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Financial market volatility and contagion effect: A copula–multifractal volatility approach

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HIGHLIGHTS

- A new approach based on the multifractal volatility method (MFV) is proposed to study the financial contagion effect.
- The tail dependence structure between the U.S. and Chinese stock market is analyzed by copulas.
- The multifractal volatility method is used to construct the marginal distributions for different kinds of copulas.

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ABSTRACT

In this paper, we propose a new approach based on the multifractal volatility method (MFV) to study the contagion effect between the U.S. and Chinese stock markets. From recent studies, which reveal that multifractal characteristics exist in both developed and emerging financial markets, according to the econophysics literature we could draw conclusions as follows: Firstly, we estimate volatility using the multifractal volatility method, and find out that the MFV method performs best among other volatility models, such as GARCH-type and realized volatility models. Secondly, we analyze the tail dependence structure between the U.S. and Chinese stock market. The estimated static copula results for the entire period show that the SJC copula performs best, indicating asymmetric characteristics of the tail dependence structure. The estimated dynamic copula results show that the time-varying t copula achieves the best performance, which means the symmetry dynamic t copula is also a good choice, for it is easy to estimate and is able to depict both the upper and lower tail dependence structure. Finally, with the results of the previous two steps, we analyze the contagion effect between the U.S. and Chinese stock markets during the subprime mortgage crisis. The empirical results show that the subprime mortgage crisis started in the U.S. and that its stock market has had an obvious contagion effect on the Chinese stock market. Our empirical results should/might be useful for investors allocating their portfolios.

1. Introduction

The successful experience of financial liberalization that started in the early 1980s in some of the emerging economies in Europe, Asia and Latin America provided a positive incentive to other emerging economies around the globe to follow the same policy initiatives. Although the benefits of economic and financial liberalization are well-documented in the literature, a surge of quick profiteering through liberalization forced the countries to implement fast but poorly managed reforms without sound frameworks of financial sector supervision and management. Many economists have now realized that contagion played a role in propagating the subprime mortgage crisis to the emerging market economies after 2008.

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The term "contagion" refers to a significant increase in cross-market linkage after a shock to one country or a group of countries [1,2]. With regard to the financial market contagion phenomenon, there are several most common methods used to investigate it. In earlier studies, a variety of papers [3–6] found that the correlations have changed over time, tending to increase during unstable periods. Furthermore, King and Wadhwani [7] and Bertero and Mayer [8] found that international correlation tends to increase during periods of market crises. However, the correlation coefficient is biased on high volatility regimes [9] and may be misleading if volatility is an important factor for contagion [2].

To avoid this bias, a common method is to use a GARCH-type model to estimate dynamic conditional correlation, and then test whether significant increases occur or not after the shock. For example, Longin and Solnik [10] use a bivariate GARCH model and found that correlations between the major stock markets rise in periods of high volatility. Ramchand and Susmel [11] used a switching ARCH model and found that the correlations between the U.S. and other world markets are on average 2 to 3.5 times higher when the U.S. market is in a high variance state as compared to a low variance state. Many other studies (such as Refs. [12–16]) have also used GARCH-type models to research the contagion phenomenon.

However, correlation is only a linear measure of dependence.[1] How can one measure nonlinear dependence? There is another way: copula approach. Recently, copulas have been widely used in contagion research (e.g. Refs. [17–21]). Copulas contain all the information about the dependence structure of a vector of random variables. They can capture nonlinear dependence, while correlation is only a linear measure of dependence. In particular, copulas contain information about the joint behavior of the random variables in the tails of the distribution, which should be of primary interest in a study of contagion during financial crises [17].

It is well-known that, when using the copula approach, we can employ a two-step procedure for the estimation of model parameters: (1) transform the raw data to independent and identically distributed (i.i.d) random variables and estimate the marginal distributions; (2) fit the copula function parameters. But there exists heteroskedasticity in financial time series, so how to filter heteroskedasticity and make the data transform to an i.i.d. series is a key issue when the copula method is applied. The most popular method to filter heteroskedasticity is using a GARCH-type model for each series, followed by a second step that is known as the copula-GARCH model. A series of papers [22–26] have found that volatility measurements based on multifractal methods obtain better forecasting accuracy than GARCH-type or RV models. So, in this paper, we use a more accurate method to filter the heteroskedasticity of financial time series, i.e., multifractal volatility (MFV), as proposed by Wei and Wang [22]. Just as the name copula-GARCH model suggests, in combination with the second step we call it the copula-MFV model.

Since the original studies of Mandelbrot [27,28], a series of studies in a line of literature known as econophysics have revealed that many financial market time series display fractal and multifractal characteristics [29–33]. Multifractal tools are also used to take into account some important stylized facts that cannot be described by traditional methods, such as the GARCH-type model [34–38], and other financial research, such as volatility forecasting [37–40], market efficiency [41–43], and portfolio allocation [44,45]. The multifractal characteristics of financial markets have not been limited to developed financial markets, but also have been noted in recent studies of the emerging stock market in China [46–48].

In this paper, we study the dependence structure between the U.S. and Chinese stock markets, and analyze the dependence variation from 2006 to 2012 by using the time varying parameters copula approach so as to detect the contagion effect after the subprime mortgage crisis. To begin with, we employ a multifractal volatility model to estimate the volatility for each market, and transform i.i.d. standardized returns to a uniform distribution on (0, 1) by the probability integral transform. Second, in order to capture the nonlinear dependence, especially the tail dependence between the U.S. and Chinese stock markets, we show several copula dependence structures. Finally, in order to test the contagion effect, we use the time varying parameters copula to analyze the variation of the tail dependence between the U.S. and Chinese stock markets. The main contribution of this paper is the combination of multifractal methods and copulas to study the contagion effect between the U.S. and Chinese stock markets after the subprime mortgage crisis. The multifractal method can filter the heteroskedasticity more accurately, and the copulas contain all the information about the dependence structure of a vector of random variables.

The rest of this paper is organized as follows. We introduce the sample data and discuss how daily returns are constructed in Section 2. A model for analyzing the multifractal spectrum of high-frequency intraday data and the multifractal volatility measure is introduced in Section 3. Section 4 presents various static copulas and shows the dependence structure between the U.S. and Chinese stock market. Time varying parameters copulas and the contagion effect test are introduced in Section 5. In Section 6 we give our conclusions.

2. Data

The data set in our empirical study consists of high-frequency (every 5 min) price quotes for the Standard & Poor's 500 (S&P 500) index and the Shanghai Stock Exchange Composite (SSEC) index during the period from January 2, 2004, to June 29, 2012. The S&P 500 is one of the most commonly followed indices and many consider it the best representation of the market and a bellwether for the U.S. economy. The National Bureau of Economic Research has classified common stocks as

[1] In fact, another method, linear regression, is also used to test contagion; for example, the vector autoregressive method is such a method using only a linear measure of dependence.

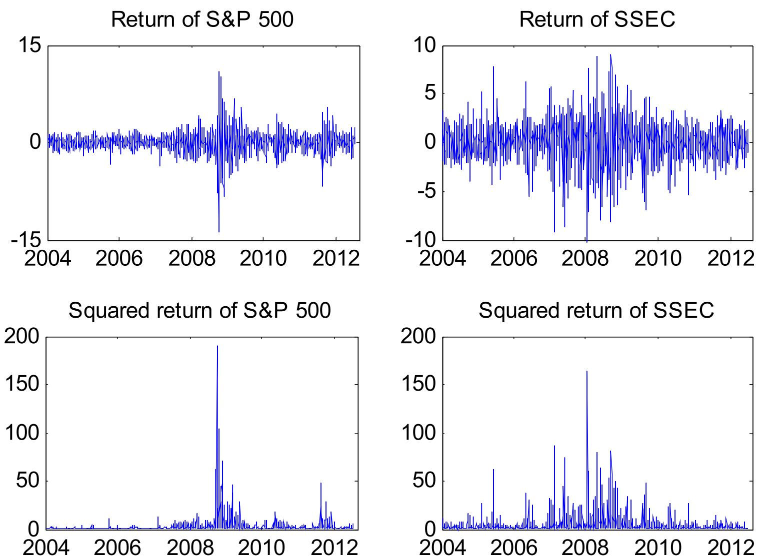

Fig. 1. Daily return and squared return of the S&P 500 and the SSEC.

a leading indicator of business cycles.[2] The SSEC is an authoritative statistical index widely followed and used at home and abroad to measure the performance of China's securities market. In order to exclude incorrect correlations between these two markets, we eliminated those observations where data were unavailable in either market, because of holidays or other reasons. Finally, we obtained 1970 observations for both the S&P 500 and SSEC. In addition, we made the data of the S&P 500 correspond to the next trading day of the SSEC, because the Chinese stock market closes before the opening of the U.S. stock market, which only affects the following day of its counterpart in China.

For the S&P 500 index, we can get 80 5 min quotes per day from 8:30 a.m. to 3:15 p.m. The Shanghai Stock Exchanges run from 9:30 a.m. to 11:30 a.m. and then from 1:00 pm to 3:00 pm, giving a total of 4 trading hours for each trading day. The SSEC is quoted once every 5 min over the course of the trading day for a total of 48 quotes (excluding the opening price) per day. The 5 min price quotes are denoted as $I_{t,d}$, $t = 1, 2, \ldots, N$ and $d = 1, 2, \ldots, M$, where $N = 1970$ and $M = 80$ are for the S&P 500 or $M = 48$ is for the SSEC. $I_{t,80}$ denotes the closing price on day t for the S&P 500 and $I_{t,48}$ means closing price on day t for the SSEC. The daily return R_t is defined as

$$R_t = 100 \times (\ln I_{t,M} - \ln I_{t-1,M}), \tag{1}$$

Fig. 1 shows these two markets' daily returns and squared daily returns along with time from January 2, 2004 to June 29, 2012. Table 1 presents some descriptive statistics for the two time series. From Fig. 1, we can easily find that these two stock markets both have stronger fluctuations after a sharp shock around the end of 2008. All four time series shown in Table 1 are stationary according to the ADF test [49] and the distributions exhibit varying degrees of skewness and leptokurticity. Skewness and kurtosis are significant at the 1% level in all four distributions, thus strongly rejecting the null hypothesis of normality according to the Jarque–Bera test [50]. Meanwhile, all series are not i.i.d. series according to the BDS test [51]. Moreover, under the null hypothesis of non-auto-correlation, the Ljung–Box Q tests show that all series are significantly auto-correlated, with time lags of 10 days. It indicates significant heteroscedasticity for all series.

3. Multifractal volatility measurement and DCCA

3.1. Multifractal volatility measurement by multifractal spectrum analysis

A number of recent studies have found that the multifractal spectrum of high-frequency price fluctuations over the course of a trading day contains valuable volatility information [52,53]. Thus, in this section, we discuss how to construct a multifractal volatility measure (MFV) from a multifractal spectrum and how to model the MFV series according to the method introduced by Wei and Wang [22].

In contrast to the realized volatility measurement proposed by Andersen and Bollerslev [54], the MFV measure is constructed on the basis of high-frequency price data as opposed to high-frequency return data. Following the steps in Wei and Wang [22], we first compute the multifractal spectrum for one day using high-frequency price data within that day before calculating the MFV from the multifractal spectrum.

[2] This brief introduction about the S&P 500 comes from Wikipedia.

Table 1
Descriptive statistics for daily returns and squared daily returns of the S&P 500 and the SSEC.

	S&P 500		SSEC	
	R_t	R_t^2	R_t	R_t^2
Mean	0.010259	2.020764	0.033553	3.388225
Std. deviation	1.421860	7.737007	1.840875	8.202425
Skewness	−0.580811*	12.968039*	−0.374640*	7.869692*
Kurtosis	12.705253*	240.066514*	3.900398*	103.376960*
ADF	−35.5785*	−5.7695*	−45.5307*	−16.2995*
Jarque–Bera	1.34×10^4*	4.79×10^6*	1.29×10^3*	8.98×10^5*
BDS	19.79269*	13.18630*	9.21396*	9.19878*
No ARCH	536.11172*	279.61134*	129.69663*	28.48607*
Q (10)	46.739*	1409.983*	23.869*	236.639*

Notes: Kurtosis is measured by the excess kurtosis coefficient; ADF is the Augmented Dickey–Fuller test; Q (·) is the Ljung–Box Q (·) statistic.
* Denotes significance at the 1% level.

We use the box-counting method proposed by Sun, Chen, Wu, and Yuan [52] to calculate the multifractal spectrum of the SSEC. To be clear, we denote the high-frequency price quotes at time t (at 5 min intervals) as $I(t)$ and t running from 1 to $End = M \times 1970$, where $M = 80$ is for the S&P 500 or $M = 48$ is for the SSEC.

The price variations within a trading day can be divided into many normalized boxes (time intervals) of size $\delta(\delta \le 1)$. In this article, for the S&P 500 the box size can be 1/80, 1/40, 1/20, 1/16, 1/10, 1/8, 1/5, 1/4, 1/2 or 1, and for the SSEC the box size can be 1/48, 1/24, 1/16, 1/12, 1/8, 1/6, 1/4, 1/3, 1/2 or 1.

Suppose we need m boxes to cover all the high-frequency quotes within a trading day $I(t)$, $t = 1, 2 \ldots, M$, and there are n quotes in each box. If P_i is the average probability in box i, then

$$P_i(\delta) = \frac{\sum_{j=1}^{n} I(i_j)}{\sum_{t=1}^{M} I(t)}, \quad i = 1, 2, \ldots, m, \tag{2}$$

where $I(i_j)$ is the jth quote in the ith box in the trading day. We can then describe the average probability in box i from a multifractal perspective as

$$P_i(\delta) \sim \delta^{\alpha}, \tag{3}$$

$$N_\alpha(\delta) \sim \delta^{-f(\alpha)}, \tag{4}$$

where α is the singularity exponent of the subset of probabilities, $N_\alpha(\delta)$ is the number of boxes of size δ with the same probability, and $f(\alpha)$ is the fractal dimension of the α subset.

According to the suggestion of Wei and Wang [22], what is important for us is statistical information about index fluctuations contained in the multifractal spectra $f(\alpha)$. We can use the partition function $S_q(\delta)$ to calculate such statistics. The partition function $S_q(\delta)$ is defined and expressed as a power law of δ, with an exponent $\tau(q)$, where q is the moment order $(-\infty < q < \infty)$:

$$S_q(\delta) = \sum_{i=1}^{m} P_i^q(\delta) \sim \delta^{\tau(q)}, \tag{5}$$

and $\tau(q)$ can be obtained from the slope of the linear part of the $\ln S_q(\delta) - \ln \delta$ curve. Via a Legendre transform, then $f(\alpha)$ can be obtained as follows:

$$\alpha = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q}, \tag{6}$$

$$f(\alpha) = \alpha q - \tau(q). \tag{7}$$

Fig. 2(a) shows the high-frequency price movements of the S&P 500 in two consecutive trading days. Fig. 2(b) indicates the multifractal spectra of the price data for the two days. The continuous bow-shaped multifractal spectra shown in Fig. 2(b) reveal that the price volatility of the S&P 500 clearly exhibits multifractal characteristics [22,52,53]. Jiang and Zhou [55,56] considered the statistical significance of multifractality and more detailed analyses of the sources of multifractality in financial markets, which can be found in Refs. [57–59]. However, we do not carry out the following analysis along with these papers. The main purpose of this manuscript is not to discuss the source of multifractality in financial markets but to focus on the statistical information obtained in the multifractal spectrum of intraday high-frequency price data, from which we can extract useful information.

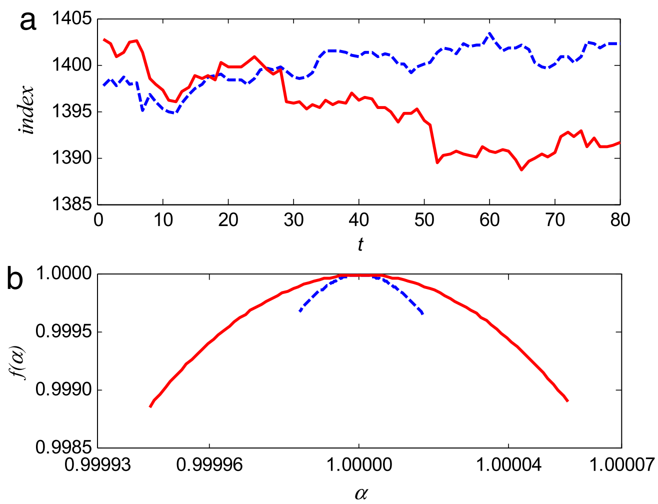

Fig. 2. Two-day price and multifractal-spectrum of the S&P 500 index (blue dash line denotes May 2, 2012, and red solid line denotes May 3, 2012).

Table 2
Statistical analysis of standardized returns in different models.

P-values	GARCH	GJR	EGARCH	IGARCH	RV	MFV
S&P500						
BDS	0.339	0.004	0.645	0.549	0.534	0.734
No ARCH	0.000	0.002	0.000	0.001	0.012	0.370
Q (10)	0.592	0.668	0.485	0.562	0.311	0.360
SSEC						
BDS	0.002	0.064	0.067	0.577	0.501	0.679
No ARCH	0.156	0.166	0.053	0.143	0.491	0.867
Q (10)	0.143	0.134	0.127	0.135	0.159	0.765

The width of a multifractal spectrum can be expressed as $\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{max} and α_{min} are respectively the maximum and minimum of α. Because the probability of each box is $P_i(\delta) \sim \delta^{\alpha}$ and $\delta \leq 1$, α_{min} and α_{max} denote the value of the maximum and the minimum probability. In other words, α_{min} indicates the highest "price level" in that trading day and α_{max} indicates the lowest "price level" in that trading day. Thus, the larger the value of $\Delta\alpha$ is, the more violent the price fluctuations over the course of the trading day are, and $\Delta\alpha$ can be treated as a measure of daily volatility. In Fig. 2(a), intraday prices oscillate more widely on May 3, 2012 than they do on the previous day. Thus, the width of the multifractal spectrum, $\Delta\alpha$, on May 3, 2012 is much larger than that on May 2, 2012 in Fig. 2(b).

However, $\Delta\alpha$ only captures the volatility during the part of the day when the market is open. In order to extend $\Delta\alpha$ to a measure of volatility for the full day and make it comparable to other commonly used measures of volatility, following the suggestion of Hansen and Lunde [60], we use a scalar parameter ψ to scale $\Delta\alpha$ as the MFV. Therefore, the multifractal volatility measure for day t is defined as

$$MFV_t = \psi \cdot \Delta\alpha_t, \tag{8}$$

where the so-called scale parameter ψ is defined as

$$\psi = \frac{N^{-1}\sum_{t=1}^{N} R_t^2}{N^{-1}\sum_{t=1}^{N} \Delta\alpha_t}. \tag{9}$$

Through the above work, we get the standardized returns series (also called innovations) after filtering heteroskedasticity, and then use a series of statistical tests to analyze the standardized returns, with the results shown in Table 2. In order to make a contrast with the GARCH-type models, we also offer the test results using GARCH, GJR, EGARCH, IGARCH, and realized volatility to filter heteroskedasticity.

The *P*-values in Table 2 show that all the null hypotheses of various statistical tests for standardized returns calculated by MFV are accepted at the 5% level or even higher levels. It means standardized returns calculated by the MFV method are i.i.d series. In most cases, the *P*-values are the highest among all methods, except for the Ljung–Box Q (10) statistics of the S&P 500.

3.2. Detrended cross-correlation analysis (DCCA)

Before analyzing the dependence structure of the U.S. and Chinese stock markets, we employ a detrended cross-correlation analysis (DCCA) to search for the cross-correlation between these two markets. Podobnik and Stanley [61] propose a detrended cross-correlation analysis which can be used to detect long-range cross-correlation between two non-stationary time series. Recently, a series of studies have greatly improved this method. For example, Horvatic, Stanley and Podobnik [62] improved DCCA to analyze non-stationary time series with periodic trends; Podobnik, Jiang, Zhou and Stanley [63] proposed an additional statistical test that can be used to quantify the existence of cross-correlations between two power-law correlated time series; Zhou [64] proposed a multifractal version and Jiang and Zhou [65] proposed a generalization of DCCA to the multifractal version. DCCA is widely applied to financial time series [64,66–68] and can be described as follows.

Step 1. Consider the two time series: $\{x_t\}$ and $\{y_t\}$ are of equal length N, where $N = 1970$. Then we describe the "profile" of each series and get two new series

$$xx_k = \sum_{t=1}^{k}(x_t - \bar{x}), \qquad yy_k = \sum_{t=1}^{k}(y_t - \bar{y}), \quad k = 1, 2, \ldots, N. \tag{10}$$

Step 2. Divide both the profiles $\{xx_k\}$ and $\{yy_k\}$ into $N_s = \mathrm{int}(N/s)$ nonoverlapping segments of equal length s. Since the length N of the series is not often a multiple of the considered time scale s, a short part at the end of each profile may remain. In order not to disregard this part of the series, the same procedure is repeated, starting from the opposite end of each profile. Therefore, $2N_s$ segments are obtained all together. We set $10 < s < N/5$ in this paper.

Step 3. We calculate the local trends $\widetilde{xx}_{(\lambda-1)s+j}$ and $\widetilde{yy}_{(\lambda-1)s+j}$ for each of the $2N_s$ segments by a least-squares fit of each series. Then we determine the co-moved variance

$$F^2(s, \lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}[xx_{(\lambda-1)s+j} - \widetilde{xx}_{(\lambda-1)s+j}][yy_{(\lambda-1)s+j} - \widetilde{yy}_{(\lambda-1)s+j}] \tag{11}$$

for $\lambda = 1, 2, \ldots, N_s$ and

$$F^2(s, \lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}[xx_{N-(\lambda-N_s)s+j} - \widetilde{xx}_{N-(\lambda-N_s)s+j}][yy_{N-(\lambda-N_s)s+j} - \widetilde{yy}_{N-(\lambda-N_s)s+j}] \tag{12}$$

for $\lambda = N_s + 1, N_s + 2, \ldots, 2N_s$. The trends $\widetilde{xx}_{(\lambda-1)s+j}$ and $\widetilde{yy}_{(\lambda-1)s+j}$ can be computed from a linear, quadratic or high order polynomial fit of each profile for segment λ.

Step 4. Average over all segments to get the fluctuation function

$$F(s) = \left\{\frac{1}{2N_s}\sum_{\lambda=1}^{2N_s}[F^2(s, \lambda)]\right\}^{1/2}. \tag{13}$$

Step 5. Analyze the scaling behavior of the fluctuation function through the log–log plots of $F(s)$ versus s. If two series are long-range cross-correlated, as a power-law

$$F(s) \sim s^H, \tag{14}$$

the scaling exponent H can be obtained by observing the slope of the log–log plot of $F(s)$ versus s through the method of ordinary least squares (OLS). If $H > 0.5$, we can say that two time series are long-range cross-correlated. An increase of one series is likely to be followed by an increase of the other. Obviously, if $\{x_t\}$ and $\{y_t\}$ are the same time series, the results of DCCA are equivalent to DFA [69,70].

Fig. 3 shows the log–log plots between $F(s)$ and the time scale s for the S&P 500 and SSEC return series and the MFV series. We can find a general power-law relationship between $F(s)$ and the time scale, s. The MFV series display stronger long-range cross-correlations than the return series in this case, implying the existence of long-range cross-correlations between the S&P 500 and the SSEC. These empirical results suggest that the S&P 500 and the SSEC are correlated and risks may be transmitted from one to another.

4. Static copulas and dependence structure

The word "contagion" became popular both in the press and in the academic literature in reference to a significant increase in cross-market linkage after a shock to one country or a group of countries [1,2]. In this paper, we use copulas to investigate the financial contagion phenomenon after obtaining standardized returns by using multifractal analysis.

Nelsen defines copulas as functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions [71]. They contain all the information about the dependence structure of a vector of

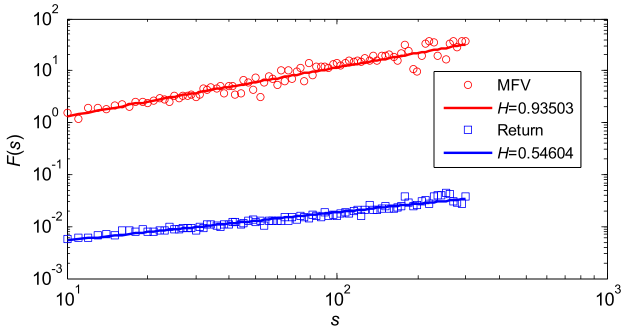

Fig. 3. DCCA of MFV and returns between the U.S. and Chinese stock markets.

random variables. According to Sklar's theorem, a two-dimensional joint distribution function D with continuous marginals F_X and F_Y has a unique copula representation, so that $D(x, y) = C(F_X(x), F_Y(y))$, and for a joint distribution function the marginal distributions and the dependence structure described by a copula can be separated. More detailed introductions of the applications of copulas in finance can be found in Refs. [72,73].

To capture different patterns of the tail dependence, in this paper we use four copulas that have been often studied in the literature: the Student, symmetrized Joe–Clayton (SJC), Clayton, and the mixture model consisting of Clayton and Frank copulas.

The margins u_1 and u_2 are i.i.d. uniform $(0, 1)$ random variables, carried out as the first step by a probability integral transform (PIT), given as follows

$$u_i = F_i(z_i), \quad i = 1, 2 \tag{15}$$

where z_i denotes the standardized returns of market i. The student copula is defined as

$$C_t(u_1, u_2; \rho, v) = \int_{-\infty}^{T_v^{-1}(u_1)} \int_{-\infty}^{T_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{s^2 - 2\rho st + t^2}{v(1-\rho^2)}\right)^{\frac{-(v+2)}{2}} \mathrm{d}s\,\mathrm{d}t, \tag{16}$$

where $-1 \le \rho \le 1$ is the dependence parameter, v is the freedom degree parameter, and $T_v^{-1}(\cdot)$ denotes the inverse function of the student distribution with freedom of v. The SJC copula is given as follows

$$C_{SJC}\left(u_1, u_2; \tau^U, \tau^L\right) = 0.5[C_{JC}(u_1, u_2; \tau^U, \tau^L) + C_{JC}(1 - u_1, 1 - u_2; \tau^U, \tau^L) + u_1 + u_2 - 1], \tag{17}$$

where C_{JC} is the Joe–Clayton (JC) copula with upper and lower tail dependence coefficients τ^U and τ^L respectively. The distribution of the JC copula is defined as

$$C_{JC}(u_1, u_2; \tau^U, \tau^L) = 1 - \left\{1 - \frac{1}{[1/(1-(1-u_1)^k)^\gamma + 1/(1-(1-u_2)^k)^\gamma - 1]^{1/\gamma}}\right\}^{1/k}, \tag{18}$$

where $k = 1/\log_2(2 - \tau^U)$ and $\gamma = -1/\log_2 \tau^L$. Both τ^U and τ^L belong to $(0, 1)$. The mixed copula which consists of Clayton and Frank is defined as

$$C_M(u_1, u_2; \theta_1, \theta_2) = \omega C_C(u_1, u_2; \theta_1) + (1 - \omega)C_F(u_1, u_2; \theta_1), \tag{19}$$

where $C_C(\cdot)$ and $C_F(\cdot)$ are the Clayton and Frank copula distribution function respectively, and $0 \le \omega \le 1$ is the Clayton copula's weight parameter. The $C_C(\cdot)$ and $C_F(\cdot)$ copulas are defined as

$$C_C(u_1, u_2; \theta_1) = \max[(u_1^{-\theta_1} + u_2^{-\theta_1} - 1)^{-1/\theta_1}, 0], \quad \theta_1 \in [-1, \infty)\backslash 0 \tag{20}$$

$$C_F(u_1, u_2; \theta_2) = -\frac{1}{\theta_2}\log\left[1 - \frac{(1 - e^{-\theta_2 u_1})(1 - e^{-\theta_2 u_2})}{1 - e^{-\theta_2}}\right], \quad \theta_2 \neq 0. \tag{21}$$

Table 3 gives the estimates of parameters for the four copulas mentioned above. As can be seen from Table 3, all parameters are significant at the 1% level, except for the upper tail dependence coefficient τ^U of the SJC copula. The SJC has the highest likelihood and lower AIC or BIC, which, generally speaking, performs best in all three copulas. We conduct the χ^2 goodness-of-fit test to evaluate the performance of the latter three estimated models, i.e., the Clayton, SJC and mixed model which consists of the Clayton and Frank.

Genest and Rivest [74], Hu [75] conducted this goodness-of-fit test method in their studies. At first, construct two $m \times m$ cross-classifications of the observed and estimated frequencies of the margins. Let A and B denote the observed and estimated contingency tables. The Pearson's χ^2 statistic is calculated as follows:

$$chi = \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{(A_{ij} - B_{ij})^2}{B_{ij}}. \tag{22}$$

Table 3
Estimated results of four copulas.

	$v/\tau^U/\omega$	τ^L/θ_1	θ_2	Log-likelihood	AIC, BIC
Student	24.8817*			33.2286	−64.4571
	(3.8818)				−58.8724
Clayton		0.2198*		34.0283	−66.0565
		(0.0298)			−60.4717
SJC	0.0054	0.0852*		36.3228	−68.6455
	(0.011)	(0.026)			−57.4760
Mixed-Clayton–Frank	0.6606*	0.2916*	0.6011*	34.7585	−63.5170
	(0.042)	(0.033)	(0.045)		−46.7626

Notes: There are four copula parameter estimates with standard errors in parentheses.
* Indicates statistical significance at the 1% level.

Table 4
The χ^2 goodness-of-fit test results for three models.

chi [*p*-values]	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$
Clayton	19.1181	13.7063	30.8964	53.3939	74.211	92.4945
	[0.2626]	[0.9666]	[0.7098]	[0.3092]	[0.1796]	[0.1800]
SJC	14.1861	9.5214	27.5209	51.2518	71.1776	89.4895
	[0.5115]	[0.9963]	[0.8121]	[0.3474]	[0.2242]	[0.2193]
Mixed-Clayton–Frank	18.0189	11.6956	29.0898	52.2763	75.0999	91.9701
	[0.2059]	[0.9749]	[0.7071]	[0.2765]	[0.1227]	[0.1509]

Notes: The first line for each model is the statistics *chi* in the table, and *p*-values of the statistics are reported in square brackets.

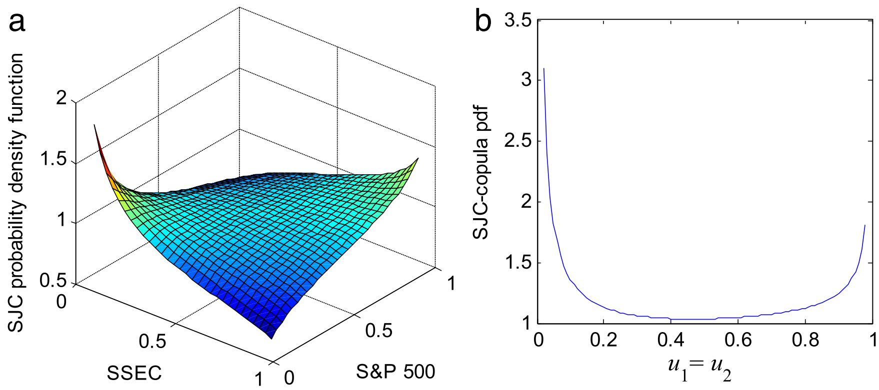

Fig. 4. SJC-copula probability function and *pdf* on the diagonal.

In computing the statistics, we pool together cells whose expected frequency is less than five. The degrees of freedom associated with the goodness-of-fit statistic *chi* are $(m - 1)^2 - p - (q - 1)$, where p is the number of parameters in the model, and q is the number of pooled cells. The χ^2 goodness-of-fit test results are shown in Table 4.

As can be seen from Table 4, the statistics *chi* of all the three models are not significant, even at a higher level of 10%; the SJC copula always has the smallest statistics *chi* and the largest *p*-values, and the *p*-values of Clayton copula are larger than the mixed Clayton and Frank copula for five of the six cases (we pay more attention to *p*-values rather than the statistics *chi*, since different models have different freedom).

We depict the dependence structure of the best performance model: SJC-copula. Fig. 4(a) shows the fitted SJC-copula dependence structure between the U.S. and Chinese stock markets. Moreover, the co-movement and tail dependence between these two markets can be visualized by plotting their densities along the diagonal $u_1 = u_2$, see Fig. 4(b). We can see a strong tail dependence between these two markets from Fig. 4. Furthermore, Fig. 4 also reveals an asymmetric characteristic with an upper and lower tail dependence.

5. Time-varying copulas and contagion effect

In the above analysis, we described how to measure the volatility using the multifractal method, and then showed the dependence, especially the tail dependence between the U.S. and Chinese stock markets. However, we have still not confirmed in that discussion whether the contagion effect exists after the subprime mortgage crisis. In this section, we conduct dynamic copulas to study the contagion effect of the subprime mortgage crisis.

Table 5
Estimated results of three dynamic copulas.

	β_1/η_1	β_2/η_2	v/η_3	Log-likelihood	AIC	BIC
DCC-dynamic-Gaussian	0.0187	0.0992		32.107	−60.2142	−49.0446
	(0.029)	(0.599)				
DCC-dynamic-*t*	0.0043*	0.9931**	29.4041**	37.131	−68.2611	−51.5068
	(0.003)	(0.006)	(3.249)			
Time-varying-Clayton	−2.2995**	−1.1808*	−0.3719	35.305	−64.6099	−47.8556
	(1.122)	(0.941)	(0.519)			

Notes: Standard errors are reported in the parentheses.
* Indicates statistical significance at the 5% level.
** Indicates statistical significance at the 1% level.

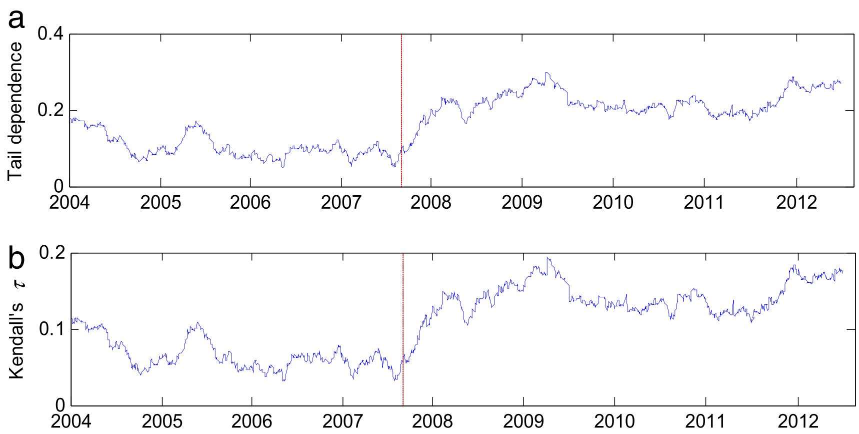

Fig. 5. The evolution of the tail dependence (a) and Kendall's τ (b) of the dynamic t copula. The red dot-line indicates the date August 31, 2007.

Rodriguez [17] has pointed out that: tests of contagion can be seriously affected by the size of the "crisis" and "non-crisis" period. An advantage of time-varying copulas in the study of contagion is that they do not rely on an ad hoc determination of the crisis period. In this paper, we estimate tail dependence parameters using dynamic copulas for full samples, and then separate "non-crisis" and "crisis" periods based on analyzing the tail dependence variation through time.

Next, we will introduce three time-varying copulas, i.e., DCC(1,1)-dynamic-Gaussian, DCC(1,1)-dynamic-t, and time-varying Clayton copula.[3] For the dynamic Gaussian and t copula (for t copula, the degree of freedom parameter is static), and we specify that the dependence parameter ρ_t evolves through time as in the DCC(1,1) model of Engle [76]:

$$\begin{aligned} Q_t &= (1-\beta_1-\beta_2)\bar{Q} + \beta_1 z_{t-1} z'_{t-1} + \beta_2 Q_{t-1} \\ \rho_t &= \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \end{aligned} \tag{23}$$

where $\bar{Q}$ is the sample covariance of the standardized return z_t, $\tilde{Q}_t$ is a square 2×2 matrix with zeros as off-diagonal elements and diagonal elements being the square root of those of Q_t.

The time-varying Clayton copula defines the evolution of the dependency parameter (i.e., Kendall's τ), according to the equations, defined in Patton [77]:

$$\tau_t = \Lambda\left(\eta_1 + \eta_2 \tau_{t-1} + \eta_3 |u_{1,t-1} - u_{2,t-1}|\right) \tag{24}$$

where Λ denotes the logistic transformation: $\Lambda(x) = 1/(1+e^{-x})$ in order to keep the parameter Kendall's τ in (0, 1).

Table 5 reports the results on the three dynamic copulas. As can be seen from this table, the dynamic t copula has the largest log-likelihood and the least AIC and BIC; moreover, all parameters are significant at the 5% level. Therefore, we will study the contagion effect based on the dynamic t copula in what follows.

Fig. 5 shows that since the end of 2007, the tail dependence parameter ρ and Kendall's τ between the U.S. and Chinese stock markets have both sharply increased. From that time on, these parameters always maintain a higher value, in which even the least value was greater than the maximum before 2008. In about March 2009, the tail dependence parameter ρ and Kendall's τ reached the maximum level in history. It is well-known that the subprime mortgage crisis began

[3] We also tried to use a time-varying SJC copula to study contagion in this paper, but too many parameters need to be estimated for this model and the parameters in this model are very hard to converge to a solid result. We estimate the time-varying SJC copula at numerous start points. Although log-likelihood converges to a constant, the value of log-likelihood is only 0.4 greater than the t-copula. Moreover, AIC and BIC are both greater than for the other three dynamic copulas and parameters with no convergence.

Table 6
Estimated results of the contagion effect test.

	λ_0	λ_1	λ_2
$d_t = \rho_t$	0.0011*	0.9886*	0.0015*
	(0.0003)	(0.0028)	(0.0004)
$d_t = \tau_t$	0.0007*	0.9887*	0.0010*
	(0.0002)	(0.0028)	(0.0002)

Notes: Standard errors are reported in the parentheses.
* Indicates statistical significance at the 1% level.

sweeping the major financial markets worldwide, including the U.S., EU and Japan, in August 2007. There were a series of major events falling in August 2007. For example, on August 6, American Home Mortgage Investment Corporation (AHMI) filed for bankruptcy. On August 7, numerous quantitative long/short equity hedge funds suddenly began experiencing unprecedented losses as a result of what was believed to be liquidations by some managers' eagerness to access cash during the liquidity crisis. On August 8, the Mortgage Guaranty Insurance Corporation (MGIC) announced it would discontinue its purchase of Radian Group after suffering a billion-dollar loss of its investment in Credit-Based Asset Servicing and Securitization (C-BASS). On August 9, the French investment bank BNP Paribas suspended three investment funds invested in subprime mortgage debt. On August 10, central banks coordinated efforts to increase liquidity for the first time since the aftermath of the September 11, 2001 terrorist attack. The United States Federal Reserve (Fed) injected a combined 43 billion USD, the European Central Bank (ECB) 156 billion euros (214.6 billion USD), and the Bank of Japan 1 trillion Yen (8.4 billion USD). Smaller amounts came from the central banks of Australia and Canada. On August 14, Sentinel Management Group suspended redemptions for investors and sold off \$312 million worth of assets, and three days later Sentinel filed for bankruptcy protection. The U.S. and European stock indices continued to fall. On August 15, the stock of Countrywide Financial, which was the largest mortgage lender in the United States, fell to around 13% on the New York Stock Exchange after Countrywide said that foreclosures and mortgage delinquencies had risen to their highest levels since early 2002. On August 17, the Federal Reserve cut the discount rate by half a percent to 5.75% from 6.25%, while leaving the federal funds rate unchanged, in an attempt to stabilize financial markets. On August 31, President Bush announced a limited bailout for U.S. homeowners unable to pay the rising costs of their debts. The red dot-line in Fig. 5 denotes the date of August 31, 2007.

Therefore, according to Fig. 5 and the above analysis, we test the tail dependence ρ and Kendall's τ between the U.S. and Chinese stock markets to see whether there is a significant increase after August 31, 2007[4] by the following model

$$d_t = \lambda_0 + \lambda_1 d_{t-1} + \lambda_2 I_t(crisis) + \varepsilon_t \tag{25}$$

where d_t denotes the time-varying tail dependence parameter ρ_t or Kendall's τ_t. I_t is an indicator of whether the date is after August 31, 2007; if not, $I_t = 0$ else $I_t = 1$. Table 6 gives the estimated results of λ_i $(i = 0, 1, 2)$.

Table 6 shows that all the parameters in Eq. (25) are significant at the 1% level. According to the statistics, we know that the tail dependence and Kendall's τ increased significantly after August 31, 2007. That means the subprime mortgage crisis has a contagion effect on the Chinese stock market.

6. Conclusions

In this paper, we propose a new approach to study the contagion effect: copula–multifractal. Based on this method, we study the contagion effect between the U.S. and Chinese stock markets following the subprime mortgage crisis. Firstly, we calculate the multifractal volatility (MFV) based on the multifractal spectrum analysis. The test results for standardized returns show that the MFV method performs best among numerous other methods. Secondly, we depict the dependence structure between the U.S. and Chinese stock markets using static copulas after we obtain the margins by the probability integral transform (PIT) for standardized returns. The estimated static copulas results show that the SJC copula performs best, which means asymmetric characteristics exist in the tail dependence structure between the U.S. and Chinese stock markets. Thirdly, we analyze the dynamic tail dependence between the U.S. and Chinese stock markets using time-varying copulas. The estimated dynamic copulas results show that the time-varying t copula performs best. The dynamic SJC copula is not employed because too many parameters need to be estimated, causing non-convergence for this model. It means the symmetry dynamic t copula is also a good choice, as it is easy to estimate and depicts both the upper and lower tail dependence structure, rather than some models that can depict just one tail dependence structure, such as the Clayton copula. Lastly, we test whether the tail dependence and Kendall's τ significantly increased after the subprime mortgage crisis. The test results show that the subprime mortgage crisis had a contagion effect on the Chinese stock market.

Our empirical results may be useful for investors selecting portfolios and international diversification, as well as having potentially important implications for risk management. The risk of a portfolio consists of a systematic risk and an unsystematic risk. We know that unsystematic risk can be eliminated by diversification while systematic risk cannot. In order to reduce risks, investors may sometimes seek to construct a portfolio which consists of stocks in the U.S. and Chinese

4 We also tested whether there is a significant increase in ρ and Kendall's τ after July 31, taking into account the contagion began in August 2007. The conclusion is the same as we reported in the following.

markets before the financial crisis. However, with the linkages between the U.S. and Chinese stock markets increasing significantly after the crisis, the risks of a portfolio would also increase greatly if the portfolio is without any other changes. In order to reduce risks, investors have to change the components and their percentage of the portfolio. For example, investors can reduce the percentage of either market, but invest in stocks of other countries which have not been or are little affected by the financial crisis. There are also other ways to reduce risks, for instance, hedging or investing in more risk-free assets, such as treasuries.

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