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RESEARCH PAPER

# Dynamic Pricing Model of Container Sea-Rail Intermodal Transport on Single OD Line

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**Abstract:** In the management of container sea-rail intermodal transportation, dynamic pricing problem with uncertain conditions has significant impacts on the benefit and competitiveness of a multimodal transport operator. Based on the revenue management theory and the features of container sea-rail intermodal transport, this paper develops a two stages optimal model which integrates dynamic pricing and slot allocation on single origin-destination line. The first stage is proposed by considering long-term slot allocation in contract market, and the second stage is set up in consideration of the dynamic pricing in free market. Because of the demand uncertainty and the statistic error characteristics, the method of chance constrained programming and a robust optimization model are used to solve the models, respectively. The simulation shows the feasibility and efficiency of the proposed models and algorithms.

**Key Words:** integrated transportation; dynamic pricing; revenue management; container; sea-rail intermodal transport; optimal model

## 1 Introduction

Due to its high efficiency, fast speed, large capacity, low cost, less pollution, and other outstanding advantages, container sea-rail intermodal transport becomes the focus model promoted by the integrated transport system in “the 12th Five-Year” National Plan. The relevant departments, such as government, railways, ports, and shipping companies, are actively making improvements and co-ordinations to container sea-rail intermodal transport from the aspects of management system, infrastructure, and operational organization. Thus, the development of the environmental and technical condition for container sea-rail intermodal transport will be gradually improved, and the market demand will grow with diverse characteristics, which will result in an enormous challenge to MTOs. Container sea-rail intermodal transport has the typical characteristics of applying the revenue management theory, such as the transport capacity in a certain period of time is fixed; transport services cannot be stored with perishability, but can be pre-sold; fixed cost is high, and marginal cost is low; market demand can be segmented, but is volatile. Therefore, employing revenue management ideas in the container sea-rail intermodal transport system is feasible. How to use the revenue management theory to respond to intermodal market demand changes flexibly and to increase transport efficiency and effectiveness are important decisions related to the future survival and development of MTOs.

Domestic as well as foreign scholars have conducted a lot of researches on container transport revenue management. Ha[1] has studied the slot control strategies of a container shipping company using the expected marginal revenue (EMR) and threshold curve (threshold calve) model; Feng *et al*.[2] have studied the optimal slot allocation problem of a container liner on specific routes by taking the cost of an empty container allocation into account, and established a mathematical programming model with the objective of maximizing liner companies’ operating profits, and the constraints on shipping capacity, container demand, and the supply of empty containers; Sebastian[3] has studied the discrete simulation of liner slot booking, and established liner slot allocation quantitative models with taking the transfer possibilities among multi sections and multi routes into account, which were simulated in different situations,

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networks, and input settings to determine the optimal slot-booking strategy for shipping companies; Pu[4] has established a series of mathematical models based on the container liner slot allocation problem with stochastic programming and dynamic programming methods in his Ph.D. thesis, and solved the models with chance-constrained programming and robust optimization methods; Yang *et al*.[5] conducted quantitative research on the pricing of container liner slots, and established a slot pricing model with the objective of maximizing the expected return and the constraints that demands obey Poisson distribution and shippers' reservation prices obey exponentially distribution, then got the optimal slot pricing equation and analyzed the nature of the optimal price; Ren[6] has studied the pricing problem of China Railway Container Transport in his master's degree thesis, and established a railway container transport pricing model without considering empty container allocation, then simulated, and analyzed the impacts of transport costs, differences of shippers' subjective value, shippers' arrival rate, and initial slot changes on the optimal pricing and the maximum expected return.

As mentioned above, existing literatures only focus on container transport revenue management of a single mode of transport (by sea or rail transport) from the perspective of a particular decision-making behavior (eg, capacity or slot allocation decisions, dynamic pricing decisions); however, the research on container sea-rail intermodal transport revenue management and the comprehensive decision making of slot allocation and dynamic pricing is still rare. Based on the business and organizational characteristics of container sea-rail intermodal transport, this article integrates the pricing strategy with slot allocation by considering the pricing differences between contract sale and free sale as well as the dynamic pricing during the free sale period from the MTOs' point of view, and establishes the dynamic pricing model of container sea-rail intermodal transport based on revenue management in order to enrich the theory and practice of container sea-rail intermodal transport revenue management and to provide a scientific decision-making tool for the operational management of MTOs.

## 2 Modeling

### 2.1 Problem description

It is assumed that an MTO enterprise in an imperfectly competitive market has a monopoly pricing power. Based on the container sea-rail intermodal transport demand between A and B, the MTO decides to operate a container sea-rail intermodal transport line between A and B as the Origin-Destination point (O-D). The MTO selects port P as the seamless transferring port of rail and sea through a path selection decision; determines a railway company and a liner company as the actual carriers of railway and maritime sections through a sub-carrier selection decision; then signs a long-term agreement with the actual carriers; and, finally, gets the operational right of the same amount of slots in both railway container trains between $A$ and $P$ and shipping container liners between $P$ and $B$, so as to ensure a stable capacity as well as the reduction of operating costs. In the agreement period, the MTO will sign sea-rail intermodal transport contracts with shippers as a contract carrier and charge for total freight at a single rate to organize the container sea-rail intermodal transport. In order to adapt to market competition and increase efficiency and effectiveness, the MTO needs to formulate a reasonable pricing strategy and a slot allocation strategy for different transport demands.

Since the demands of container sea-rail intermodal market between A and B is in a one-way direction, the MTO controls slots sale at the originating point, that is to say, the MTO sells slots at A and B separately. The selling process can be divided into two stages: In the first stage, the MTO sells a part of the slots in advance according to the requests of large customers, which are manifested as a series of sale contracts; in the second stage, the MTO sells the remaining bit of slots freely at public price according to demand forecasting, and accepts the booking from a variety of scattered customers.

In the first stage, the intermodal price for contract customers who have a strong bargaining power is certain; thus, the MTO needs to decide how many slots at most can be allocated to these contract customers at a negotiated price to make maximum revenue. In the second stage, since scattered customers who do not have bargaining power have to book slots at the public price announced by the MTO, the MTO may divide freight solicitation time $T$ into $t$ periods and determine the intermodal price and slot allocation in each period, respectively, according to the forecast of demands to make maximum revenue. The revenue management problem of the MTO is depicted in Fig. 1.

### 2.2 Model

The objective in the first stage is to determine the appropriate slot number for contact sale to maximize the revenue of the MTO, as the model (M1)

$$\text{Objective: } \max z = p^{\mathrm{I}} \cdot x^{\mathrm{I}}$$

$$\text{s.t. } \begin{cases} x^{\mathrm{I}} \le D^{\mathrm{I}} & \text{①} \\ x^{\mathrm{I}} \le C & \text{②} \\ x^{\mathrm{I}} \in N \cup \{0\} & \text{③} \end{cases}$$

where $x^{\mathrm{I}}$ is a decision variable that represents the slot number allocated to contract customers for contact sale; $D^{\mathrm{I}}$ represents the random demands of contract customers; $p^{\mathrm{I}}$ indicates the negotiated price for contract customers; $C$ represents the total slot capacity of the intermodal line.

Constraint ① shows that the slot number for contact sale cannot be greater than the random demands of contract customers; constraint ② expresses that the slot number for

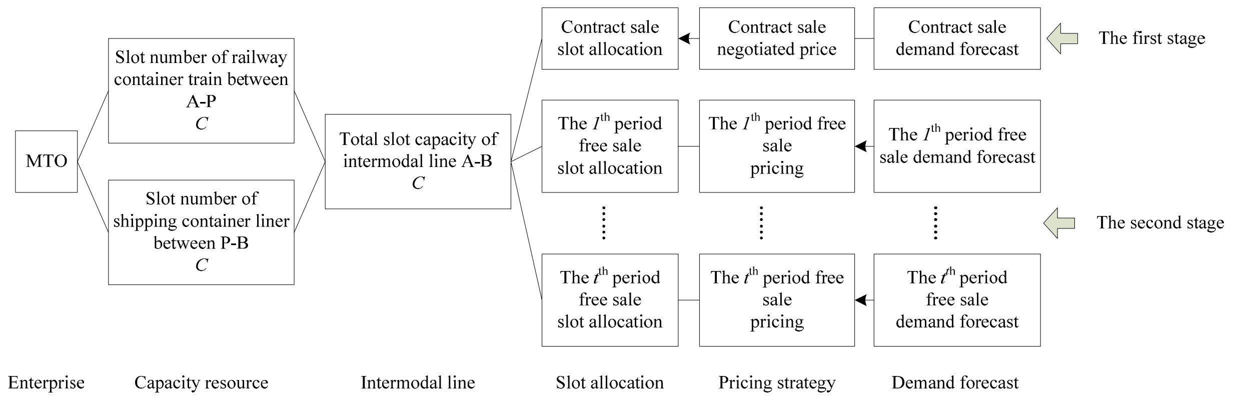


Fig. 1 Dynamic pricing problem of the MTO based on revenue management

contact sale cannot exceed the total slot capacity of the intermodal line; and constraint ③ is an integer constraint of the decision variable, that is to say, this model is a random integer programming model.

The objective in the second stage is to determine the intermodal price and slot number in each period of freight solicitation time for free sale. In the freight solicitation time for free sale, the demands of scattered customers change with the price changes. It is assumed that the freight solicitation time of the MTO for free sale is *T*, which can be divided into t periods according to weeks (or days).

$p_t^{\text{II}}$ is the intermodal price at the $t^{\text{th}}$ period of free sale, and $p_t^{\text{II}}$ is a decision variable;

$x_t^{\text{II}}$ is the slot demands in the $t^{\text{th}}$ period of free sale, and $x_t^{\text{II}}$ is a linear function of $p_t^{\text{II}}$, that is,

$$x_t^{\text{II}} = \alpha_t - \beta_t \cdot p_t^{\text{II}}, \qquad t = 1,2,\cdots,T \tag{1}$$

where the coefficients $\alpha_t, \beta_t$ need to be estimated using statistical methods.

The mathematical model in the second stage (M2) is as follows:

$$\text{Objective:} \quad \max z = \sum_{t=1}^{T} p_t^{\text{II}} \cdot x_t^{\text{II}} = \sum_{t=1}^{T} p_t^{\text{II}} \cdot (\alpha_t - \beta_t \cdot p_t^{\text{II}})$$

$$s.t.\begin{cases} \sum_{t=1}^{T} x_t^{\text{II}} + x^{\text{I}} = \sum_{t=1}^{T}\left(\alpha_t - \beta_t p_t^{\text{II}}\right) + x^{\text{I}} \le C & \text{①} \\ p^{\text{I}} \le p_t^{\text{II}} \le \bar{P}, \forall t & \text{②} \end{cases}$$

Constraint ① indicates that the slot number for contact sale and free sale in the sum cannot exceed the total slot capacity of the intermodal line; constraint ② represents that the price for free sale at any period cannot be less than the price for contact sale, and cannot be more than a price upper limit $\bar{P}$ as well.

## 3 Model solution

### 3.1 Model solution in the first stage

Model (M1) is a random integer programming model because of the existence of the random demand variable $D^{\text{I}}$; thus, the chance-constrained programming method is used in this article[4]. Considering the decision made in the adverse situation may not satisfy the constraint, it is allowed that the decision does not satisfy the constraint to a certain extent, but the decision should make the probability of satisfying the constraint to be not less than a certain confidence level $\alpha$; thus, the constraint ① in model (M1) can be transformed into a chance constraint, that is,

$$\Pr\left(x^{\text{I}} \le D^{\text{I}}\right) \ge \alpha \tag{2}$$

Let $\Phi(\bullet)$ to be the distribution function of $D^{\text{I}}$; thus, the certainty equivalence constraint of chance constraint (2) is

Model (M1) is converted into an equivalent deterministic model (M3):

$$\text{Objective:} \quad \max z = p^{\text{I}} \cdot x^{\text{I}}$$

$$s.t.\begin{cases} x^{\text{I}} \le K_\alpha & \text{①} \\ K_\alpha = \sup\left\{K \middle| K = \Phi^{-1}(1-\alpha)\right\} & \text{②} \\ x^{\text{I}} \le C & \text{③} \\ x^{\text{I}} \in N \cup \{0\} & \text{④} \end{cases}$$

Solving model (M3) with Lingo software packages can result in an optimal slot allocation strategy in the first stage.

### 3.2 Model solution in the second stage

In the free sale model (M2), the actual demands fluctuate randomly; thus, the optimal solution depends very much on the coefficients of $x_t^{\text{II}}$. If the estimation of coefficients $\alpha_t$ and $\beta_t$ in Eq. (1) is not accurate, the optimal solution may not satisfy the constraint of the slot capacity limit; therefore, the goal of obtaining the maximum revenue will not be met. As a result, this article employs the robust dynamic pricing model[7] to fit the uncertainty of demands.

Let $\tilde{\alpha}_t \in [\alpha_t - \hat{\alpha}_t, \alpha_t + \hat{\alpha}_t]$, $\quad \tilde{\beta}_t \in \left[\beta_t - \hat{\beta}_t, \beta_t + \hat{\beta}_t\right]$

where $\tilde{\alpha}_t, \tilde{\beta}_t$ represent the actual value of the demand function coefficients $\alpha_t, \beta_t$, and $\hat{\alpha}_t > 0, \hat{\beta}_t > 0$ indicate the variation in the coefficients $\tilde{\alpha}_t, \tilde{\beta}_t$. Supposing $\xi_t$ and $\eta_t$ are decision variables whose values are in a closed interval [–1,1], $\xi_t$ is the deviation degree between the actual value $\tilde{\alpha}_t$ and the estimation $\alpha_t$, $\eta_t$ is the deviation degree between the actual value $\tilde{\beta}_t$ and the estimation $\beta_t$, that is, $\tilde{\alpha}_t = \alpha_t + \hat{\alpha}_t \xi_t, \tilde{\beta}_t = \beta_t + \hat{\beta}_t \eta_t$. Thus, the absolute value of the differences between actual demands and nominal demands in the t^th period is $\left| \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right|$.

The parameter $\Gamma$, which is a given non-negative real number, is introduced to constrain the deviation between total actual demands and total nominal demands in each period, that is, $\left| \sum_{t=1}^{T} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \right| \le \sum_{t=1}^{T} \left| \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right| \le \Gamma$, $\Gamma$ can be valued on $\left[ 0, \sum_{t=1}^{T} \left( \hat{\alpha}_t + \hat{\beta}_t \overline{P} \right) \right]$; the larger value of $\Gamma$, the less demand function information is mastered by the MTO; on the contrary, the smaller value of $\Gamma$, the more demand function information is mastered by the MTO. Model (M2) can be transformed into a dynamic pricing robust model (M4), as follows:

Objective:

$$\max z = \sum_{t=1}^{T} p_t^{\mathrm{II}} (\alpha_t - \beta_t p_t^{\mathrm{II}}) + \min \left( \sum_{t=1}^{T} p_t^{\mathrm{II}} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \right)$$

$$\text{s.t.} \begin{cases} \sum_{t=1}^{T} \left( \alpha_t - \beta_t p_t^{\mathrm{II}} \right) + \sum_{t=1}^{T} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \le C - x^{\mathrm{I}} & ① \\ \left| \sum_{t=1}^{T} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \right| \le \Gamma & ② \\ |\xi_t| \le 1, |\eta_t| \le 1, \forall t & ③ \\ p^{\mathrm{I}} \le p_t^{\mathrm{II}} \le \overline{P}, \forall t & ④ \end{cases}$$

Merge constraint ① and ② in model (M4) into a new constraint:

$$\sum_{t=1}^{T} \left( \alpha_t - \beta_t p_t^{\mathrm{II}} \right) \le C - \left( x^{\mathrm{I}} + \Gamma \right) \tag{4}$$

Thus model (M4) can be slacked into the following robust model (M5):

Objective:

$$\max z = \sum_{t=1}^{T} p_t^{\mathrm{II}} (\alpha_t - \beta_t p_t^{\mathrm{II}}) + \min \left( \sum_{t=1}^{T} p_t^{\mathrm{II}} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \right)$$

$$\text{s.t.} \begin{cases} \sum_{t=1}^{T} \left( \alpha_t - \beta_t p_t^{\mathrm{II}} \right) \le C - \left( x^{\mathrm{I}} + \Gamma \right) & ① \\ \sum_{t=1}^{T} \left( \hat{\alpha}_t \xi_t - \hat{\beta}_t \eta_t p_t^{\mathrm{II}} \right) \ge -\Gamma & ② \\ |\xi_t| \le 1, |\eta_t| \le 1, \forall t & ③ \\ p^{\mathrm{I}} \le p_t^{\mathrm{II}} \le \overline{P}, \forall t & ④ \end{cases}$$

Model (M5) is a bi-level programming problem, and its inner minimization problem can be seen as a linear programming based on the decision variables $\xi_t, \eta_t$; using the strong duality theorem, model (M5) is equivalent to the following convex programming problem model (M6):

Objective:

$$\max z = \sum_{t=1}^{T} p_t^{\mathrm{II}} \left( \alpha_t - \beta_t p_t^{\mathrm{II}} \right) - \left( \Gamma y + \sum_{t=1}^{T} \left( \hat{\alpha}_t - \hat{\beta}_t p_t^{\mathrm{II}} \right) \left( p_t^{\mathrm{II}} - y \right) \right)$$

$$s.t. \begin{cases} \sum_{t=1}^{T} \left( \alpha_t - \beta_t p_t^{\mathrm{II}} \right) \le C - \left( x^{\mathrm{I}} + \Gamma \right) & ① \\ p^{\mathrm{I}} \le p_t^{\mathrm{II}} \le \overline{P}, \forall t & ② \\ y \ge 0 & ③ \end{cases}$$

where $y$ is the decision variable in the dual programming of the inner programming of model (M5).

Solving model (M6) with Lingo software packages can result in an optimal pricing strategy in the second stage.

## 4 Simulation example analysis

It is assumed that an MTO operates a domestic container sea-rail intermodal transport on a single OD line with the total slot capacity of $C$=100 TEU.

In the first stage, the price for contract sale is known as $p^I$=3871 yuan/TEU, the slot demand of contract customers has been obtained through historical data, which is a random variable following a normal distribution of $D^{\mathrm{I}} \sim N(54, 2^2)$. With given confidence level of 95%, model (M3) is solved with the Lingo software package to obtain the result of slot allocation in the first stage. The result is that the slot number allocated to contract customers is $x^I$=57 TEU.

In the second stage, it is assumed that the freight solicitation time for free sale is divided into three periods on an average. The greater of t, the closer to the canvassing deadline, and the less sensitive of shippers' demands to price changes. Through a statistical analysis of relevant data, the estimation and variation of demand function coefficients in different periods are shown in Table 1.

If the price cap limit $\overline{P} = 4324$ yuan, then $\Gamma \in [0, 125]$. Providing the MTO can grasp more information on the demand function through an extensive demand survey of the new pricing system, it is assigned $\Gamma = 2$ in this example. Model (M6) is solved with the Lingo software package to obtain the pricing strategy in the second stage, as shown in Table 2.

The strategy in the second stage is integrated with the strategy in the first stage to obtain the slot allocation strategy and pricing strategy of the intermodal line in Table 3.

Table 1 Estimation and variation of demand distribution in different periods

| Freight solicitation periods of free sale | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|
| Estimation of demand function coefficients $\alpha_t, \beta_t$ | 200, 0.044 | 126, 0.027 | 52, 0.011 |
| Variation of demand function coefficients $\hat{\alpha}_t, \hat{\beta}_t$ | 20, 0.005 | 20, 0.005 | 20, 0.005 |

Table 2 Pricing strategy in the second stage

| Freight solicitation periods of free sale | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|
| Pricing of free sale (yuan/TEU) | 3982 | 4230 | 4324 |

Table 3 Pricing strategy and slot allocation of the intermodal line

| Strategy | Two-stage strategy integrating slot allocation with dynamic pricing | | | | Slot allocation strategy | | General strategy |
|---|---|---|---|---|---|---|---|
| | The first-stage contract sale | The second-stage free sale | | | The first-stage contract sale | The second-stage unified sale | |
| | | $t=1$ | $t=2$ | $t=3$ | | | |
| Pricing (yuan/TEU) | 3871 | 3982 | 4230 | 4324 | 3871 | 3982 | 3871 |
| Slot (TEU) | 57 | 25 | 12 | 6 | 57 | 43 | 100 |
| Revenue (yuan) | 220647 | 99550 | 50760 | 25944 | 220647 | 171226 | 387100 |
| Total revenue(yuan) | 396901 | | | | 391873 | | 387100 |

From Table 3, it can be observed that if the MTO adopts a two-stage strategy that integrates slot allocation with dynamic pricing, the total revenue is 396,901 yuan. If the MTO adopts the slot allocation strategy only, that is to say, the MTO adopts slot allocation in the first stage while it sells the remaining slots at a unified price in the second stage, the total revenue is 391,873 yuan. If the MTO adopts the general strategy, that is to say, the MTO sells all slots at the same negotiated price to all shippers without dividing the stages, the total revenue is only 387,100 yuan. Thus, the two-stage strategy that integrates slot allocation with dynamic pricing can increase the revenue as well as satisfy the shippers' demands for the MTO.

## 5 Conclusions

Based on the revenue management theory, this article establishes a two-stage optimal model that integrates dynamic pricing and slot allocation on a single O-D line from the viewpoint of the different pricing strategy of MTOs, and solves the models with methods of chance-constrained programming and robust optimization. In the first stage, the model solves the problem of slot allocation for contract customers at a negotiated price. In the second stage, the model considers price as a decision variable, and solves the problem of dynamic pricing and slot allocation in free sale according to the rules of scattered shippers' demands changing with price. The prices during different booking periods are different, which makes the container sea-rail intermodal pricing more flexible, thereby increasing the revenue of MTOs. The simulation verifies both the feasibility and effectiveness of the models and algorithms.

Only one single O-D intermodal line is considered in this model, and also only one value of confidence level and demand variance is used to calculate the slot allocation and pricing results in the simulation example. It should be noted that, in the first-stage model solution, if the confidence level values are larger, the actual demand of contact sale deviates larger from the mean, and the calculated slot number allocated to contact customers in the first stage is more; thus, the remaining slot number for free sale in the second stage becomes less, but there is no effect on the negotiated price. Conversely, if the confidence level values are smaller, the calculated slot number allocated to contact customers in the first stage is less; thus, the remaining slot number for free sale in the second stage becomes more, but there is no effect on the negotiated price. The research on container sea-railway intermodal transport revenue management will be further completed wiht considering the actual situiation, such as the multi O-D intermodal line with several railway and shipping points, different types of containers, unsubscribing and overbooking, the impacts of different values of demand variance and the parameter $\Gamma$ on the results of slot allocation, and pricing.

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