

# Mathematical Modeling of Upflow Anaerobic Sludge Blanket Reactor in Domestic Wastewater Treatment

Mostafa Seddek<sup>a</sup>, Hishama Heglim<sup>b</sup>, Agalwan Saieed<sup>c</sup>

**Abstract:** This paper introduces a dynamic model to describe the behavior of an upflow anaerobic sludge blanket (UASB) reactor. The model is based on the mass and energy balances of the reactor. The reactor is divided into two zones: a suspended zone and a sludge zone. The suspended zone is modeled as a stirred tank reactor, while the sludge zone is modeled as a fixed bed reactor. The model is used to study the effect of various operating parameters on the reactor performance. The results show that the reactor performance is highly sensitive to the inlet substrate concentration and the reactor height. The model is validated against experimental data, and the results show a good agreement between the model and the experimental data.

**Indexing:** UASB reactor, dynamic model, wastewater treatment.

$$S = S_0 - \frac{1}{Y} \left( \frac{dX}{dt} + \frac{dX_s}{dt} \right) + \frac{1}{Y} \left( \frac{dX_p}{dt} + \frac{dX_{ps}}{dt} \right)$$

**A** mathematical model is developed for the upflow anaerobic sludge blanket (UASB) reactor. The reactor is divided into two zones: a suspended zone and a sludge zone. The suspended zone is modeled as a stirred tank reactor, while the sludge zone is modeled as a fixed bed reactor. The model is based on the mass and energy balances of the reactor. The reactor is divided into two zones: a suspended zone and a sludge zone. The suspended zone is modeled as a stirred tank reactor, while the sludge zone is modeled as a fixed bed reactor. The model is based on the mass and energy balances of the reactor. The reactor is divided into two zones: a suspended zone and a sludge zone. The suspended zone is modeled as a stirred tank reactor, while the sludge zone is modeled as a fixed bed reactor. The model is based on the mass and energy balances of the reactor.

<sup>a</sup>Civil Engineering, Institute of Aviation Engineering and Technology (IAET), Giza, Egypt (E-mail: [mostafaalseddek@yahoo.com](mailto:mostafaalseddek@yahoo.com))

<sup>b</sup>Sanitary and Environmental Engineering, Public Works Department, Civil Engineering, Faculty of Engineering, Cairo University, Egypt (E-mail: [hishama.heglim2011@gmail.com](mailto:hishama.heglim2011@gmail.com))

<sup>c</sup>Sanitary and Environmental Engineering, Public Works Department, Civil Engineering, Faculty of Engineering, Cairo University, Egypt (E-mail: [agalwanwan@ieee.org](mailto:agalwanwan@ieee.org))

$$\frac{dS}{dt} = \frac{1}{V} (Q S_0 - Q S - V r_s)$$

$$\phi = \sqrt{\frac{1}{Y} \left( \frac{dX}{dt} + \frac{dX_s}{dt} \right) + \frac{1}{Y} \left( \frac{dX_p}{dt} + \frac{dX_{ps}}{dt} \right)}$$

$f_{CO_2} = 0.10$   
 $f_{CH_4} = 0.15$   
 $f_{H_2O} = 0.20$

Substrate concentration degradation at (a) various dispersion coefficients, and (b) different fractions of reactor volume occupied by granules

Fig.1. Substrate concentration degradation at (a) various dispersion coefficients, and (b) different fractions of reactor volume occupied by granules

$$V_1 \frac{dC_{S1}}{dt} = F_o C_{S1} - F_{12} C_{S1} - R_{S1} V_1$$

$$V_2 \frac{dC_{S2}}{dt} = F_{12} C_{S1} + SF F_m C_{S1} - F_{21} C_{S2} - F_m C_{S2} - R_{S2} V_2$$

$$R_S = (\mu - K_d) \frac{C_X}{Y}, \quad \frac{dC_{S3}}{dt} = D \frac{\partial^2 C_{S3}}{\partial z^2} - W \frac{\partial C_{S3}}{\partial z}$$

$$F_m = SF F_m + F_o, \quad F_{12} = F_{21} + F_o$$

$$D_{80} \sim \dots \sim X \cdot F_{21} = q_{gb} \left( \frac{10}{10 + h_2 + h_3} \right)$$

B. Model proposed by Parsamehr, Sweden

$$R_{CH_4} = f_{CH_4} \frac{(1-Y) \mu_1 C_{X1}}{Y}$$

$$f = \frac{f_{max} C_{CH_4}}{K_{CH_4} + C_{CH_4}}$$

$$C_{HS} = \frac{10^{-pH} C_S}{K_A + 10^{-pH}} \quad (13)$$

Substrate concentration degradation at (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time, and gases concentration produced (c) in reactor bed zone versus time, and (d) in reactor blanket zone versus time

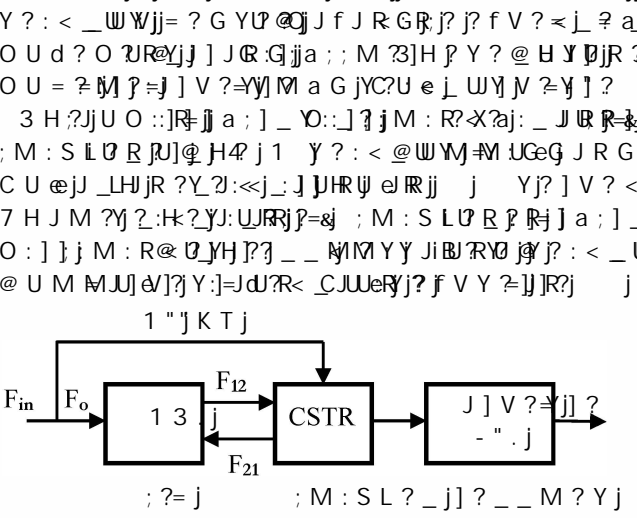


Fig.2. A schematic block diagram for hydraulic model of the UASB reactor

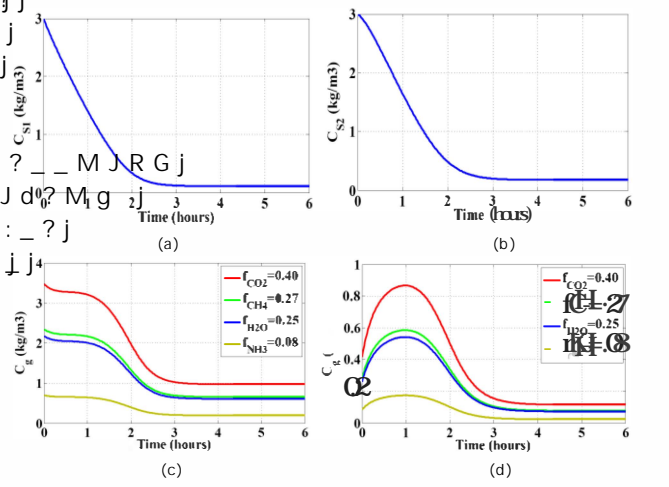


Fig.3. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time, and gases concentration produced (c) in reactor bed zone versus time, and (d) in reactor blanket zone versus time

1

J. 60,03

Time (hrs) (a) Time (hrs) (b)

Fig.4. (a) Substrate concentration degradation in the UASB reactor versus time, and (b) The effective biomass concentration in the UASB reactor versus time

$$C_{Si} = C_{Si0} \exp(-\mu_{max} t)$$

$$X_{eff} = X_{max} \left( \frac{C_{Si}}{C_{Si0}} \right)^n$$

A. Previous experimental results

The effect of bacterial growth rate on reactor bed zone performance is shown in Fig. 5(a). The substrate concentration  $C_{Si}$  (kg/m<sup>3</sup>) decreases over time (0 to 6 hours) for three different  $\mu_{max}$  values: 0.040 day<sup>-1</sup> (green), 0.035 day<sup>-1</sup> (blue), and 0.030 day<sup>-1</sup> (red). Higher growth rates result in faster substrate degradation.

$$E_{COD} = \frac{C_{Si0} - C_{Si}}{C_{Si0}}$$

$$E_{COD} = \frac{C_{Si0} - C_{Si}}{C_{Si0}} + (j - s) j$$

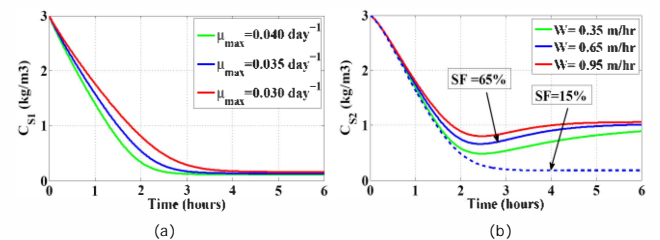


Fig.5. (a) Effect of bacterial growth rate on reactor bed zone performance, and (b) Effect of upflow velocity and bypass flow fraction on reactor blanket zone performance

B. Developed model for VASE reactor

The developed model for VASE reactor describes substrate concentration degradation in the reactor bed zone and effective biomass concentration in the reactor blanket zone. The model equations are:

C. Sensitivity analysis for developed model

Sensitivity analysis was performed for the developed model to determine the effect of various parameters on substrate concentration degradation. The parameters analyzed include the maximum growth rate ( $\mu_{max}$ ), upflow velocity ( $W$ ), and bypass flow fraction ( $SF$ ). The results show that substrate concentration degradation is most sensitive to changes in  $\mu_{max}$  and  $W$ .

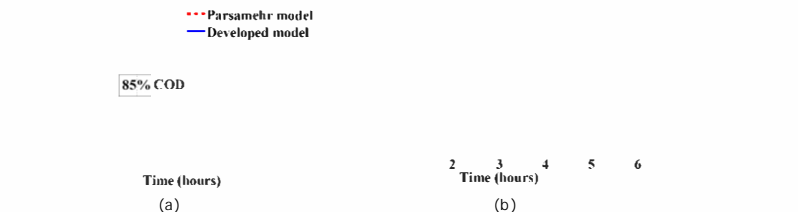


Fig.6. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time

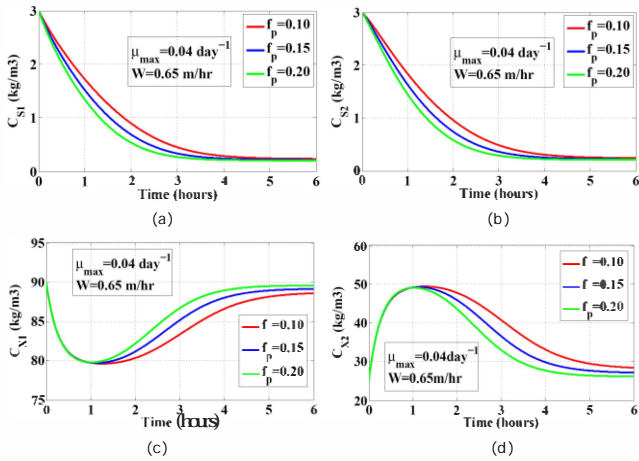


Fig.7. Substrate concentration in the UASB reactor (a) bed zone versus time and (b) blanket zone versus time, and Biomass concentration in the UASB reactor (c) bed zone versus time and (d) blanket zone versus time

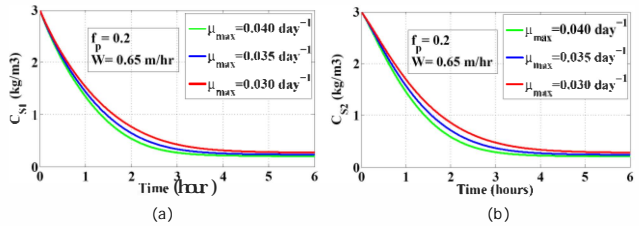


Fig.8. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

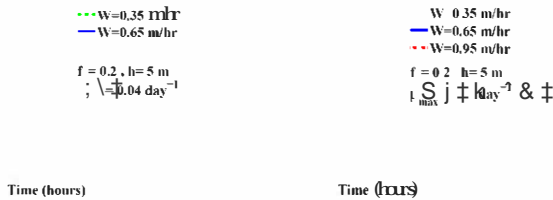


Fig.9. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

$J, \phi$	fraction of volume fraction occupied by granules	—
$V_H$	height of reactor	m
$K, k$	chemical reaction constant	hr <sup>-1</sup>
$k_L$	mass transfer coefficient	m/hr
$K_A, K_1, K_s$	substrate inhibition constants	kg/m <sup>3</sup>
$K_d$	decay coefficient	hr <sup>-1</sup>
$U$	upflow velocity	m/hr
$> \phi$	organic degradation BOD	kg/m <sup>3</sup> day
$r_{dr}$	dragging efficiency	%
$A$	radius of biological floc	m
$R_{CH}, R_s$	methanogenesis rate	kg/m <sup>3</sup> hr
$S$	sludge thickness	cm
$e$	various hydraulic time	hours
$G$	volume of reactor	m <sup>3</sup>
$H_c, J$	settling and upflow velocities	m/hr
$X$	drag coefficient of biomass	m <sup>3</sup> /m <sup>3</sup>
$h$	yield coefficient of substrate	kg/kg
$Z$	vertical height of reactor	m

REFERENCES

- [1] Raul Rodriguez Gomez (2011), Upflow Anaerobic Sludge Blanket Reactor: Modeling thesis, KTH Chemical Science and Eng., Sweden.
- [2] Rodriguez R., L. Moreno (2009, 2010), Modeling of Substrate Degradation and Microorganism Growth in an UASB Reactor, In Proceedings of the International Conference on Chemical, Biological & Environmental Engineering (CBEE 2009), Singapore, October 9-11, 2009, Singapore; Kai Ed., pp 76-80.
- [3] Larisa Korsak (2008), Anaerobic Treatment of Wastewater in a UASB Reactor, licentiate thesis in Chemical Science and Engineering, Royal Institute of Technology, Stockholm, Sweden.
- [4] Mohammad Parsamehr (2012), Modeling and Analysis of a UASB Reactor, M.Sc. thesis, Department of Environmental Engineering, Lulea University of Technology, Sweden.
- [5] T. Coskun, H.A. Kabuk (2012), Antibiotic Fermentation Broth Treatment by a pilot upflow anaerobic sludge bed reactor and kinetic modeling, Bioresource Technology Journal, Environmental Engineering Department, Yildiz University, Turkey.
- [6] Olafadehan A., Olafadehan, Adetunji T., Alabi, June (2009), Modelling and Simulation of Methanogenic Phase of an Anaerobic digester, Journal of Engineering Research, Vol. 13, No. 2, Department of Chemical Eng., University of Lagos, Akoka-Yaba, Lagos, Nigeria.
- Ehab Helmy (2004), A Pilot Plant Study for the Use of UASB Reactor for Domestic Wastewater Treatment, Ph.D. thesis, Cairo University, Faculty of Engineering, Environmental and Sanitary Engineering Division, Public Works Department, Cairo, Egypt.

ABBREVIATIONS

Symbol	Definition	Unit
$5 \phi$	cross-sectional area of reactor	m <sup>2</sup>
$CH_3S, 'CS_0, CS_m$	unionized, volatile and non-volatile substrate concentrations	kg/m <sup>3</sup>
$C_X, C_{X0}$	various time and initial biomass concentrations	kg/m <sup>3</sup>
$D, D_A$	dispersion and effective diffusion coefficients	m <sup>2</sup> /hr
$E$	substrate conversion efficiency	%
$F_{in}$	influent wastewater discharge	m <sup>3</sup> /hr