Wireless Pers Commun (2014) 78:1297–1312
DOI 10.1007/s11277-014-1818-8

Robust Probabilistic Distributed Power Control Algorithm for Underlay Cognitive Radio Networks under Channel Uncertainties

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Published online: 3 May 2014

Abstract Due to limited cooperation among users and erratic nature of wireless channel, it is difficult for secondary users (SUs) to obtain exact values of system parameters, which may lead to severe interference to primary users (PUs) and cause communication interruption for SUs. In this paper, we study robust power control problem for spectrum underlay cognitive radio networks with multiple SUs and PUs under channel uncertainties. Precisely, our objective is to minimize total transmit power of SUs under the constraints that the satisfaction probabilities of both interference temperature of PUs and signal-to-interference-plus-noise ratio of SUs exceed some thresholds. With knowledge of statistical distribution of fading channel, probabilistic constraints are transformed into closed forms. Under a weighted interference temperature constraint, a globally distributed power control iterative algorithm with forgetting factor to increase convergence speed is obtained by dual decomposition methods. Numerical results show that our proposed algorithm outperforms worst case method and non-robust method.

Keywords Cognitive radio network · Robust power control · Probability constraints · Spectrum underlay

1 Introduction

Cognitive radio (CR) is a highly promising technology to solve spectrum resource shortage problem [1]. To improve spectrum efficiency, cognitive users (i.e., secondary users or SUs) in cognitive radio networks (CRNs) are allowed to utilize licensed spectrum as long as they do not affect normal communication of licensed users (i.e., primary users or PUs). Currently, there are three main spectrum sharing techniques: interweave, overlay and underlay

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models [2]. In underlay CR scenario, SUs can simultaneously communicate with PUs when interference received at PU-receiver (PU-Rx) is below a certain interference temperature (IT) level. In this paper, we concentrate on underlay spectrum sharing model because of its high efficiency.

Due to PUs and mutual interference, resource allocation problem in CRNs is no longer the same as the fixed spectrum management mechanism in traditional communication networks. Power control policy as an efficiency method can achieve spectrum sharing by adjusting transmit power of users. With perfect parameter information (e.g., channel gain or interference power), power control algorithms for CRNs are proposed in [3,4]. However, since practical channel state information (CSI) is inevitably affected by quantization errors, estimation errors and uncertain interference, system performance will degrade in practice. For example, inaccurate channel estimation between SU-transmitters (SU-Txs) and PU-Rxs may make the interference at PU-Rx exceed IT threshold. Interference uncertainty of PU-transmitters (PU-Txs) and channel uncertainty among SUs (e.g., direct channel gain and mutual interference gain) may cause actual signal-to-interference-plus-noise ratio (SINR) of SUs below target value. These factors will increase outage probability of SUs and PUs. To tackle uncertain parameters, there are two common methods: robust optimization [5] and stochastic optimization [6]. In robust optimization, uncertain parameter is described by a deterministic uncertainty model, such as worst case approach. Under worst case approach, uncertainty is modeled by a bound and closed set for any error realization. This approach can guarantee system performance under worst errors and provide seamless communication. And it also leads to a very conservative design against uncertainties of communication system, since worst error does not always appear in the system. But, for stochastic optimization, uncertain parameter is modeled by probabilistic constraint (i.e., chance constraint), such as Bayesian approach. In probabilistic optimization problem, the statistic feature of error is assumed to be known.

1.1 Related Works

Since parametric uncertainty exists in communication system, robust power control (RPC) problems for CRNs have been concerned in robust optimization (e.g., [7–12]) and stochastic optimization (e.g., [13–18]). In [7], in a multiuser orthogonal frequency division multiplexing (OFDM) based CRN, a robust iterative waterfilling algorithm (IWFA) under noise plus interference uncertainty is proposed to maximize data rate of each SU under the constraints on sum transmit power of SU and IT of PU. For a multiuser CRN, a robust resource allocation problem based on non-cooperative game for single input single output (SISO) frequency selective channel or multiple input multiple output (MIMO) channel is studied to maximize transmission rate of each SU subject to transmit power and robust interference constraints while channel uncertainty is modeled by weighted Euclidean norm [8]. The problem is solved by asynchronous distributed algorithms with analysis of convergent properties. In [9], for an underlay CRN, a robust worst case interference control problem is studied to maximize throughput of SU while both transmit power constraint of SU and IT constraint of PU are considered. In [10], a robust distributed uplink power allocation algorithm is presented to maximize social utility of SUs when channel gain from SU to primary station and interference power from PU to secondary station are uncertain. RPC problem is converted into a geometric programming (GP) solved by Lagrange dual decomposition in a distributed way. However, most of the existing works under worst case approach concentrate on user's rate/throughput maximization without robust SINR constraint, and direct channel uncertainty and fairness are less considered. Although energy robustness problem in cellular network is studied in

[11], it can not simply extend to CR scenario. An initiative work about energy efficiency for CRNs is given in [12]. But it does not consider direct channel gain uncertainty and multiple PUs case.

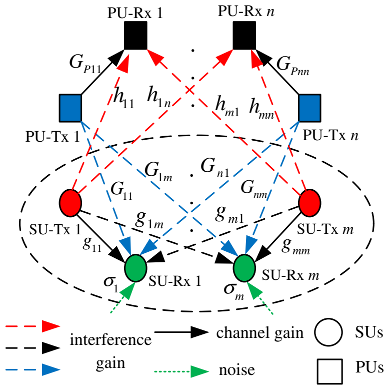
Worst case approach requires prior knowledge on upper bound of estimation error which is not pragmatic due to random nature of radio environment. A better formulation of uncertainty, the focus of this paper, is probability constraint. Since probabilistic approach guarantees performance on average, it yields a less conservative result compared to the works in [7–12]. Only considering interference outage probability constraints of PU, RPC algorithms are proposed in [13–16]. In [13], for an OFDM-based CRN, an improved IWFA with partial CSI is proposed to maximize throughput of SU with total transmit power and average interference outage probability constraints. In [14], for a single-user CRN, power allocation strategies based on convex optimization techniques are proposed to achieve ergodic/outage capacity under outage probability constraint of PU and average/peak transmit power constraint of SU. In [15], for an OFDM-based CRN with one SU and one PU, a resource allocation problem with full and partial inter-system CSI is studied to maximize capacity of SU under peak/average interference probability constraints. The problem is solved by a modified IWFA. Extend to multiple SUs and one PU scenario, in [16], considering an OFDMA uplink CRN, a RPC scheme is proposed to maximize weighted sum rate under probabilistic interference constraint and maximum transmit power constraint of SUs where probabilistic constraint is transformed into a convex constraint by Bernstein method. In [17], an asynchronous distributed RPC algorithm is proposed to minimize total transmit power of SUs while outage probability constraints of SU and PU are considered. But user fairness is not considered. In [18], RPC algorithms are proposed to maximize sum rate of SUs with probabilistic interference constraint of PU and probabilistic SINR constraint of SU. Through Fenton-Wilkinson-type approximation, the stochastic optimization problem is converted into a tractable form solved by sequential GP. However, most of the above mentioned works with probability constraints only consider one kind of probabilistic constraints, such as probabilistic interference constraints (e.g., [13,15,16]), or outage capacity under outage probability constraint of PU [14]. And outage probability constraint of SU is less considered. Although probabilistic SINR and interference constraints are simultaneously considered in [18], the proposed centralized algorithm is not suitable for large scale networks. Additionally, the existing works focus on sum rate or network utility maximization for single user scenario. Some robust beamforming problems with power minimization are given in [19,20], few work has been done for RPC problem with energy efficiency by stochastic optimization approach in multiuer CRNs.

1.2 Main Contributions and Organization

In this paper, we consider power minimization problem for multiuser underlay CRNs and propose a robust distributed power control algorithm based on stochastic optimization. Specifically, considering channel gain uncertainties (e.g., SU-to-PU channel gain and SU-to-SU channel gain), both SINR constraints of SUs and IT constraints of PUs are formulated into probabilistic forms. Our objective is to minimize total transmit power of SUs while keeping satisfaction probability constraints above predetermined levels. To reduce computational complexity and obtain distributed solutions, coupled IT constraint is modified by weighted IT constraint. With exponential distribution model of Rayleigh fading channel, the robust optimization problem with probabilistic constraints is converted into a deterministic form solved by Lagrange dual method in a distributed way.

The rest of the paper is organized as follows. Section 2 introduces system model and non- robust resource allocation problem. Section 3 gives robust power control formulation.

Fig. 1 Multiuser underlay CRNs



A robust distributed power control algorithm is developed in Sect. 4. In Sect. 5, simulation results are presented to demonstrate the effectiveness of the proposed algorithm by comparing with several existing algorithms. Finally, Sect. 6 concludes the paper.

2 System Model

In this paper, we consider resource allocation problem for underlay CRNs with M secondary links and N primary links as shown in Fig. 1. Let $i \in A = \{1, 2, \ldots, M\}$ and $k \in B = \{1, 2, \ldots, N\}$. Each link consists of a pair of transmitter and receiver, and each node has single antenna. From Fig. 1, g_{ii} denotes direct channel gain from the ith SU-Tx to the ith SU-Rx, g_{ji} is interference gain from the jth SU-Tx to the ith SU-Rx, σ_i is background noise at the ith SU-Rx, h_{ik} is link gain from the ith SU-Tx to the kth PU-Rx, G_{ki} is interference gain from the kth PU-Tx to the ith SU-Rx. For each SU, actual SINR at the SU-Rx of link i is

$$\gamma_i = \frac{p_i g_{ii}}{\sum_{j \in A, j \neq i} p_j g_{ji} + \sum_{k \in B} P_k G_{ki} + \sigma_i} \tag{1}$$

where p_i denotes transmit power of the ith SU-Tx. P_k is transmit power of the kth PU-Tx. Let $n_i = \sum_{k \in B} P_k G_{ki} + \sigma_i$ denote interference plus noise except for SUs. First item is interference power from other SUs. Obviously, actual QoS of each SU is not only decided by its own transmit power p_i but also affected by other users.

To keep QoS of PUs, the aggregated interference from SUs at the kth PU-Rx should not exceed IT threshold

$$\sum_{i \in A} p_i h_{ik} \leq I_k, \quad k \in B \tag{2}$$

where I_k denotes maximum tolerated interference at the kth PU-Rx. The IT level is obtained by the method [21].

The optimization objective is to adjust transmit power of SUs in such a way that the following goals are simultaneously satisfied: (1) Total transmit power of SUs is minimized while transmit power of every SU-Tx is limited by battery capacity $p_i^{\max}$, and actual received SINR of each SU is maintained above a give value γ_i^d. (2) Interference to the kth PU-Rx is kept below a given threshold I_k. To realize these goals, resource allocation problem with perfect system information can be mathematically formulated as

$$\min \sum_{i\in A} p_i$$
$$\text{subject to} \begin{cases} p_i \le p_i^{\max}, & \forall i \in A \\ \sum_{i\in A} p_i h_{ik} \le I_k, & \forall k \in B \\ \gamma_i \ge \gamma_i^d, & \forall i \in A \end{cases} \tag{3}$$

where γ_i^d denotes minimum SINR requirement of each SU for keeping normal communication of SU. When transmit power of SUs reaches steady state (i.e., equilibrium point), actual received SINR of SU should be above γ_i^d. Otherwise, when actual SINR of SU is below γ_i^d, communication outage may appear. In addition, if channel gain and interference are accurately obtained by channel training, estimation and feedback mechanisms [14], problem (3) can be transformed into a GP problem solved by distributed way [22]. However, in practical communication environment, related parameter information is impossible to be exactly obtained due to random nature of wireless channels and errors. Therefore, the robustness of power allocation problem has to be considered.

3 Robust Problem Formulation

In practice, PUs do not have any obligation to provide any channel information to SUs [10]. Since channel gain is often obtained by channel state feedback or channel estimation techniques, quantization errors arise from limited feedback information and time-delay. Channel gain may be imperfectly known at SUs, since channel gain may be outdated due to rapid movement of users. If SUs utilize the outdated or un-exact CSI to update transmit power, the QoS of SUs and PUs may degrade.

To deal with these problems, there are some RPC algorithms with probability constraints [16] and [19]. However, channel uncertainties of SUs are not considered. Different from those works, in this section, a RPC problem under probabilistic SINR constraints and probabilistic interference constraints is formulated as

$$\min \sum_{i\in A} p_i$$
$$\text{subject to} \begin{cases} \Pr\left\{\sum_{i\in A} p_i h_{ik} \le I_k\right\} \ge \alpha_k, & (4a) \\ \Pr\left\{\gamma_i \ge \gamma_i^d\right\} \ge \beta_i, & (4b) \\ \forall i \in A, \forall k \in B \end{cases} \tag{4}$$

where $\Pr\{\cdot\}$ denotes probability operator. Transmit power of every SU should simultaneously satisfy $0 \le p_i \le p_i^{\max}, \forall i \in A$ [12]. (4a) and (4b) denote that satisfaction probabilities are bigger than α_k and β_i, respectively. $\alpha_k \in [0, 1]$ denotes the predetermined probability threshold that the received interference power at PU-Rx is limited by I_k.$\beta_i \in [0, 1]$ denotes the given probability that actual SINR of SU is maintained above γ_i^d. From (4a),the bigger α_k is, the more protection PUs has. Maximum allowable transmit power of SUs becomes small to overcome channel uncertainties between SU-Txs and PU-Rxs. From (4b), the bigger β_i is, the more transmit power the system provides. The received SINR of SUs stays above γ_i^d with probability β_i.

Since problem (4) is difficult to solve, similarly to [16], we need to convert it into a deterministic form where analytical solutions are available. Bernstein-type inequalities have been used to transform probability constraints into deterministic forms [16] and [19], however, the complexity and distributed solution of the algorithms are not considered.

4 Robust Distributed Power Control Algorithm

In this section, a robust distributed power control algorithm is proposed to solve problem (4). Since traditional centralized solutions are computationally intensive and require excessive coordination among users, we solve problem (4) in a distributed manner.

Due to near-far effect, average IT constraint (e.g., [22,23]) is rather conservative for QoS requirement of each SU. Thus we consider weighted IT constraint with user fairness as

$$p_i h_{ik} \le \omega_{ik} I_k, \quad \forall i \in A, \quad \forall k \in B \tag{5}$$

where $\omega_{ik} = d_i d_{ik} / (\sum_{j \in A} d_j d_{jk})$ represents weighted factor to balance performance of user. d_i denotes the distance between the ith SU-Tx to the ith SU-Rx. d_j denotes the distance between SU-Tx and SU-Rx of link j. d_{ik} is the distance between the ith SU-Tx to the kth PU-Rx. d_{jk} is the distance between the jth SU-Tx to the kth PU-Rx. All distance information of SUs can be obtained by global positioning system (GPS) or location awareness [24].

From (5), it is easy to know that ω_{ik} is proportional to the distance of the active user i (i.e., the ith SU). When channel state of the active user i is good (i.e., g_{ii}/n_i is big), ω_{ik} should be small to avoid harmful interference to PU. When channel environment of some user is bad, ω_{ik} should be bigger to provide higher transmit power of SUs. When SUs have same distance to primary base station, ω_{ik} keeps same for every SU in the network. The user under bad channel condition needs more relaxed upper transmit power level to guarantee its QoS requirement.

To avoid the impact of channel uncertainties, we guarantee performance of PUs by keeping the probability so that interference power becomes smaller than threshold I_k with certain probability α_{ik}. Under Rayleigh fading channel, h_{ik} is assumed to follow the exponential distribution given in [25] with mean $\bar{h}_{ik}$, i.e. $h_{ik} \sim \exp(\bar{h}_{ik})$. Therefore, (4a) becomes

$$\Pr_{h_{ik} \sim \exp(\bar{h}_{ik})} (p_i h_{ik} \le \omega_{ik} I_k) \ge \alpha_{ik} \tag{6}$$

From (6), we have

$$\begin{aligned} \Pr_{h_{ik} \sim \exp(\bar{h}_{ik})} (p_i h_{ik} \le \omega_{ik} I_k) &= \Pr_{h_{ik} \sim \exp(\bar{h}_{ik})} \left(h_{ik} \le \frac{\omega_{ik} I_k}{p_i} \right) \\ &= \int_0^{\frac{\omega_{ik} I_k}{p_i}} \frac{1}{\bar{h}_{ik}} \exp\left(-\frac{h_{ik}}{\bar{h}_{ik}} \right) dh_{ik} \\ &= 1 - \exp\left(-\frac{\omega_{ik} I_k}{p_i \bar{h}_{ik}} \right) \end{aligned} \tag{7}$$

Combing (6) with (7), we get

$$\ln\left(\frac{1}{1 - \alpha_{ik}} \right) p_i \bar{h}_{ik} \le \omega_{ik} I_k \tag{8}$$

Since $\ln\left(\frac{1}{1-x}\right)$ is monotonically increasing function with x, p_i becomes small with increasing α_{ik} to give more protection to PUs from (8).

In the same way, channel gain of SUs is assumed to follow exponentially independent distributed random variables with mean $\bar{g}_{ji}$. Then, we have

$$\Pr\left\{\gamma_i \geq \gamma_i^d\right\} = \prod_{j=1, j\neq i}^{M}\left(1+\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d\right)^{-1}\exp\left(-\frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}}\right) \tag{9}$$

The proof is given in "Appendix".

Combing (4b) and (9), probabilistic SINR constraint is rewritten as

$$\exp\left(-\frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}}\right)\prod_{j=1, j\neq i}^{M}\left(1+\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d\right)^{-1} \geq \beta_i \tag{10}$$

Taking natural logarithm of both sides, (10) becomes

$$\ln\frac{1}{\beta_i} \geq \frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}} + \sum\nolimits_{j\in A, j\neq i}\ln\left(1+\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d\right) \tag{11}$$

Since $\log(1+x) \leq x$, for $x > 0$ is hold, we have

$$\sum\nolimits_{j\in A, j\neq i}\ln\left(1+\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d\right) \leq \sum\nolimits_{j\in A, j\neq i}\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d \tag{12}$$

Therefore, we have

$$\frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}} + \sum\nolimits_{j\in A, j\neq i}\ln\left(1+\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d\right) \leq \frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}} + \sum\nolimits_{j\in A, j\neq i}\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d \tag{13}$$

Since $\frac{\gamma_i^d n_i}{p_i\bar{g}_{ii}} + \sum_{j\in A, j\neq i}\frac{p_j\bar{g}_{ji}}{p_i\bar{g}_{ii}}\gamma_i^d = \frac{\gamma_i^d}{p_i\bar{g}_{ii}}\left(n_i + \sum_{j=1, j\neq i}^{M} p_j\bar{g}_{ji}\right) = \frac{\gamma_i^d}{\bar{\gamma}_i}$ holds, according to (11) and (13), the probabilistic constraint can be simplified to a determinate form as

$$\frac{\gamma_i^d}{\bar{\gamma}_i} \leq \ln\frac{1}{\beta_i} \tag{14}$$

where $\bar{\gamma}_i$ is the estimated SINR at the ith SU-Rx. Under the nominal problem, actual SINR satisfies $\gamma_i \geq \gamma_i^d$. To maintain the performance of SUs under channel perturbation, there is $\ln\frac{1}{\beta_i} \leq 1$, or $\beta_i \geq \exp(-1) \approx 0.3679$. If β_i is bigger, it means that γ_i is greater than γ_i^d with high probability. As a result, more transmit power for the ith active SU is required to satisfy QoS of SUs under channel uncertainty.

According to (8) and (14), problem (4) becomes a convex form [26] as

$$\begin{aligned} &\min \sum\nolimits_{i\in A} p_i \\ &\text{subject to} \begin{cases} \hat{\alpha}_{ik} p_i \bar{h}_{ik} \leq \omega_{ik} I_k, & (15a) \\ \frac{\gamma_i^d}{\bar{\gamma}_i} \leq \hat{\beta}_i, & (15b) \\ \forall i \in A,\ \forall k \in B \end{cases} \end{aligned} \tag{15}$$

where $\hat{\alpha}_{ik} = \ln\left(\frac{1}{1-\alpha_{ik}}\right)$ and $\hat{\beta}_i = \ln\frac{1}{\beta_i}$. Due to the effect of parametric perturbation, transmit power of SUs needs to be carefully adjusted. From (15a), if $\hat{\alpha}_{ik}$ is small, maximum transmit power of SUs becomes bigger. From (15b), if $\hat{\beta}_i$ is small, there needs more transmit power to satisfy SINR constraint of SUs.

Considering the convexity of problem (15), the optimal solution can be obtained by Lagrangian dual function [26] as

$$J\left(\{p_i\},\{\lambda_{ik}\},\{\mu_i\}\right) = \sum\nolimits_{i\in A} p_i + \sum\nolimits_{k\in B}\sum\nolimits_{i\in A} \lambda_{ik}\left(\hat{\alpha}_{ik} p_i \bar{h}_{ik} - \omega_{ik} I_k\right) + \sum\nolimits_{i\in A} \mu_i \left(\frac{\gamma_i^d}{\bar{\gamma}_i} - \hat{\beta}_i\right) \quad (16)$$

where $\lambda_{ik} \geq 0$ and $\mu_i \geq 0$ denote Lagrange multipliers. The dual function of original problem is expressed as

$$D\left(\{\lambda_{ik}\},\{\mu_i\}\right) = \min_{0\leq p_i \leq p_i^{\max},\forall i} J\left(\{p_i\},\{\lambda_{ik}\},\{\mu_i\}\right) = \sum\nolimits_{i\in A} \min_{0\leq p_i\leq p_i^{\max}} J_i\left(p_i,\lambda_{ik},\mu_i\right) - \sum\nolimits_{k\in B}\sum\nolimits_{i\in A}\lambda_{ik}\omega_{ik} I_k - \sum\nolimits_{i\in A}\mu_i \hat{\beta}_i \quad (17)$$

with each user optimization problem as

$$J_i\left(p_i,\{\lambda_{ik}\},\mu_i\right) = p_i + \sum\nolimits_{k\in B}\lambda_{ik}\hat{\alpha}_{ik} p_i \bar{h}_{ik} + \frac{\mu_i \gamma_i^d}{\bar{\gamma}_i} \quad (18)$$

Lagrange dual function $D(\{\lambda_{ik}\}, \{\mu_i\})$ in (17) can be evaluated for fixed$(\{\lambda_{ik}\}, \{\mu_i\})$. We need to solve

$$\begin{aligned} &\max\ D\left(\{\lambda_{ik}\},\{\mu_i\}\right) \\ &\text{subject to}:\quad \lambda_{ik}\geq 0,\ \mu_i \geq 0,\ \forall i\in A, \forall k \in B \end{aligned} \quad (19)$$

It is shown that (18) is a convex function with respect to p_i. According to KKT condition [26], the optimal transmit power can be obtained by solving $\partial J_i(p_i, \{\lambda_{ik}\}, \mu_i)/\partial p_i = 0$. On the basis of the maximum transmit power constraint, the optimal power allocation can be expressed as

$$p_i^{opt} = \min\left(p_i^{\max}, \sqrt{\frac{\mu_i\gamma_i^d z_i}{1+\sum_{k\in A}\lambda_{ik}\hat{\alpha}_{ik}\bar{h}_{ik}}}\right) \quad (20)$$

where $z_i = (\sum_{j\in A, j\neq i} p_j \bar{g}_{ji} + n_i)/\bar{g}_{ii}$ denotes the estimated CSI of link i.

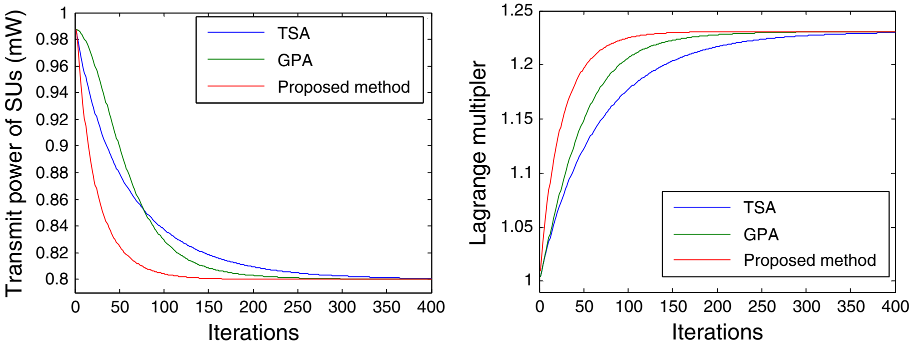
To obtain distributed solutions of problem (15), dual variables can be updated by using sub-gradient method as

$$\lambda_{ik}(t+1) = \left[\lambda_{ik}(t) + a_1\left(\hat{\alpha}_{ik} p_i \bar{h}_{ik} - \omega_{ik} I_k\right)\right]^+ \quad (21)$$

$$\mu_i(t+1) = \left[\mu_i(t) + a_2\left(\gamma_i^d/\bar{\gamma}_i(t) - \hat{\beta}_i\right)\right]^+ \quad (22)$$

where $[\mathrm{X}]^+ = \max\{0, \mathrm{X}\}$. a_1 and a_2 are nonnegative step sizes. While t denotes step time. Note that as long as a_1 and a_2 are sufficiently small, λ_{ik} and μ_i can converge to the optimal value λ_{ik}^*, μ_i^* as $t\to\infty$ [27]. To improve convergence speed of algorithm, transmit power is updated by using forgetting factor

$$p_i(t+1) = \min\left\{p_i^{\max}, \max\left\{0, (1-a_3)p_i(t) + a_3\sqrt{\frac{\mu_i\gamma_i^d z_i}{1+\sum_{k\in B}\lambda_{ik}\hat{\alpha}_{ik}\bar{h}_{ik}}}\right\}\right\} \quad (23)$$



(a) Transmit power **(b)** Lagrange multiplier

Fig. 2 Convergence analysis between different algorithms

where $a_3 \in (0, 1)$ denotes the forgetting factor. The convergence of the update approach is given in [28].

Lagrange multipliers λ_{ik} and μ_i are locally updated without cooperation of users by (21) and (22). The robust distributed power control algorithm can be summarized as follows

1. Initialization: $t = 0, \lambda_{ik}(0) \geq 0, \mu_i(0) \geq 0, 0 < p_i(0) < p_i^{\max}, a_1, a_2, a_3 \in (0, 1), \alpha_{ik}, \beta_i \in (0, 1), \forall i, k$.
2. Calculation at SU-Rx: measure SINR γ_i, update Lagrange multiplier $\mu_i(t+1)$ by (22), feedback information γ_i and μ_i to the SU-Tx of same link.
3. Calculation at SU-Tx: receive γ_i and μ_i and estimate $\bar{h}_{ik}$, update $\lambda_{ik}(t+1)$ by (21), calculate $p_i(t+1)$ by (23), and broadcast p_i.
4. Convergence: if the transmit power vector $\mathbf{p} = [p_1, \ldots, p_M]^T$ satisfies $\|\mathbf{p}(t+1) - \mathbf{p}(t)\| \leq \zeta$ (ζ is an error tolerance factor), stop iteration; Otherwise, go to step 2

5 Simulation Results and Discussion

In this section, the effectiveness of the proposed RPC algorithm is demonstrated by comparison with the worst case approach [12] and non-robust approach. In the following simulation, for convenience but without losing generality, we assume α_{ik} and β_i are equal for all users, and denoted as α and β, respectively. The maximum transmission power is $p_i^{\max} = 1$ mW. $\bar{g}_{ii}$, $\bar{g}_{ji}$ and $\bar{h}_{ik}$ are randomly chosen from intervals [0,1], [0,0.1], and [0,1], respectively. The allowable deviation is $\zeta = 10^{-5}$ and background noise is $\sigma_i = 0.01$ mW.

Figure 2 shows convergent performance of transmit power of SUs and Lagrange multipliers λ of the different algorithms. Specifically, the classical distributed algorithm with coupled IT constraint of [22] is denoted by traditional sub-gradient algorithm (TSA). The distributed iterative gradient based update algorithm of [10] is expressed as gradient projection algorithm (GPA). We assume there are two SUs and one PU in the network. The target SINR is $\gamma_i^d = [4, 5]^T$ dB. The IT threshold is 0.02 mW. The satisfaction probability of SU and PU is $\alpha = 0.1$ and $\beta = 0.9$, respectively.

From Fig. 2, the proposed algorithm can converge to the equilibrium point faster than both TSA and GPA, since traditional dual update method may not guarantee the dual gap of SINR constraint shrinking to zero due to discrete rate operation. Moreover, the introduction of auxiliary variables needs more time to exchange information. And the coupled IT constraint

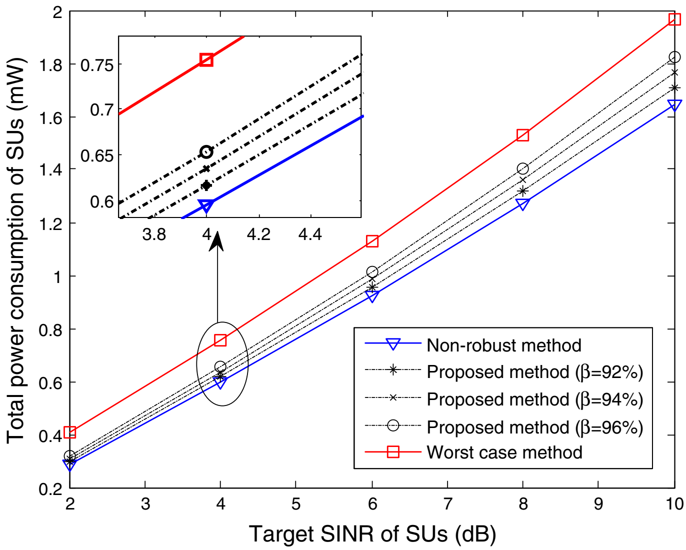


Fig. 3 Total transmit power versus target SINR γ^d

forces each user waiting for channel gain of other SUs before updating power command. In addition, GPA needs to measure interference and channel gain from other users and project a vector related with the feasible set. It is also clear that the proposed method can achieve perfect convergence performance because of the globally distributed solutions. Each SU does not need to collect all CSI in the network.

Figure 3 shows total power consumption versus target SINR for different β. In this simulation, each SU has same QoS requirement. The interference from PU-Tx to SU-Rx is $2\sigma_i$ for every PU. The IT constraint is 0.02 mW. The probability value of PUs is $\alpha = 0.8$. There are two SUs and three PUs in the network.

From Fig. 3, total power consumption of SUs monotonously increases with the increasing target SINR γ^d, since it needs more transmit power to satisfy basic QoS requirement under the uncertainty. Moreover, high satisfaction probability can guarantee low outage probability of SUs, which needs more transmit power of SU. The worst case method needs the most transmit power to overcome the channel uncertainty effect, since it is more conservative than the statistical approach. From Fig. 3, the non-robust method can achieve minimization of transmit power. But the communication quality may not be satisfied due to channel perturbation, which is verified in the following results.

Figure 4 depicts satisfaction probability of PU versus target SINR. For a simple scenario, we assume there are one SU and one PU in the network. And the satisfaction probability of PU is $\alpha = 75\,\%$. The interference threshold is 0.006 mW.

From Fig. 4, the satisfaction probability of PU is high when the minimum required SINR at SU is small, since the probability SINR constraint limits the transmit power of SU. Specifically, in the low SINR regime, the satisfaction probability is $\alpha = 100\,\%$, much higher than $\alpha = 75\,\%$. The reason is that small SINR threshold requires low transmit power to maintain QoS requirement of SU. In addition, the interference received at PU-Rx is very small due to the low transmit power of SU, which can keep the probability $\Pr\left(\sum_{i=1}^{M} p_i h_{ik} \le I_k\right)$ satisfying all channel realization. If the target γ_i^d continuously increases under interference

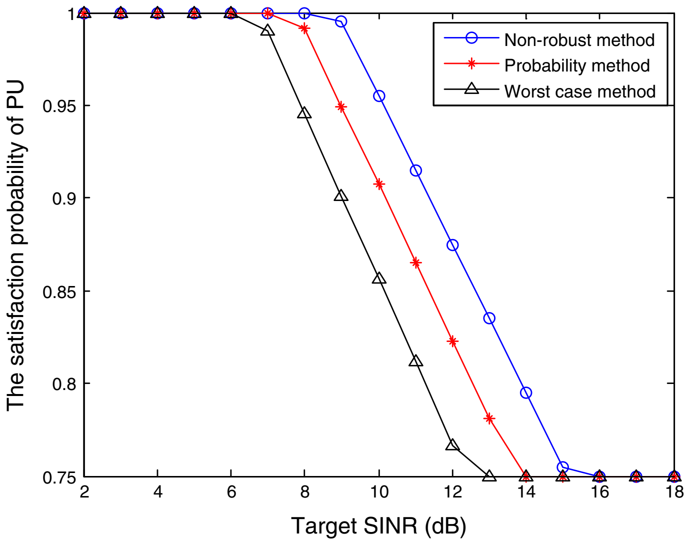


Fig. 4 Satisfaction probability of PU versus target SINR

temperature limited regime, the satisfaction probability is saturated to the target. Under worst case channel uncertainty in SINR, the method needs to adjust transmit power to suppress the channel uncertainty effect from Fig. 3, which may bring more harmful interference to PU and result in worst satisfaction probability of PU or lead to a higher outage probability of PU.

But the non-robust method can improves satisfaction probability of PU. And we can find that it not only consumes less power from Fig. 3, but also has high satisfaction probability from Fig. 4. Unfortunately, it can not keep QoS requirement of SUs all time, which will be discussed in following case. Moreover, the final satisfaction probability tends to the target value due to the target probability constraint of PU.

In order to investigate the impact of channel estimation errors, both the channels between SU-Tx and PU-Rx and the channels among SUs are considered. The channel model can be expressed as

$$h_{ik} = \bar{h}_{ik} + \Delta h_{ik} \tag{24}$$

$$g_{ji} = \bar{g}_{ji} + \Delta g_{ji} \tag{25}$$

where h_{ik} and $\bar{h}_{ik}$ are the true and the estimated fading channel coefficients between SU-Tx and PU-Rx, respectively. Similarly, g_{ji} and $\bar{g}_{ji}$ are the true and the estimated coefficients of the fading channel among SUs. Δh_{ik} and Δg_{ji} denote the estimated errors assumed to be uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. If the value ε is small, the estimated channel $\bar{h}_{ik}$ and $\bar{g}_{ji}$ are close to the true channel coefficients, and vice versa. Here we assume $\beta = 90\,\%$ which implies that the actual SINR satisfaction probability is not less than 90 %. The minimum SINR is $\gamma_i^d = 4\,\text{dB}$. After the system goes in steady state, we assume there are some randomly channel uncertainty with an upper bound $\varepsilon = 0.05$. The received SINR of SUs is given in Fig. 5.

From Fig. 5, it is obvious that the performance of non-robust method is the worst one. When the actual SINR (i.e, γ_i) at SU-Rx is below the threshold γ_i^d, communication outage may appear. For example, if direct channel gain is overestimated, e.g., $\bar{g}_{ii} > g_{ii}$, the SINR

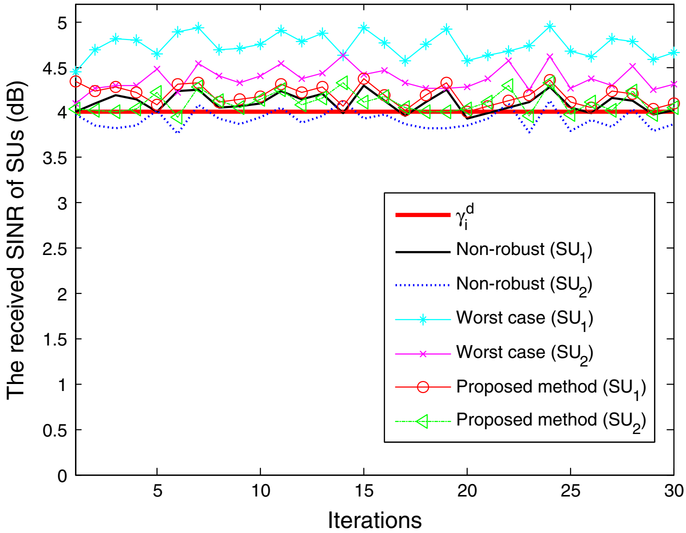


Fig. 5 Actual received SINR at SU-Rx under channel realizations

without consideration of uncertainty (i.e., non-robust method) may be below the target value. From Fig. 5, the average outage probability of non-robust method is 41.67 % under most of channel realizations. The QoS of SUs under this method seriously degrades. While the worst case method can reach actual SINR satisfaction probability to 100 %, namely, the outage probability is zero. Thus the worst case approach can protect the performance of all SUs under any error at the cost of higher power consumption (e.g., Fig. 3). But our proposed method is less conservative according to the power consumption in Fig. 3, and the actual SINR satisfaction probability can reach 93.33 %.

To show the effect of interference weighted factor ω and the satisfaction probability α, it is important to examine the SINR performance of SUs and transmission rate of users. Due to the near-far effect and the existence of uncertainty, it is necessary for us to consider user fairness and system robustness. And, it is also necessary to avoid that selfish user under good channel state transmits more power which will affect the performance of user under imperfect channel. However, for transmit power minimization problem, each SU transmits as less power as possible to induce less mutual interference.

Figure 6 shows the throughput of SUs versus the channel estimated error between SU-Tx and PU-Rx. Channel gains among SUs are assumed to be perfect. The SINR threshold is 4 dB. The IT level is 0.2 mW and the satisfaction probability of PU is $\alpha = 65\,\%$. The perturbation model (24) is considered with an upper bound of worst case error $\varepsilon = \max\{\Delta h_{ik}\}$.

From Fig. 6, the throughput of SUs decreases with the increasing channel estimation error, since the range of transmit power becomes tightened to protect the QoS of PUs. Since the non-robust method does not consider the estimation error between SU and PU, although the throughput of the non-robust method is bigger than that of two robust methods due to the big range of transmit power. However, the robustness of the non-robust method is low shown in Fig. 5. Specifically, when channel gain h_{ik} is underestimated (i.e., $\bar{h}_{ik} < h_{ik}$), actual interference power at PU-Rx may exceed the IT level. Since the worst case method considers any possible error in a bounded set, it is more conservative than the proposed method. As a result, the throughput of the proposed method is bigger than that of the worst case approach.

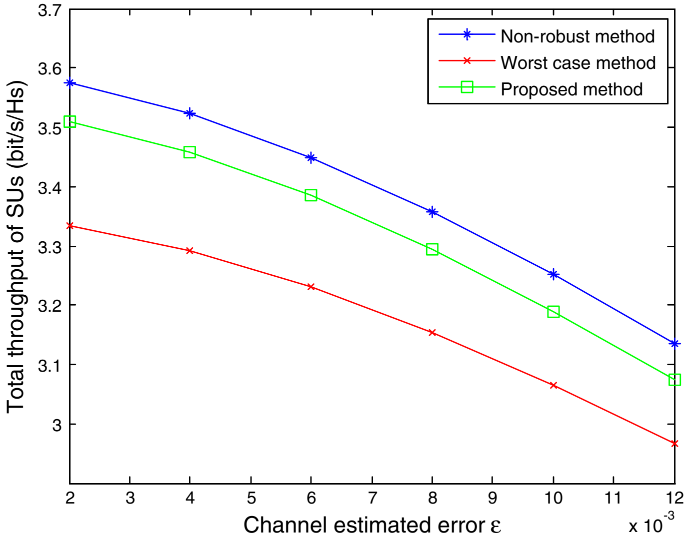


Fig. 6 Throughput of secondary system versus channel estimation error

6 Conclusions

In this paper, we solve robust transmit power minimization problem for underlay CRNs with multiple SUs and multiple PUs under parametric uncertainties. In the problem formulation, the introduced probability constraint of PUs represents the aggregated interference power from SUs below an IT threshold to guarantee QoS of PUs. The probabilistic SINR constraint of SUs is maintained at a certain level. In order to obtain a globally distributed solution, the coupled IT constraint is modified by the weighted IT constraint with fairness. Under exponential distribution of stochastic channel gain, RPC problem is transformed into a tractable convex one solved by dual decomposition method. Simulation results show that in terms of the total transmit power and QoS of SUs, the performance of our approach is better than that of the worst case method and non-robust design.

Acknowledgments This work is supported by the National Natural Science Foundation of China under Grant No. 61171079. We thank the reviewers for their detailed, constructive and valuable reviews and comments.

Appendix

Proof of Equation (9)

For simplicity, we define two independent random variables $A = \frac{p_i g_{ii}}{n_i}$ and $B_j = \frac{p_j g_{ji}}{n_i}$. According to probability distribution, A and B_j follow exponential distribution with mean $\delta_i = \frac{p_i \bar{g}_{ii}}{n_i}$ and $\delta_j = \frac{p_j \bar{g}_{ji}}{n_i}$ respectively. The probabilistic SINR constraint is expressed as

$$\Pr\left(\frac{p_i g_{ii}}{\sum_{j=1, j\neq i}^{M} p_j g_{ji} + n_i} \geq \gamma_i^d\right) = \Pr\left(\frac{A}{1 + \sum_{j=1, j\neq i}^{M} B_j} \geq \gamma_i^d\right)$$

$$
\begin{aligned}
&= \Pr\left(A \geq \gamma_i^d + \gamma_i^d \sum\nolimits_{j=1, j\neq i}^{M} B_j\right) \\
&= \int_0^{+\infty} \cdots \int_0^{+\infty} \left(\int_{\gamma_i^d + \gamma_i^d \sum_{j=1, j\neq i}^{M} b_j}^{+\infty} \frac{1}{\delta_i} \exp\left(-\frac{a}{\delta_i}\right) da \right) \\
&\quad \times \left\{ \prod_{j=1, j\neq i}^{M} \frac{1}{\delta_j} \exp\left(-\frac{b_j}{\delta_j}\right) \right\} db_1 \cdots db_M \\
&= \int_0^{+\infty} \cdots \int_0^{+\infty} \exp\left(-\frac{\gamma_i^d \left(1 + \sum_{j=1, j\neq i}^{M} b_j\right)}{\delta_i}\right) \\
&\quad \times \left\{ \prod_{j=1, j\neq i}^{M} \frac{1}{\delta_j} \exp\left(-\frac{b_j}{\delta_j}\right) \right\} db_1 \cdots db_M \\
&= \exp\left(-\frac{\gamma_i^d}{\delta_i}\right) \int_0^{+\infty} \cdots \int_0^{+\infty} \left(\prod_{j=1, j\neq i}^{M} \frac{1}{\delta_j} \right. \\
&\quad \times \left. \exp\left(-\left(\frac{\gamma_i^d \sum_{j=1, j\neq i}^{M} b_j}{\delta_i} + \frac{b_j}{\delta_j}\right)\right)\right) db_1 \cdots db_M \\
&= \exp\left(-\frac{\gamma_i^d}{\delta_i}\right) \prod_{j=1, j\neq i}^{M} \int_0^{+\infty} \frac{1}{\delta_j} \exp\left(-\left(\frac{1}{\delta_j} + \frac{\gamma_i^d}{\delta_i}\right) b_j\right) db_j \\
&= \exp\left(-\frac{\gamma_i^d}{\delta_i}\right) \prod_{j=1, j\neq i}^{M} \frac{1}{\delta_j} \left(\frac{1}{\delta_j} + \frac{\gamma_i^d}{\delta_i}\right)^{-1} \\
&= \exp\left(-\frac{\gamma_i^d}{\delta_i}\right) \prod_{j=1, j\neq i}^{M} \left(1 + \frac{\delta_j \gamma_i^d}{\delta_i}\right)^{-1}
\end{aligned} \tag{26}
$$

Substituting the variables $\delta_i = \frac{p_i \bar{g}_{ii}}{n_i}$ and $\delta_j = \frac{p_j \bar{g}_{ji}}{n_i}$ into (26), we get

$$
\Pr\left\{ \frac{p_i g_{ii}}{\sum_{j=1, j\neq i}^{M} p_j g_{ji} + n_i} \geq \gamma_i^d \right\} = \exp\left(-\frac{\gamma_i^d n_i}{p_i \bar{g}_{ii}}\right) \prod_{j=1, j\neq i}^{M} \left(1 + \frac{p_j \bar{g}_{ji}}{p_i \bar{g}_{ii}} \gamma_i^d\right)^{-1} \tag{27}
$$

The proof is completed.

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