	0.0039	0.1828	0.1843
10	0.1709	0.0040	0.2046	0.1874
11	0.1845	0.0033	0.2751	0.1970
12	0.2021	0.0054	0.3090	0.2287

machines are used. Therefore, in the studied factories many technical features of diamond circular saw such as the diameter and peripheral speed of the saw in were nearly similar. In this study, the characteristics of diamond circular saw were considered to be constant and were not used in the ranking model. For laboratory tests, rock blocks were collected from the studied factories. Each block sample was inspected for macroscopic defects so that it would provide test specimens free from fractures, partings or alteration zones. Then, test samples were prepared from these block samples and standard tests were completed to measure the above-mentioned parameters following the suggested procedures by the ISRM standards. The results of field and laboratory studies are listed in Table 3.

5.5. TOPSIS method

After determining the weights of the criteria with FDAHP method and laboratory studies, ranking the sawability of carbonate rocks is performed by TOPSIS method. First, the amount of each criterion is filled in decision matrix for each criterion. Decision matrix is obtained with respect to important rock properties (Table 4). Decision matrix is normalized via Eq. (9) (Table 5). Then, weighted normalized matrix is formed by multiplying each value with their weights (Table 6). Positive and negative ideal solutions are determined by taking the maximum and minimum values for each criterion:

$$A^{*} = \{0.1737, 0.2721, 0.0719, 0.0419\} \tag{21}$$

$$A^{-} = \{0.0532, 0.0012, 0.0306, 0.0184\} \tag{22}$$

Then, the distance of each method from PIS (positive ideal solution) and NIS (negative ideal solution) with respect to each criterion are calculated with the help of Eqs. (13) and (14). Then closeness coefficient of each rock (CC_j) is calculated by using Eq. (15) and the ranking of the rocks are determined according to these values. The ranking of the sawability of ornamental stone are also shown in Table 7 in the descending order of priority.

6. Discussion

In the present study, a new hierarchical model is developed to evaluate and ranking the sawability of ornamental stone with the use of effective criteria and considering of decision makers' judgments. The proposed approach is based on the combination of Fuzzy Delphi and analytic hierarchy process (FDAHP) methods. Technique for order preference by similarity to ideal solution (TOPSIS) is also used in this study. FDAHP was used for determining the weights of the criteria according to decision makers then rankings of carbonate rocks were determined by TOPSIS. The proposed method was applied for Iranian ornamental stone to evaluation the production rate in rock sawing process. For validation of applied ranking system, experimental procedure was carried out. The sawability ranking results of studied rocks are shown in Table 8. According to Table 8, the first rock in ranking is Sefid Nehbandan granite. It has a maximum value of production rate among other rock samples. On the opposite side,

Table 6
Weighted normalized matrix.

	UCS C_1	SF-a C_2	YM C_3	MH C_4
1	0.1737	0.0850	0.0719	0.0419
2	0.1426	0.1598	0.0645	0.0388
3	0.1456	0.2721	0.0525	0.0365
4	0.1335	0.1169	0.0428	0.0359
5	0.1255	0.0956	0.0462	0.0356
6	0.0718	0.0015	0.0481	0.0222
7	0.0748	0.0012	0.0497	0.0203
8	0.0532	0.0014	0.0306	0.0184
9	0.0617	0.0014	0.0311	0.0184
10	0.0632	0.0014	0.0348	0.0187
11	0.0683	0.0012	0.0468	0.0197
12	0.0748	0.0019	0.0525	0.0229

Table 7
Rankings of the sawability of ornamental stone according to CC_j values.

Rank	Granite rock samples	dj*	dj⁻	CC_j
1	Morvarid Mashhad (granite)	0.185	0.121	0.396
2	Chayan (granite)	0.187	0.154	0.452
3	Khoramdare (granite)	0.163	0.142	0.466
4	Ghermez Yazd (granite)	0.117	0.186	0.615
5	Sefid Nehbandan (granite)	0.035	0.288	0.893
Rank	Carbonate rock samples	dj*	dj⁻	CC_j
1	Azarshahr (travertine)	0.300	0.000	0.000
2	Hajiabad (travertine)	0.297	0.009	0.028
3	Darebokhari (travertine)	0.296	0.011	0.035
4	Salsali (marble)	0.293	0.022	0.070
5	Harsin (marble)	0.291	0.026	0.081
6	Anarak (marble)	0.289	0.029	0.091
7	Haftoman (marble)	0.289	0.031	0.097

Table 8
The sawability ranking results of studied rocks.

Rank	Granite rock samples	CC_j	P_h
1	Morvarid Mashhad	0.396	6.5
2	Chayan	0.452	5
3	Khoramdare	0.466	6
4	Ghermez Yazd	0.615	5.5
5	Sefid Nehbandan	0.893	5
Rank	Carbonate rock samples	CC_j	P_h
1	Azarshahr (travertine)	0.000	11
2	Hajiabad (travertine)	0.028	10
3	Darebokhari (travertine)	0.036	10
4	Salsali (marble)	0.070	9
5	Harsin (marble)	0.081	8.5
6	Anarak (marble)	0.091	9
7	Haftoman (marble)	0.097	8

Morvarid Mashhad granite has a minimum value of production rate and maximum value of CC_j in the granite group. As can be seen from Table 8, there are same results for carbonate rock group. The relationship between hourly production rate and closeness coefficient of carbonate and granite rock (CC_j) was also investigated graphically for fitting a line to the set of the experimental data. Based on this analysis, among the many functions tested (linear, power, logarithmic, exponential), the exponential curve relation was fitted to the experimental data with higher correlations than all the other relationships. This relationship is presented in Fig. 4. As can be seen from Fig. 4 that there is a highly statistically significant relationship between hourly production rate and CC_j value. As hourly production rate increases, CC_j value decreases. It means that the new developed ranking is correct. According to Fig. 4 meaningful correlations between hourly production rate and CC_j value was obtained with the prediction equations given by

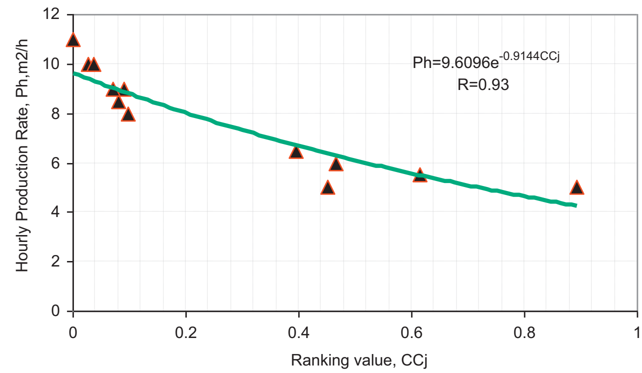

Fig. 4. Graph of production rate against CC_j.

$$P_h = 9.6096 \times \exp(-0.9144 \times CC_j) \tag{23}$$

where P_h is the hourly production rate, m^2/h, and CC_j is the ranking value.

7. Conclusion

In this paper, a decision support system was developed for ranking the sawability of ornamental stone. This system designed to eliminate the difficulties in taking into consideration many decision criteria simultaneously in the rock sawing process and to guide the decision makers for ranking the sawability of ornamental stone. In this study, FDAHP and TOPSIS methods was used to evaluate the sawability of ornamental stone. FDAHP is utilized for determining the weights of the criteria and TOPSIS method is used for ranking the sawability of ornamental stone. During this research, twelve types of ornamental stone belonging to granite and carbonate rock were tested in some factories located in Iran. The production rate of each sawn rock was determined to verify the result of applied approach for ranking them by sawability criteria. The experimental results confirmed the new ranking results precisely. This new ranking method can be used for evaluating the production rate of ornamental stone at any stone factory with different rock. Some parameters such as uniaxial compressive strength, Schmiazek F-abrasivity, mohs hardness and young's modulus must be obtained for the best sawability ranking.

References

[1] Burgess RB Circular sawing granite with diamond saw blades. In: Proceedings of the fifth industrial diamond seminar, 1978. p. 3–10.
[2] Ertingshausen W. Wear processes in sawing hard stone. Ind Diamond Rev 1985;5:254–8.
[3] Wright DN, Cassapi VB. Factors influencing stone sawability. Ind Diamond Rev 1985;2:84–7.
[4] Birle JD, Ratterman E. An approximate ranking of the sawability of hard building stones based on laboratory tests. Dimensional Stone Mag 1986:3–29.
[5] Hausberger P. Causes of the different behaviour of rocks when machined with diamond tools. Ind Diamond Rev 1989;3:1–25.
[6] Jennings M, Wright D. Guidelines for sawing stone. Ind Diamond Rev 1989;49:70–5.
[7] Clausen R, Wang CY, Meding M. Characteristics of acoustic emission during single diamond scratching of granite. Ind Diamond Rev 1996;3:96–9.
[8] Tonshoff HK, Asche J. Wear of metal-bond diamond tools in the machining of stone. Ind Diamond Rev 1997;1:7–13.
[9] Luo SY. Investigation of the worn surfaces of diamond sawblades in sawing granite. J Mater Process Technol 1997;70:1–8.
[10] Webb SW, Jackson WE. Analysis of blade forces and wear in diamond stone cutting. ASME J Manuf Sci Eng 1998;120:84–92.

[11] Özçelik Y Investigation of the working conditions of diamond wire cutting machines in marble industry. PhD thesis. Hacettepe University, Ankara, Turkey; 1999.
[12] Xu X. Friction studies on the process in circular sawing of granites. Tribol Lett 1999;7:221–7.
[13] Xu X, Li Y, Malkin S. Forces and energy in circular sawing and grinding of granite. J Manuf Sci Eng 2001;123:13–22.
[14] Özçelik Y, Kulaksız S, Çetin MC. Assessment of the wear of diamond beads in the cutting of different rock types by the ridge regression. J Mater Proc Technol 2002;127(3):392–400.
[15] Brook B. Principles of diamond tool technology for sawing rock. Int J Rock Mech Min Sci 2002;39:41–58.
[16] Konstanty J. Theoretical analysis of stone sawing with diamonds. J Mater Proc Technol 2002;123:146–54.
[17] Eyuboglu AS, Ozcelik Y, Kulaksiz S, Engin IC. Statistical and microscopic investigation of disc segment wear related to sawing Ankara andesites. Int J Rock Mech Min Sci 2003;40:405–14.
[18] Xu X, Li Y, Yu Y. Force ratio in the circular sawing of granites with a diamond segmented blade. J Mater Process Technol 2003;139:281–5.
[19] Ozcelik Y, Polat E, Bayram F, Ay AM. Investigation of the effects of textural properties on marble cutting with diamond wire. Int J Rock Mech Min Sci 2004;41:398.
[20] Gunaydin O, Kahraman S, Fener M. Sawability prediction of carbonate rocks from brittleness indexes. J South Afr Inst Min Metall 2004;104:239–44.
[21] Ersoy A, Atici U. Performance characteristics of circular diamond saws in cutting different types of rocks. Diamond Relat Mater 2004;13:22–37.
[22] Ersoy A, Buyuksagis S, Atici U. Wear characteristics of circular diamond saws in the cutting of different hard and abrasive rocks. Wear 2005;258:1422–36.
[23] Delgado NS, Rodriguez R, Rio A, Sarria ID, Calleja L, Argandona VGR. The influence of microhardness on the sawability of Pink Porrino granite (Spain). Int J Rock Mech Min Sci 2005;42:161–6.
[24] Özçelik Y. Effect of mineralogical and petrographical properties of marble on cutting by diamond wire. CIM Bull 2005;98(1085):1–6.
[25] Özçelik Y. Optimum working conditions of diamond wire cutting machines in the marble industry. Ind Diamond Rev 2005;1:58–64.
[26] Özçelik Y. The effect of marble textural characteristics on the sawing efficiency of diamond segmented frame saws. Ind Diamond Rev 2007;2:65–70.
[27] Mikaeil R, Ataei M, Hoseinie SH. Predicting the production rate of diamond wire saws in carbonate rocks cutting. Ind Diamond Rev 2008;3:28–34.
[28] Yousefi R, Mikaeil R, Ataei M Study of factors affecting the sawability of ornamental stone. In: Proceedings of the eighth international scientific conference SGEM, Bulgaria; 2010, p. 533–541.
[29] Wei X, Wang CY, Zhou ZH. Study on the fuzzy ranking of granite sawability. J Mater Process Technol 2003;139:277–80.
[30] Tutmez B, Kahraman S, Gunaydin O. Multifactorial fuzzy approach to the sawability classification of building stones. Constr Build Mater 2007;21:1672–9.
[31] Kahraman S, Ulker U, Delibalta S. A quality classification of building stones from P-wave velocity and its application to stone cutting with gang saws. J South Afr Inst Min Metall 2007;107:427–30.
[32] Mikaeil R, Yousefi R, Ataei M, Abbasian R. Development of a new classification system for assessing of carbonate rock sawability. Arch Min Sci 2011;56(1):57–68.
[33] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338–53.
[34] Tsaur SH, Chang TY, Yen CH. The evaluation of airline service quality by fuzzy MCDM. Tourism Manag 2002;23:107–15.
[35] Ertugrul I, Tus A. Interactive fuzzy linear programming and an application sample at a textile firm. Fuzzy Optim Decis Making 2007;6:29–49.
[36] Bojadziev G, Bojadziev M. Fuzzy sets fuzzy logic applications. Singapore: World Scientific; 1998.
[37] Lai YJ, Hwang CL. Fuzzy multiple objective decision making. Berlin: Springer; 1996.
[38] Saaty TL. The analytic hierarchy process. New York: McGraw-Hill; 1980.
[39] Saaty TL, Vargas LG. The analytic hierarchy process series. Pittsburgh: University of Pittsburgh; 1990.
[40] Kahraman C, Ruan D, Dogan I. Fuzzy group decision making for facility location selection. Inf Sci 2003;157:135–53.
[41] Wang TC, Chen YH. Applying consistent fuzzy preference relations to partnership selection. Omega Int J Manage Sci 2007;35:384–8.
[42] Deng H. Multicriteria analysis with fuzzy pair-wise comparison. Int J Approx Reason 1999;21:215–31.
[43] Liu YC, Chen CS. A new approach for application of rock mass classification on rock slope stability assessment. Eng Geol 2007;89:129–43.
[44] Hoseinie SH, Ataei M, Osanloo M. A new classification system for evaluating rock penetrability. Int J Rock Mech Min Sci 2009;46:1329–40.
[45] Cheng JH, Lee CM, Tang CH. An application of Fuzzy Delphi and fuzzy AHP for multi-criteria evaluation on bicycle industry supply chains. WSEAS Trans Syst Control 2009;1:4.
[46] Cheng JH, Lee CM, Tang CH. An application of Fuzzy Delphi and fuzzy AHP on evaluating wafer supplier in semiconductor industry. WSEAS Trans Info Sci 2009;5:6.
[47] Ishikawa A, Amagasa T, Tamizawa G, Totsuta R, Mieno H. The max–min Delphi method and Fuzzy Delphi method via fuzzy integration. Fuzzy Sets Syst 1993:55.
[48] Hwang CL, Lin ML. Group decision making under multiple criteria. Berlin: Springer; 1987.
[49] Kaufmann A, Gupta MM. Fuzzy mathematical models in engineering and management science. Amsterdam: North-Holland; 1988.
[50] Yoon K, Hwang CL. Manufacturing plant location analysis by multiple attribute decision making: Part II. Multi-plant strategy and plant relocation. Int J Prod Res 1985;23(2):361–70.
[51] Hwang CL, Yoon K. Multiple attributes decision making methods and applications. Berlin: Springer; 1981.
[52] Benitez JM, Martin JC, Roman C. Using fuzzy number for measuring quality of service in the hotel industry. Tourism Manag 2007;28(2):544–55.
[53] Wang MY, Elhag TMS. Fuzzy T.O.P.S.I.S. method based on alpha level sets with an application to bridge risk assessment. Expert Syst Appl 2006;31:309–19.
[54] Wang YJ. Applying FMCDM to evaluate financial performance of domestic airlines in Taiwan. Expert Syst Appl 2008;34:1837–45.
[55] Bieniawski ZT. Engineering rock mass classifications. New York: Wiley; 1989.
[56] Ersoy A, Waller MD. Textural characterization of rocks. Eng Geol 1995;39:123–36.
[57] International society for rock mechanics. Rock characterisation, testing and monitoring: ISRM suggested methods. Oxford: Pergamon; 1981.