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Mathematical Modeling of Upflow Anaerobic Sludge Blanket Reactor in Domestic Wastewater Treatment

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***Abstract*— This paper introduces a dynamic model to adequately describe an Upflow Anaerobic Sludge Blanket (UASB) reactor. Some available models of a UASB reactor are discussed in order to modify their drawbacks and propose a new improved model with less complexity and more reliability. The developed model is a combination of two recent models introduced in Sweden. According to this model, a UASB rector is divided hydraulically into three compartments with integration of a kinetic model. Simulations are performed to investigate the validity of the developed model which indicates a good agreement with experimental data. Moreover, the developed model appears to be sensitive to variation of hydraulic and kinetic parameters through sensitivity analysis.**

***Index Terms*—modeling; UASB reactor; Wastewater treatment; sensitivity analysis.**

I. INTRODUCTION

A model is proposed for the Up-flow Anaerobic Sludge Blanket (UASB) reactor treating domestic wastewater. The proposed model describes the substrate degradation, the microorganism concentration growth and the particle amount and size distribution in the reactor. This type of reactor is an attractive alternative for hot climate regions since it works better under mesophilic conditions and it does not need any supporting structure for the development of microorganisms, which grow in the form of biological granules [1], [2].

Mathematical models like the one proposed here are useful tools in the development and design of UASB reactors. It allows researchers to study the consequences for the reactor performance of different types of substrate, different inlet substrate concentration, different flow rates and different kinds of biomass. Several models for the UASB studied in Sweden show uncertainty to the accurate results. Among these models, Larisa [3] and Parsamehr [4] models are illustrated in the next section. The two models are deeply discussed concerning the hydraulic and kinetic models followed to describe the UASB reactor. The hydraulic model is established to include the flow behavior of the UASB reactor and the kinetic model quantifies the growth rate of biomass and utilization of the substrate.

In this paper, a new model based on mass balances for the substrate and microorganisms in the reactor is introduced. For the substrate, the processes included in the model are dispersion, advection in reactor top zone and degradation of the organic matter in the substrate in bed and blanket zones of the reactor. The reaction rate for microorganisms includes growth and decay of microorganisms which is described by Andrews type equation and also considers the substrate diffusion in the granule through first order kinetics [2].

This paper is organized in four sections; where previous models are firstly discussed and then compared with a proposed model which is validated using results reported in the literature from experiments carried out at pilot scale. Simulations are performed in the Matlab software using Runge-Kutta 4th order. The results of the proposed model show sensitivity to the parameters applied in the simulation.

II. PREVIOUS MODELS

A. *Model proposed by Larisa Korsak, Sweden*

Larisa Korsak in 2008 [3] describes the physical and biological processes in a UASB reactor by a novel one-dimensional model. According to the presented model biological reactions which result in degradation of substrate and production of biogas take place within the granules. The author introduced the granule structure as a perfect spherical biocatalyst so that the substrate diffuses into the granule through a thin stagnant liquid layer around the granule surface. The mass transport phenomenon along the UASB reactor was described by a dispersion-advection mechanism. The substrate (C_S) and biomass (C_X) mass balance equations of the reactor are given by (1) and (2) respectively while the chemical reaction constant (K), Thiele modulus (ϕ), substrate volumetric conversion rate (k) and reaction rate (R_S) are given in (3), (4) respectively:

$$\frac{dC_S}{dt} = D\frac{\partial^2 C_S}{\partial z^2} - W\frac{\partial C_S}{\partial z} - R_S \tag{1}$$

$$\frac{dC_X}{dt} = F_{in}\frac{(C_{Xo} - C_X)}{V_b} + R_S Y \tag{2}$$

$$K = \frac{3k_L f(D_A(\phi\cosh\phi - \sinh\phi))}{R(D_A(\phi\cosh\phi - \sinh\phi) + Rk_L\sinh\phi)} \tag{3}$$

$$\phi = \sqrt{\frac{k}{D_A}}R \quad , \quad k = \mu_{\max}\frac{C_X}{K_S} \quad , \quad R_S = KC_S \tag{4}$$

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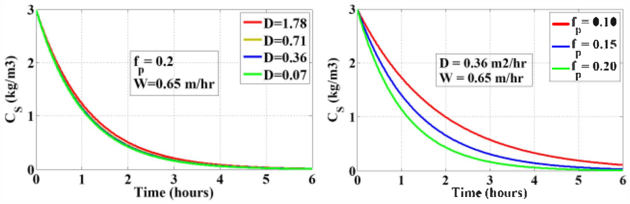


(a) (b)

Fig.1. Substrate concentration degradation at (a) various dispersion coefficients, and (b) different fractions of reactor volume occupied by granules

Fig.1(a) illustrates that the substrate concentration as COD (chemical oxygen demand) in UASB reactor decreases with time in an exponential phase achieving 100 % COD removal efficiency after 4 hours reactor operation. The variation of dispersion coefficients (D) as well as upflow velocity (W) of domestic wastewater doesn't affect the reactor performance as shown in Fig.1 (a). The neglected error considers this model to be not sensitive to the hydraulic parameters as flow dispersion and up flow velocity. While Fig.1(b) shows the effect of fraction of granule particles (f_p) on UASB reactor performance indicating a high UASB reactor performance as the amount of granules increases in the reactor.

B. Model proposed by Parsamehr, Sweden

In this section a dynamic model is developed in 2012 to adequately describe a UASB reactor [4]. This model is comprised of a bio kinetic and hydraulic model. According to this model a UASB reactor can be divided hydraulically into three different compartments; bed, blanket and settler that are denoted by 1, 2 & 3 respectively as shown in Fig.2.

The bed zone is located at the bottom of the reactor where granulated sludge produces biogas through biodegradation of organic materials. Biogas production and influent flow create a high degree of mixing in the reactor bottom so that this region can be modeled as a continuous stirred tank reactor (CSTR). Fully mixed flow can also be presumed to the blanket zone. Biomass transport and rising bubbles are two important factors to form turbulence in this compartment. Finally in the reactor top a degree of mixing can be expected due to movement of rising gas bubbles. Therefore this zone is modeled as a dispersed plug flow reactor (Dispersed PFR).

The biomass and substrate mass balance equations in bed & blanket zones of the UASB reactor follow dragging-settling flow with kinetic reaction as shown in (5) – (8) respectively. While the reaction rate in bed & blanket zones and substrate mass balance for the settler (clarifier) zone of reactor that follows dispersion- advection flow are expressed in (9).

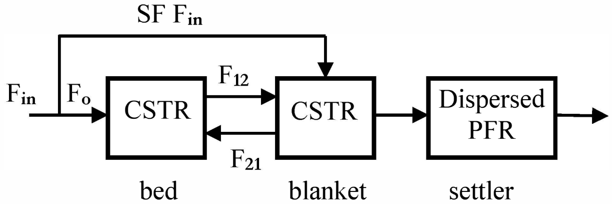


Fig.2. A schematic block diagram for hydraulic model of the UASB reactor

In addition, the influent wastewater flow and flow from bed to blanket zones of UASB reactor are estimated by (10). Also, the fraction of influent flow that bypasses the reactor bed zone can be shown in (11). According to Van Der Meer [5], the back mixing flow from blanket to bed zones in reactor is also illustrated in (11). While the amount of methane gas produced in the bed zone of reactor can be estimated in (12). And the specific growth rate of bacteria is expressed by (13) using Andrews kinetics.

$$V_1 \frac{dC_{X1}}{dt} = F_o C_{Xin} - \eta_{dr} x q_{gb} C_{X1} + A C_{X2} v_s + R_{S1} Y V_1 \tag{5}$$

$$V_2 \frac{dC_{X2}}{dt} = \eta_{dr} x q_{gb} C_{X1} - A C_{X2} v_s - (1-\eta) F_{in} C_{X2} + R_{S2} Y V_2 \tag{6}$$

$$V_1 \frac{dC_{S1}}{dt} = F_o C_{Sin} + F_{21} C_{S2} - F_{12} C_{S1} - R_{S1} V_1 \tag{7}$$

$$V_2 \frac{dC_{S2}}{dt} = F_{12} C_{S1} + SF\, F_{in} C_{Sin} - F_{21} C_{S2} - F_{in} C_{S2} - R_{S2} V_2 \tag{8}$$

$$R_S = (\mu - K_d)\frac{C_X}{Y} \quad , \quad \frac{dC_{S3}}{dt} = D\frac{\partial^2 C_{S3}}{\partial z^2} - W\frac{\partial C_{S3}}{\partial z} \tag{9}$$

$$F_{in} = SF\, F_{in} + F_o \quad , \quad F_{12} = F_{21} + F_o \tag{10}$$

$$SF = 0.13 + 0.17 h_1 \quad , \quad F_{21} = q_{gb}\left(\frac{10}{10 + h_2 + h_3}\right) \tag{11}$$

$$q_{gb} = R_{CH4}\frac{V_1}{\rho_{CH4}} \quad , \quad R_{CH4} = f_{CH4}\frac{(1-Y)\mu_1 C_{X1}}{Y} \tag{12}$$

$$\mu = \frac{\mu_{max}}{1 + \frac{K_S}{C_{HS}} + \frac{C_{HS}}{K_I}} \quad , \quad C_{HS} = \frac{10^{-pH} C_S}{K_A + 10^{-pH}} \tag{13}$$

Parsamehr model simulates the COD substrate degradation with time in reactor bed and blanket zones as shown in Fig.3 (a) and (b). While Fig.3(c) and (d) discuss gas concentration of each gas produced by methanogenic bacteria in reactor bed & blanket zones respectively after hydrolysis of complex organics & fermentation of volatile fatty acids by acidogenic bacteria[5]. The fraction of each gas is estimated from the ratio of gas molecular weight to the whole gases produced.

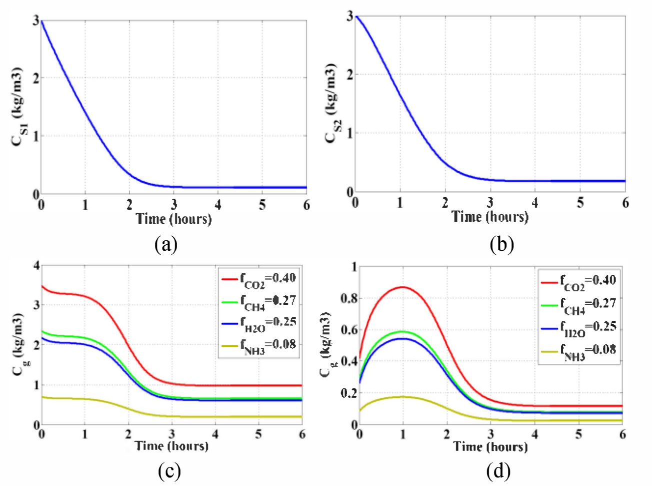
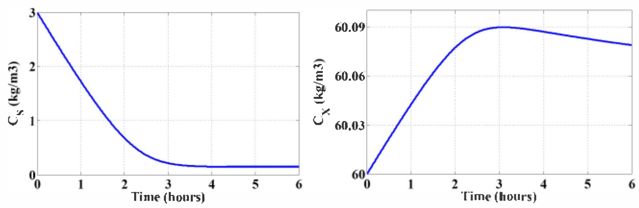


Fig.3. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time, and gases concentration produced (c) in reactor bed zone versus time, and (d) in reactor blanket zone versus time



(a) (b)

Fig.4. (a) Substrate concentration degradation in the UASB reactor versus time, and (b) The effective biomass concentration in the UASB reactor versus time

A model proposed by Olafadehan in Nigeria [6] achieved similar results to Parsamehr model as shown in Fig.4(a) However, this model is not accurate as it considers the UASB reactor a zone with constant biomass concentration as shown in Fig.4 (b). The effect of variation of maximum specific growth rate of bacteria on reactor bed zone performance is shown in Fig.5 (a) where the reactor performance increases as bacterial growth rate increases. While Fig.5(b) shows that reactor efficiency increases as the upflow velocity decreases (higher hydraulic retention time). Fig.3(b) and Fig.5(b) also show that the reactor blanket zone efficiency decreases from 90 % to 65 % by increasing the bypass flow fraction (SF) from 15% to 65%. This model is sensitive to both hydraulic and kinetic parameters. However, it doesn't consider the other kinetics as sludge thickness as reported in [4] that requires model verification.

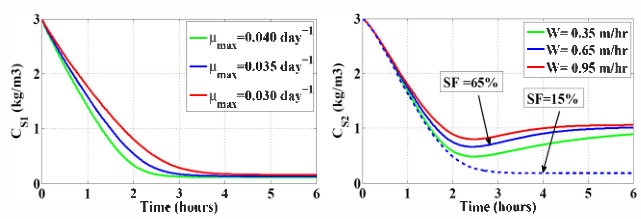
III. PROPOSED MODEL

A. *Previous experimental results*

An experimental work of UASB reactor [7] had been taken to be the data used to estimate the % removal efficiency (E) of COD from domestic wastewater as a function of hydraulic retention time (T) in hour, organic load (L) in kg BOD/m3/day (kg biochemical oxygen demand/m3/day) and sludge thickness (s) in cm as shown in (14) and (15). According to these equations, the removal efficiency of the COD substrate concentration reaches about 80 % for T of 8 hours, s of 100 cm and L of 6 kg BOD/m3/day which show that Parsamehr model implies inaccurate results.

$$E_{COD} = 25.683 + 1.034T + 0.45s \tag{14}$$

$$E_{COD} = 8.749 - 20.175L + 1.937s \tag{15}$$



(a) (b)

Fig.5. (a) Effect of bacterial growth rate on reactor bed zone performance, and (b) Effect of upflow velocity and bypass flow fraction on reactor blanket zone performance

B. *Developed model for UASB reactor*

This model follows Parsamehr model considering Larisa kinetic model. A combination between the biomass reaction rates of these models is illustrated in the proposed model which is compared with Parsamehr model as shown in Fig.6. This model appears to be more accurate than other models considering all parameters affecting the UASB reactor performance. The reaction rate can be expressed in (16) which is used to estimate biomass and substrate mass balance in UASB reactor through (5) - (8). It implies an equal weight of the reaction rates of the previous models.

$$R_S = 0.5(\mu - K_d)\frac{C_X}{Y} + 0.5KC_S \tag{16}$$

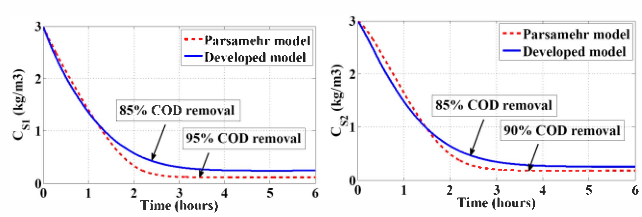
Fig.6(a) and (b) indicate that substrate concentration in reactor bed & blanket zones decreases with time in a linear phase achieving 90 % - 95% COD removal efficiency after 3 hours reactor operation according to Parsamehr model. While the proposed model shows that substrate concentration in reactor bed and blanket zones decreases with time in a mixed linear exponential phase achieving COD removal efficiency of about 85 % after 4 hours reactor operation.

C. *Sensitivity analysis for developed model*

In this section the UASB reactor is discussed for the variation of some hydraulic and kinetic parameters such as the fraction of volume of reactor occupied by granules particles (sludge thickness), maximum specific growth rate of bacteria and wastewater upflow velocity in reactor.

The effect of variation of sludge thickness on reactor bed and blanket zones performance shown in Fig.7 (a), (b), (c) and (d) indicates that substrate removal efficiency and biomass concentration increase as the fraction of volume of reactor occupied by granules particles increases. Fig.8 (a) and (b) also illustrates the effect of bacterial growth rate on reactor bed and blanket zones performance which increases as the growth rate of bacteria increases.

On the other hand, the effect of variation of wastewater upflow velocity on reactor bed & blanket zones performance is discussed for same reactor height as shown in Fig.9 (a) and (b). It is found that COD removal efficiency increases from 80% to 90% as domestic wastewater upflow velocity decreases from 0.95 to 0.35 m/hr. As a result, this model achieves high sensitivity to kinetic and hydraulic parameters.



(a) (b)

Fig.6. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time

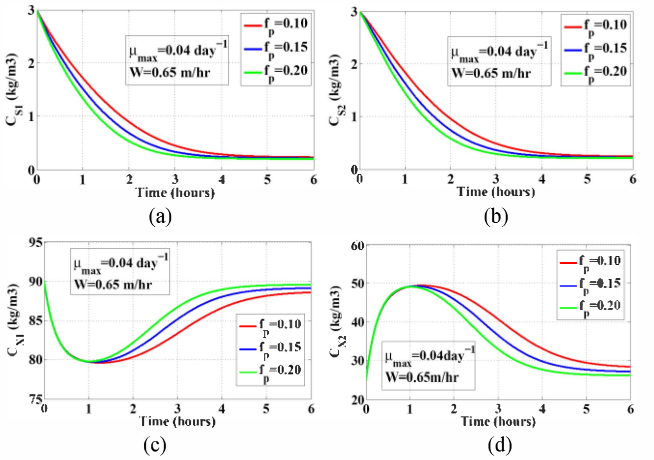


Fig.7. Substrate concentration in the UASB reactor (a) bed zone versus time and (b) blanket zone versus time, and Biomass concentration in the UASB reactor (c) bed zone versus time and (d) blanket zone versus time

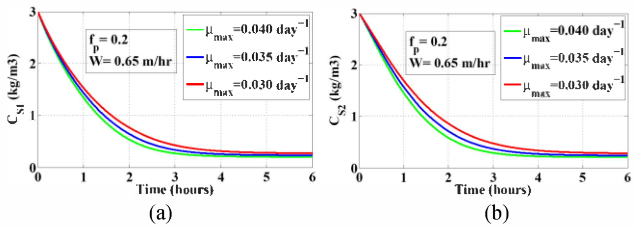


Fig.8. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

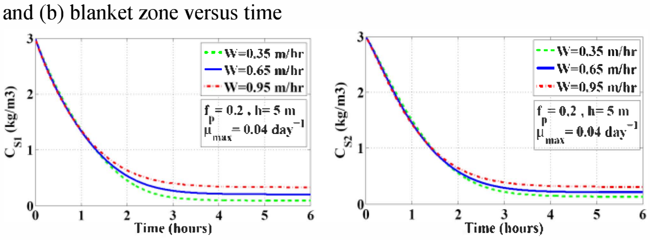


Fig.9. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

IV. CONCLUSIONS

The figures of the developed model of UASB reactor show 85% removal efficiency of the COD substrate concentration for T of 8 hours, s of 100 cm and L of 6 kg BOD/m3/day which approach approximate results as that of (14) & (15). The developed model appears to be a reliable accurate model for describing UASB reactor performance.

ABBREVIATIONS

Symbol	Definition	Unit
A	cross sectional area of reactor	m^2
$C_{HS}, C_S, C_{So}, C_{Sin}$	unionized, various, initial and inlet substrate concentration in reactor	kg/m^3
$C_X, C_{Xo}\ C_{Xin}$	various, initial and inlet biomass concentration in reactor	kg/m^3
D, D_A	dispersion and effective diffusion coefficients	m^2/hr
E	substrate removal efficiency	%
F_{in}	influent wastewater discharge	m^3/hr
f, ϕ	fraction of volume of reactor occupied by granules and Thiele modulus	——
h	height of reactor	m
K, k	chemical reaction constant and substrate volumetric conversion rate	hr^{-1}
k_L	mass transfer coefficient	m/hr
$K_A, K_I\ K_S$	substrate equilibrium, inhibition and Monod half saturation constants	kg/m^3
$K_d, \mu, \mu_{\max}$	microorganism decay, specific growth and maximum specific growth rates	hr^{-1}
L	organic loading rate as BOD	kg/m^3/ day
η_{dr}, η	dragging and settler efficiencies	%
R	radius of the biological granule	m
R_{CH_4}, R_S	methane production and biomass reaction (substrate utilization) rates	kg/m^3/ hr
s	sludge thickness	cm
t, T	various and hydraulic retention time	hours
V	volume of reactor	m^3
v_s, W	settling and upflow flow velocities	m/hr
x	drag coefficient on biomass base	m^3/m^3
Y	yield coefficient of biomass per substrate	kg /kg
z	vertical height of reactor	m

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