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MADM in the case of simultaneous equations models and economic applications

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Abstract

In this paper we will solve MADM (Multiple Attribute Decision Making) problems in the case of the simultaneous equations models. The *p* dependent variables are considered the stochastic criteria, and the alternative decisions are given by points in R^k, where *k* is the number of explanatory variables. The set of alternative decisions contains the points from the dataset from which we estimate the regression coefficients, but it is completed by simulation on an interval in R^k. The senses are given by the economic interpretation of the variables $Y_i,\ i=\overline{1,p}$. We consider as economic application the GDP/capita and long term unemployment rate in terms of computer skills

Keywords: MADM; multi-linear regression; simultaneous equations; GDP/capita; computer skills; long term unemployment rate;

1. Introduction

A MADM (Multiple Attribute Decision Making) can be formulated as follows (Văduva and Resteanu, 2009; Văduva, 2012). There are *m* decision alternatives to be taken and *n* criteria or attributes used to determine the best (optimum) alternative decision. To select the best decision according these attributes there is defined for each

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attribute the sense: minimum, if the attribute is a loss, respectively maximum if it is a gain. Because in general there exist no decision optimal with respect each criterion, we need an importance vector, P, that express the importance given for each criterion. Usually P is a probability vector.

The data of a MADM problem can be represented as in Table 1, where A_1, A_2,…, A_m are the decision alternatives, C_1, C_2,…, C_n are the criteria, the $m\times n$ matrix of a_{ij}, $1\le i\le m$; $1\le j\le n$ is the matrix of entries (the values of criterion j if we take the decision i), P is the importance vector and *sense* is the sense vector.

Table 1: The MADM problem.

	C_1	C_2	…	C_n
A_1	a_{11}	a_{12}	…	a_{1n}
A_2	a_{21}	a_{22}	…	a_{2n}
…	…	…	…	…
A_m	a_{m1}	a_{m2}	…	a_{mn}
P	p_1	p_2	…	p_n
sense	***sense***$_1$	***sense***$_2$	…	***sense***$_n$

One of the steps in solving a MADM problem is to estimate the vector P of importance weights. The first method presented in Văduva (2012) is the method of eigenvector. We have given in this case the matrix B of relative importance: $B=\begin{pmatrix}1 & \frac{p_1}{p_2} & \cdots & \frac{p_1}{p_n}\\ \frac{p_2}{p_1} & 1 & \cdots & \frac{p_2}{p_n}\\ \cdots & \cdots & \cdots & \cdots\\ \frac{p_n}{p_1} & \frac{p_n}{p_2} & \cdots & 1\end{pmatrix}$. In the above formula $b_{ij}=\frac{p_i}{p_j}$ is given by the decedent, and it represents the relative importance of the criterion i with respect the criterion j. These values can be any positives ones, but, in the ideal case (when the decedent is not contradictory) there must be fulfilled some conditions (Văduva, 2012): $\begin{cases}b_{ji}=\frac{1}{b_{ij}}\\ b_{ik}=b_{ij}b_{jk}\end{cases}$. We compute the maximum eigenvalue of B, $\lambda_{\max}$, and P is its corresponding eigenvector.

Another method to estimate P is the least squares method (Văduva, 2012). We have to solve the minimum problem, considering the above matrix B: $\begin{cases}\min\sum_{i=1}^{n}\sum_{j=1}^{n}\left(b_{ij}p_j-p_i\right)^2\\ \sum_{j=1}^{n}p_j=1\end{cases}$, where $B=\left(b_{ij}\right)_{i,j=\overline{1,n}}$ is the above matrix of relative importance. Using the Lagrange multipliers' method we solve first a linear system of equations, obtaining $p_i=p_i\left(\lambda\right)$. Next we compute λ from $\sum_{j=1}^{n}p_j=1$, and, using this value, we estimate p_i.

In the ideal case (when the decedent is not contradictory), the eigenvector value leads to $\lambda_{\max}$ lambda max=k, and the weights are proportional to the values of one row (all the rows of matrix are proportional). The same weights are obtained by the least squares method, and the minimum sum is zero.

Another step is to transform the entries a_{ij} such that all entries are of the same type, i.e. all fuzzy, or all cardinal. In this paper we consider the transformation such that all entries become cardinal. If the corresponding entries of the criterion j, a_{ij}, $i=\overline{1,m}$ are stochastic, characterized by the random variable X, we make two cardinal criteria: the first one is the expectation of the random entry, and second one is informational, as follows. The entry is normalized (by dividing to standard deviation), and, for second criterion, it is computed the Shannon entropy (Văduva and Resteanu, 2009; Văduva, 2012). The Shannon entropy can be replaced by Onicescu informational energy (Onicescu, 1966; Onicescu and Ştefănescu, 1979; Petrică and Ştefănescu, 1982). The first cardinal criterion made from the stochastic criterion j has the entry on the row I (corresponding to the decision i) equal to $E\left(X\right)$, and the second one has the entry on the same row either equal to the above Shannon entropy, either equal to the above Onicescu's informational energy. The sense of expectation is the same as for original criterion, while for the informational one, the sense is max for entropy, and min for informational energy. These reverse senses can be explained by the fact that the entropy and the informational energy have inverse variation (Onicescu, 1966).

After we have made all the entries cardinal, we need to normalize the entries such that all the criteria have the same range (Văduva and Resteanu, 2009; Văduva, 2012). First we add a positive constant (large enough) such that all entries a_{ij} become positives. First normalization method is based upon vectorial norms (Paltineanu, Matei and Mateescu, 2010). The infinite norm and the p norm, with $0 < p < \infty$ of a n-component vector x are.

$$\begin{cases} \|x\|_\infty = \max\limits_{i=\overline{1,n}} |x_i| \\ \|x\|_p = \left(\sum\limits_{i=1}^{p} |x_i|^p \right)^{\frac{1}{p}} \end{cases}$$

Using the above norms, in Văduva and Resteanu (2009) and Văduva (2012) there are presented three methods for normalization: if the sense is maximum $r_{ij} = \frac{a_{ij}}{\left\| a^{(j)} \right\|}$, where $a^{(j)}$ is the colon j (corresponding to criterion j), and $\|x\|$ is one of the above norms. The first normalization method using linear transforms is the mentioned normalization method using $\|x\|_\infty$. There are used also $\|x\|_1$ and $\|x\|_2$. If $sense_j = \max$ there is used the second normalization method by linear transforms is (Văduva, 2012): $r_{ij} = \frac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}$. In all the four cases, if $sense_j = \min$ we take $r_{ij} = 1 - r_{ij}^{'}$, where $r_{ij}^{'}$ is the normalized entry if the sense would be maximum.

A method to solve a cardinal MADM problem is SAW (Simple Addidive Weighting). For this method the sense of each criterion j must be max. If for one value j the corresponding criterion is min, we make it max using the substitution $r_{ij} \leftarrow \frac{1}{r_{ij}}$. The method is as follows (Văduva and Resteanu, 2009; Văduva, 2012; Zanakis et al., 1998) .

1) Normalize the entries a_{ij}, obtaining the new entries r_{ij}.
2) Denoting by A the set of decision alternatives, compute the function $f : \mathbf{A} \to R$, $f_i = f(A_i) = \frac{\sum_{j=1}^{n} p_j r_{ij}}{\sum_{j=1}^{n} p_j}$.
3) Order increasing the values f_i, $i = \overline{1,m}$, and denote by $f_{(i)}$, $i = \overline{1,m}$ with their corresponding decision alternatives $A_{(i)}$, $i = \overline{1,m}$, such that $f_{(1)} \le f_{(2)} \le \cdots \le f_{(n)}$. The best decision is $A_{(n)}$.

Another method to solve a cardinal MADM problem is TOPSIS (Thechnique for Order Preference by Similarity to Ideal Solution). The method is as follows (Văduva and Resteanu, 2009; Văduva, 2012).

1) Normalize the entries a_{ij}, obtaining the new entries r_{ij}.
2) Build up the weighted normalized matrix $V = (v_{ij})$, $v_{ij} = p_j r_{ij}$.
3) Build up the ideal positive solution, $V^+ = (v_1^+, v_2^+, \ldots, v_n^+)$ and the ideal negative solution $V^- = (v_1^-, v_2^-, \ldots, v_n^-)$', defined as $v_i^+ = \begin{cases} \max\limits_{i=\overline{1,m}} v_{ij} & \text{if } sense_j = \max \end{cases}$, and $v_i^- = \begin{cases} \min\limits_{i=\overline{1,m}} v_{ij} & \text{if } sense_j = \max \end{cases}$.
4) Compute the distances between weighted normalized entries v_{ij} and each of the ideal solution (using the Euclidean distance), namely $\begin{cases} D_i^+ = \sqrt{\sum\limits_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2} \\ D_i^- = \sqrt{\sum\limits_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2} \end{cases}$.
5) Compute the relative closeness to the ideal solution for each alternative as $Q_i = \frac{D_i^-}{D_i^+ + D_i^-}$. Note that $0 < Q_i < 1$.

6) Order the decision alternatives A_i as in the SAW method, but the order criterion is Q_i. The best decision is also $A_{(n)}$.

In the case of multi-linear regression, the method to estimate the variance of the estimator $\hat{Y}$ from regression equation, depending on the variance-covariance matrix of the coefficients and the variance of the residues, is presented in Jula and Jula (2012) and in Voineagu et al. (2007). The problems that arise if we treat separately the regressions are also identified, namely: the identification problem, and the distorsion of simultaneity. Due to the last problem, the estimators are biased and inconsistent, the forecast is biased and inconsistent, and the (signification) tests on the parameters are not valid. To avoid the above identification problem, in each of the p regression equations there must be omitted at least $p-1$ explanatory variables. More exactly, we have three possible situations (Jula and Jula, 2012; Voineagu et al., 2007).

1) If the number of omitted explanatory variables is exactly $p-1$, the equation is identified.
2) If the number of omitted explanatory variables is less than $p-1$, the equation is non-identified.
3) If the number of omitted explanatory variables is greater than $p-1$, the equation is super-identified.

A method to solve a simultaneous equation model with all equations identified is the least squares method in two stages.

In the first stage we estimate the parameters of the p linear regressions as if the other dependent variables are explanatory ones. In the second stage we estimate $\hat{Y}_1$, $\hat{Y}_2$,..., $\hat{Y}_p$ using these linear regression, and next we replace the values of Y_j as explanatory variables (in the right sides) with $\hat{Y}_j$. Finally we consider the model (17) with the last computed coefficients, and we solve the dependent linear system with p equations.

In Mladenović (2009) it is taken into account the relation between inflation and inflation uncertainty in the case of Serbia. For modeling this relation it is used the GARCH model

$$\pi_t = a_{10} + \sum_{i=1}^{k} a_{1i}\pi_{t-i} + \sum_{j=1}^{k} b_{1j}\sigma_{t-j}^2 + e_{1t}\text{, where} \quad (1)$$

$$\sigma_t^2 = a_{20} + \sum_{i=1}^{k} a_{2i}\pi_{t-i} + \sum_{j=1}^{k} b_{2j}\sigma_{t-j}^2 + e_{2t}, \quad (1')$$

and e_{it} are Gaussian white noise processes uncorrelated at nonzero lags.

The Friedman-Ball hypothesis of causality running from inflation to inflation uncertainty cannot be rejected if inflation is a Granger cause for inflation uncertainty. This implies the null hypothesis $H_0 : a_{2i} = 0$ for any $i = \overline{1,k}$ is accepted (Mladenović, 2009).

Analogously, the Cukierman-Melzer hypothesis of causality from inflation uncertainty to inflation cannot be rejected if inflation uncertainty is a Granger cause for inflation. This implies the null hypothesis $H_0 : b_{1i} = 0$ for any $i = \overline{1,k}$ is accepted. If the null hypothesis is rejected, the sum $\sum_{j=1}^{k} b_{1j}$ shows that inflation uncertainty leads to increase/ decrease of the inflation rate if the sum is positive/ negative.

In Mitruţ and Bratu (2013) there is studied the inadequacy of the indicators for describing a certain economic phenomenon due to uncertainty. There are considered also the forecast of unemployment rate using an econometric model (Strategy 1), and the use of some weighting coefficients to aggregate the predictions from the regions. Note the analogy to the weighting vector P in a MADM problem.

2. The methodology

Consider *n* stochastic criteria. For each such criterion, *i*, the entries a_{ij} are the values $\hat{Y}_i$, if the decision alternative is given by the current values of the explanatory variables.

To compute the entries for the corresponding cardinal criterion, we take into account that the Shannon entropy of the random variable $X \sim N\left(m, \sigma^2\right)$ is

$$H = \frac{1}{2} + \frac{1}{2}\ln\left(2\pi\right) + \ln\left(\sigma\right), \tag{2}$$

and the Onicescu informational energy is (Onicescu, 1966; Onicescu and Ştefãnescu, 1979; Petrică and Ştefãnescu, 1982)

$$e = \frac{1}{2\sigma\sqrt{\pi}}. \tag{2'}$$

Because the variance is essential in computing the entropy, respectively the informational energy, we do not standardize the random variable, as in Văduva and Resteanu (2009). Finally, we solve the cardinal MADM problem using the SAW method, or the TOPSIS method. We notice also that the connection between the informational criterion and its variance is analogue to the coefficients of the variances in the GARCH model from Mladenović (2009). Analogous to the use of the above variance, in Mitruţ and Bratu (2013) it is considered the RMSE as measure of accuracy of the forecast.

If we have a simultaneous equation model we estimate first the importance vector *P* with *p* components, corresponding to the *p* stochastic criteria. We use for this one of the three methods mentioned in Section 1. The importance vector *P* has $2 \cdot p$ **c**omponents (we obtain $2 \cdot p$ cardinal criteria), and is obtained taking into account the degrees of uncertainty for each of the *p* criteria, denoted by α_j. The difference between this case and that of the multi-linear model case is that the importance weight p_j of the stochastic criterion *j* splits in $\left(1-\alpha_j\right)p_j$ for the criterion $\hat{Y}_i$ and $\alpha_i \cdot p_i$ for the corresponding informational criterion.

For computing the variances of the estimators in the model with simultaneous equations, we start from the regression equations with which we start at the last (the third) stage. But, in the formula $D\left(\hat{A}\right) = \sigma_u^2\left(X'X\right)^{-1}$, where σ_u^2 is the variance of residues, we use $\sigma_{u,i}^2$ and X_i for the equation *i*, and for the covariance between the coefficients from positions *k* from equation *i* and *l* from position *j* we use $\sigma_{u,i,j}^2$ i.e. the covariance between u_i and u_j. $\left(X'X\right)^{-1}$ becomes also $\left(X_i'X_i\right)^{-1} X_i'X_i\left(X_i'X_i\right)^{-1}$. Next we make zeroes for rows and columns corresponding to Y coefficients.

We reduce simultaneously the two matrices in the canonical (diagonal) form. Next we compute the variance-covariance matrices for regression coefficients, taking into account that

$$\Sigma_1 = C * \Sigma * C', \tag{3}$$

where the normal variable X with the variance-covariance matrix Σ is replaced by $\mathbf{Y} = \mathbf{C} * \mathbf{X}$.

The variance-covariance matrix of $\hat{Y}$ is

$$\Sigma_{\hat{Y}} = \left(C^{-1}\right)' * \Lambda_1 * C^{-1}. \tag{4}$$

3. Applications

Example 1. *Consider the GDP per capita (the value for EU27 being considered 100), the long term unemployment rate, and the percentage of total number of individuals aged from 16 to 64 years who have carried one or two, three or four, respectively five or six of the computer related activities.*

The data for 30 European countries are, according EUROSTAT (The GDP per capita of the EU countries, EUROSTAT; *The individual level of computer skills of the EU countries*, EUROSTAT; *The total employment rate of the EU countries*, EUROSTAT*).*

Consider X_1 *the percentage of individuals that have not carried at least one computer related activities, i.e. 100 minus the sum of the above mentioned percentages, and* X_2 *the percentage of total number of individuals aged from*

16 to 64 years who have carried five or six of the computer related activities. Y_1 *is the GDP per capita, depending on* X_2 *, and* Y_2 *the long term unemployment rate, depending on* X_1 [†]*.*

The considered uncertainty degrees are 0.3 for GDP/ capita, respectively 0.4 for long term unemployment rate. We have chosen these values because a government is more interested to decrease the fingers on unemployment than to increase those on GDP. Because in crises periods each government pretends that "he has to take austerity measures", we take $b_{1,2} = 0.8$ and $b_{2,1} = 1.2$.

We notice that $b_{1,2} * b_{2,1} = 0.96$, closed to one. Using the eigenvector method, we obtain $\lambda_{\max} = 1.9798$ (the ideal case is $\lambda_{\max} = 2$), and the weights 0.44949 for GDP, respectively 0.55051 for long term unemployment. Taking into account the uncertainty degrees, the weight of GDP splits in 0.31464 for the expectation criterion and 0.13485 for the informational one, and the weight of long term unemployment splits in 0.33031 for average criterion and 0.2202 for informational one.

In the case of minimum squared method, we obtain the minimum sum $1.98021*10^{-4}$ for the weights 0.4505 and 0.5495. These weights split in 0.31535 and 0.13485, respectively 0.3297 and 0.2198.

For the first stage we obtain $\begin{cases} Y_1 = 21.24966 - 0.17691X_1 + 3.40335X_2 \\ Y_2 = 2.78177 + 0.02589X_1 - 0.04139X_2 \end{cases}$.

For the second stage we obtain $\begin{cases} Y_1 = 202.19325 - 2.65759X_1 - 0.06511X_2 \\ Y_2 = 0.56815 + 0.05658X_1 - 0.00053X_2 \end{cases}$.

For the third stage we obtain $\begin{cases} Y_1 = 188.80251 - 2.47242X_1 + 0.18919X_2 \\ Y_2 = -3.64689 + 0.11504X_1 + 0.08031X_2 \end{cases}$.

When we make the rotation, we obtain $C_1 = \begin{pmatrix} 0.03168 & 0.01227 \\ -0.00055 & 0.70376 \end{pmatrix}$, $C_2 = \begin{pmatrix} 0.72751 & -0.6861 \\ 0.6861 & 0.72751 \end{pmatrix}$, and from here $C = \begin{pmatrix} 0.03146 & -0.01281 \\ 0.48245 & 0.51237 \end{pmatrix}$. Next we generate up to 1000 pairs (970 new ones) on (0,81.95693) for X_1, respectively on (0.11113,50.15553) for X_2.

We consider now the case of eigen value method to estimate the weights, and the first linear transform normalization method. If we use the SAW method and Shannon entropy, we obtain the optimal pair (1.216714,2.141058), and F=0.90589. The same optimal solution we obtain if we use SAW method and Onicescu informational Energy, or TOPSIS and Shannon Entropy. If we use the TOPSIS method and Onicescu informational Energy, the above solution is on the second place, and the optimal solution becomes that from position two in the case of two other three cases: (0.46959,9.60303). None of the 30 countries is on the first place in the four cases: the pairs are obtained by simulation.

The order of countries in the case of Shannon entropy and SAW method (ignoring the simulated values) is: Iceland, Norway, Luxembourg, Netherlands, Denmark, Finland, Germany, Sweden, France, Slovakia, Austria and UK (same values for explanatory variables, 29 and 29), Belgium, Hungary, Slovenia, Spain, Estonia, Ireland, Czechia, Lithuania and Portugal (same values for explanatory variables, 46 and 27), Latvia, Cyprus, Poland, Malta, Italy, Croatia, Greece, Bulgaria and Romania. Same order, except few interchanges, is obtained for the other three cases.

In the case of Onicescu informational energy and SAW method, Germany and Finland (positions 7 and 6 in the above first case) switch their positions, and the same thing we can say about Poland and Cyprus (positions 24 and 23).

In the case of Shannon entropy and TOPSIS method Denmark and Netherlands (positions 5 and 4 in the first case) switch their positions; Germany and Sweden (positions 7 and 8) increase one position, to positions 6 and 7, while Finland (position 6) goes to position 8; Lithuania and Portugal (positions 20-21 with the same values of

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[†] Remember that we have to omit p-1=2-1=1 explanatory variable for each dependent variable.

explanatory variables) go to positions 21-22 (decrease one position), while Latvia (position 22) goes to position 20; and Poland and Malta (positions 24 and 25) increase one position, to positions 23 and 24, while Cyprus (position 23) goes to position 25. In the case of Onicescu informational energy and TOPSIS method Norway and Iceland (positions 2 and 1) switch their positions, and it is the only modification from the first case's order.

If we use the minimum squared method we have some switches (from the same informational criterion and the same method to solve the cardinal MADM problem, but using the eigenvector method to estimate weights) only between some simulated values. But we have the same optimal solution, and the same order as in the case of eigenvector method, in all four above cases.

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Conclusions

The considered six (the maximum number) computer skills considered in EUROSTAT database are: copy or move a file or folder; use copy/ paste tools to duplicate or move information within a document; use basis arithmetic formula (add, subtract, multiply, divide) in a spread sheet; compress files; connect and install new devices (e.g. printer, modem); write a computer program using a specialized programming language. The individuals who have carried out one or two of the mentioned computer-related items are considered with low level of computer skills, those who have carried out three or four computer-related items are considered with medium level of computer skills, and those who have carried five or six computer-related items are considered with high level of computer skills.

The last positions (Romania and Bulgaria) are due to high level of X_1 (percentage of individuals that have not carried at least one computer related activity: 64). The difference is at X_2: 9 for Bulgaria and 7 for Romania.

As we can see from the example, if we use the first linear transform normalization method, the order does not depend on the method we use to estimate the weights. It slightly depend on the informational criterion and on the method to solve the cardinal MADM problem. An open problem is to check if the same thing we can say about the other normalization methods, and how the solution depends on this method.

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