Economic Modelling 33 (2013) 926–939

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Can signal extraction help predict risk premia in foreign exchange rates

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ARTICLE INFO

Article history:
Accepted 5 June 2013

JEL classification:
C5
F31
G12

Keywords:
Forward foreign exchange rates
Non-normality
Risk premium
Spot foreign exchange rates
State space model
Volatility persistence

ABSTRACT

The present study investigates possible existence of time varying risk premia in Brazilian real, Chinese yuan; Cypriot pound, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar. Exchange rates in these series are modeled using non-Gaussian state space models that encompass non-normality and GARCH-like affects.

The results show statistically significant evidence of time varying risk premium in all the series. Following Wolff (1987), the results from Gaussian versions of the state space models are not much different. Moreover, statistically significant evidence of volatility clustering is realized in all the series. Additional tests reveal that exclusion of conditional heteroskedasticity from the forecasting models leads to false statistical inferences in favor of no time varying risk premium in most of the series.

1. Introduction

During the fixed exchange rates system of Bretton Woods era, there was no demand for predicting forward foreign exchange rates. However, soon after the breakdown of the Bretton Woods system in 1973, when the fixed exchange rates system was replaced with a flexible exchange rate system, the quest for exchange rates forecasts intensified over time because of the risks associated with the variations in flexible exchange rate system.

There is a wide body of empirical literature that reveals existence of risk premia in forward foreign exchange rates (Lewis, 1995; and Engel, 1996 among others). However, the empirical models used by the studies seeking to predict possible existence of risk premia in forward foreign exchange rates do not necessarily account for the presence of time varying volatility and non-normality that might be present in the series. This is probably because the inclusion of such features in the forecasting models raises convergence issues that cause difficulties in estimation. Therefore, the present contribution seeks to predict possible existence of risk premia in the selected foreign exchange rate markets using state space model that incorporates non-normality and GARCH-like affects.

Because of their flexibility, the state space models were used for a diverse nature of applied work. For instance, Chan et al. (2001) used them for detecting rational bubbles in residential housing markets of Hong Kong, Pollock (2001) for trend estimation, and Lungu et al. (2008) in a rational expectations macroeconomic model. State space models were also used for forecasting exchange rates. For example, Wolff (1987–2000), Hai et al. (1997), Bhar et al. (2002), Bidarkota (2004), and Kiani (2009) employed state space models for measuring and characterizing the time series properties and the size of risk premia in foreign exchange rates.

The regression based approach[1] requires regressing changes in spot exchange rates on forward premium although the choice of the explanatory variables is often arbitrary (Hansen and Hodrick (1980). On the other hand, Wolff (1987) and Nijman et al. (1993) obviated from the specification of explanatory variables assuming that the errors come from the normal distribution. However, Boothe and Glassman (1987), Tucker and Pond (1988) and So (1987) showed that forward foreign exchange rates were non-Gaussian. Moreover, Frankel and Rose (1997) demonstrated that time varying volatility was present in forward foreign exchange rates. Therefore, excluding the features that account for non-normality and time varying volatility from the models employed for forecasting foreign exchange rates would result in estimation inefficiencies. Therefore, Backus et al. (1993), and Engel (1996) recommend including these features in forecasting models to obtain accurate estimation and characterization of time series properties of risk premia in foreign exchange rates.

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[1] While Fama (1984), Lewis (1995), Berkowitz and Giorgianni (2001), and McCraken and Stephen (2002) used regression techniques, Canova and Ito (1991) vector autoregressions, and Froot and Frankel (1989) employed survey based methods for characterizing time series properties and the size of risk premia in foreign exchange rates.

0264-9993/$ – see front matter
http://dx.doi.org/10.1016/j.econmod.2013.06.005

Considering the foregoing deliberations on the predictability of foreign exchange rates, and the fact that nonlinearities are prevalent in foreign exchange rate markets (Hong and Lee, 2003; Hsiech, 1989) the present study seeks to predict possible existence of risk premium in Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against the US dollar[2] using non-Gaussian state space models that encompass features to account for non-normality and time varying that may be present in the series. The remaining study is organized as follows. In Section 2 state space or signal plus noise or time varying risk premium model is discussed. Empirical results are shown in Section 3 and hypothesis tests are presented in Section 4. Finally, Section 5 shows the conclusion.

2. Time series models for forecasting risk premia

This section explains the non-Gaussian state space model that encompasses non-normality and GARCH-like effects. The most general model and its restricted versions are elaborated in the following sub-sections.

2.1. A state space model

State space models have become widespread in finance for well over the last decade and a half. These models are elaborated in Brockwell and Davis (1991), Harvey (1989), Hamilton (1994), Kitagawa and Gersch (1996), West and Harrison (1997), Kim and Nelson (1999), and Shumway and Stoffer (2000) including others. Many dynamic time series models in economics and finance may be represented in state space form. Some common examples of these models are autoregressive moving average (ARMA) models, time varying regression models, dynamic linear models with unobserved components, discrete version of continuous time diffusion process, stochastic volatility (SV) models, and nonparametric and spline regressions. The linear Gaussian state space model is represented in the system of equations as follows:

$$y_t = Z_t\alpha_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t), \tag{1}$$

$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t, \quad \eta_t \sim N(0, Q_t), \quad i-1,2, \ldots\ldots n \tag{2}$$

$$\alpha_1 \sim N(a_1, P_1),$$

In the above equations, the matricesH_t, Z_t, Q_t, and T_t are known. In this set-up in the beginning a_1 and P_1 are assumed to be known. Here y_t is a vector of observations of dimension $p * 1$ where α_t is $m * 1$ vector of unobserved values that are known as state. The disturbances ε_t and η_t are independent of each other and are normally distributed. The matrix R_t is a selection matrix particularly when it is not an identity matrix. This matrix is employed only when the dimensions of the α_t matrix are greater than the disturbance matrix η_t. In the above set−up, Eq. (1) is an observation equation and Eq. (2) is the state equation. Although very simple, it is a flexible model that has a wide range of applications in empirical time series analysis.

The major advantage of space state models is that they enable modeling behavior of diverse components of the series under investigation. These are of a general class of models that can clasp a wide variety of applications. State space models encompass two classes of variables one of which is the observed state variable that describe the development of the system over time. The present work considers departure from normality both for the state variables and for the conditional distribution of the observation given the state. The primary intent for the state is the heavy-tailed distributions which enables to model structural shifts. The class of observation that is considered for the conditional densities includes both exponential family distribution and heavy-tailed densities since the heavy-tailed densities allow to model outliers. The exponential densities allow modeling count data as well as skewed data.

Assuming that f_t denotes contemporaneous forward exchange rates, s_t spot exchange rates, the risk premia in forward foreign exchange rates can be elaborated as follows. The difference between f_t^{t+1} (future forward rates) and s_{t+1} (future spot rates) can be represented by an unobserved component p_t and an error term v_{t+1} which is shown in Eq. (3). Accordingly, Eq. (3) is an observation equation where Eq. (4) is the state equation that jointly constitutes a state space model.

$$f_t^{t+1} - s_{t+1} = p_t + v_{t+1} \tag{3}$$

$$(p_t - \mu) = \phi(p_{t-1} - \mu) + \eta_t \tag{4}$$

Replacing $f_t^{t+1} - s_{t+1}$ with y_{t+1} in the left hand side of Eq. (3) yields Eq. (5).

$$y_{t+1} = p_t + v_{t+1} \tag{5}$$

$$(p_t - \mu) = \phi(p_{t-1} - \mu) + \eta_t \tag{6}$$

where, Eq. (5) above shows that the observed variable y_{t+1} comprises of a signal of interest p_t and a noisy component v_{t+1} which essentially is the difference between $E(s_{t+1})$ and s_{t+1}. Finally the error term v_{t+1} shown in Eq. (5) and the error term η_t presented in Eq. (6) are mutually exclusive.

Canova and Ito (1991), as well as Engel (1996) showed the existence of risk premium in forward foreign exchange rates. Wolff (1987) and Nijman et al. (1993) including others showed that dynamics of the risk premium (p_t) can be characterized by first order autoregressive process. Evidence of volatility clustering and fat tails in spot exchange rates was established by Boothe and Glassman (1987), and Tucker and Pond (1988). Likewise, the existence of risk premia and volatility clustering in forward foreign exchange was established by So (1987). Therefore, to account for fat tails and time varying volatility that might be present in the data series stable distribution and GARCH-like effects are incorporated in the state space models employed in the present study. Consequently, the errors v_{t+1} are modeled as $v_{t+1} \equiv c_t z_{1t+1}$ and $z_{1t+1} \sim \text{iid} \quad S_\alpha(0,1)$ where, S_α denotes symmetric stable distribution as was modeled by Bidarkota (2004).

A symmetric stable distribution $S_\alpha(0,1)$ would exist for a random variable X, if its log−characteristic function could be expressed as $\ln E \exp(iXt) = i\delta t - |ct|^\alpha$. The parameter $c > 0$ measures scale, $\delta \in (-\infty, \infty)$ measures the location, and the characteristic exponent $\alpha \in (0,2]$ governs the tail behavior of the distribution. Small values of α show thicker tails, but when α equals 2 normal distribution results with a finite variance that equals to $2c^2$. The term c_t captures volatility clustering which can be shown from the following equation representing GARCH (1, 1)-like process.

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |y_t - E(y_t | y_1, y_2, \ldots\ldots, y_{t-1})|^\alpha \tag{7}$$

where, in Eq. (7) above the restrictions $\omega > 0, \beta \geq 0,$ *and* $\delta \geq 0$ are imposed for obtaining log likelihood estimates of the restricted models to construct likelihood ratio test statistics for testing various hypotheses of interest. However, GARCH-like formulation of Eq. (7) reduces to GARCH-normal process when errors are normal. The error term η_t shown in the state Eq. (6) is modeled as $\eta_t \equiv c_t\eta_t z_{2t}$ where $z_{2t} \sim iid \; S_\alpha(0{,}1)$. This term is independent of z_{1t+1} at all

[2] Initially I started analyzing exchange rate series for 62 currencies against US dollar, however, exchange rates for the ten economies i.e. Brazil, China, Cyprus, Denmark, Eurozone, France, India, Japan, Pakistan, and United Kingdom are included in the study since most of the employed state space models and their restricted versions converged, therefore, the remaining series were not included. After adopting Euro as their official currency the discussion about Cypriot pound becomes redundant, however, there being insufficient data for euro, the exchange rate data from Cyprus and France are also included in the analysis.

leads and lags. The term c_η is the signal to noise scale ratio which can either be equal to or greater than zero.

2.2. Non-Gaussian state space model

The most general non-Gaussian state space model that encompasses stable distribution, GARCH-affects, and predictable component p_t is shown in the following three equations.

$$y_{t+1} = p_t + v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim iid\, S_\alpha(0,1), \tag{8}$$

$$(p_t - \mu) = \phi(p_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_t z_{2t}, \quad z_{2t} \sim iid\, S_\alpha(0,1) \tag{9}$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |y_t - E(y_t | y_1, y_2, \ldots\ldots, y_{t-1})|^\alpha \tag{10}$$

where, Eq. (8)in the model shown above is an observation equation, Eq. (9) is a state equation, and Eq. (10) represents the GARCH formulation of the non-Gaussian state space model.

Model 2 which is shown in the following two equations is obtained by restricting predicable component (ϕ) in the most general state space model.

$$y_{t+1} = \mu_t + v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim iid\, S_\alpha(0,1) \tag{11}$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |y_t - E(y_t | y_1, y_2, \ldots, y_{t-1})|^\alpha \tag{12}$$

Restricting constancy in risk premium ($\mu = 0$) in model 2 results in model 3 which is presented in the following two equations.

$$y_{t+1} = v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid}\, S_\alpha(0,1) \tag{13}$$

$$c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta |y_t|^\alpha \tag{14}$$

Model 4 is obtained restricting time varying volatility $\beta = \delta = 0$ in model 3. Model 4is shown in Eq. (15).

$$y_{t+1} = v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid} S_\alpha(0,1) \tag{15}$$

Restricting time varying volatility in the most general model gives model 5 which is presented in the following two equations.

$$y = p_t + v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid}\, S_\alpha(0,1)\,, \tag{16}$$

$$(p - \mu) = \phi(p_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_t z_{2t}, \quad z_{2t} \sim \text{iid}\, S_\alpha(0,1) \tag{17}$$

Finally, restricting constancy of risk premium in model 2 gives model 6 which is shown in Eq. (18).

$$y_{t+1} = p_t + v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid}\, S_\alpha(0,1) \tag{18}$$

2.3. Estimation issues

The non-Gaussian state model which is shown in Eqs. (8)–(10) poses difficulty in estimation even when the features to account for conditional heteroskedasticity are not included in the model. Moreover, because of the non-Gaussian nature of the models, the errors come from a non-normal family. Hence, the powerful Kalman filter is not an appropriate estimation algorithm anymore. Therefore, a smoother version of the formulae due to Kitagawa (1987) is employed to implement the general recursive algorithm due to Sorenson and Alspach (1971) for filtering and predicting densities for an error under any given distribution. This algorithm also provides a formula for computing log likelihood function. The recursive equation for computing the filtering and predictive densities is given in the form of integrals of which the close form analytical expressions are generally intractable except in very special cases. Therefore, these integrals are numerically evaluated as was done in Bidarkota (2004).

Evidence of asymmetries in risk premia was established by Canova and Ito (1991), Hsiech (1989), as well as Hong and Lee (2003). However, Zolotarev (1986) showed that integral representation can be used to evaluate stable distribution and density function. Stable distribution can also be evaluated using inverse Fourier transformation of the characteristic function. However, the present study restricts itself to symmetric stable distribution which employs fast numerical approximations due to McCulloch (1996) that have an expected relative density to the precision of 10^{-6} for $\alpha \in [0.84,2]$. Therefore, the characteristic exponent (α) is restricted in this range which provides computational suitability.

In GARCH (1,1) models the impact of the initial values of GARCH volatility on the parameter estimates is asymptotically negligible as was demonstrated by Lumsdaine (1996). Nevertheless, Diebold and Lopez (1995) suggested that at the first iteration, the initial conditional variance be set equal to sample variance and at the following iterations equal to the sample variance from a simulated realization with the estimated parameters from the previous iteration equal to $2c_0^2$, when it does exist. Alternately, Engle and Bollerslev (1986) suggested initialization of the GARCH process using the estimate of c_0 based on sample values. Therefore, the value of c_0 is set equal to its unconditional value that is obtained from the volatility process shown in Eq. (10).

3. Empirical results

3.1. Data sources

The present research employs daily foreign exchange rates for Brazilian real, Chinese yuan, Cypriot pound[3], Danish krone, Eurozone euro, French franc[4], Indian rupee, Japanese yen, Pakistani rupee, and British pound against US dollar that are obtained from DataStream. The corresponding forward foreign exchange rates for Brazilian real, Chinese yuan, Cypriot pound, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound against US dollar are also obtained from DataStream. Additional information on both the daily spot as well as forward foreign exchange rates for all the series is presented in Table 1.

3.2. Estimation results

The maximum likelihood (ML) estimates for the most general non-Gaussian state space model and its restricted versions (models 2–6) are shown respectively in columns 2–7 row 9 in Tables 2.1–2.10 for non-Gaussian state space models and in Tables 3.1–3.10 for Gaussian state space models for Brazilian real, Chinese yuan, Cypriot pound, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound against the US dollar. The results presented in these tables show estimates of mean risk premium μ ARCH parameter δ volatility persistence parameter β autoregressive coefficient of the risk premium = ϕ signal to noise scale ratio c_η and characteristic exponent α.

4. Hypothesis tests

The hypothesis tests employed in the present study include the null hypothesis of constant risk premium, no risk premium, and homoscedasticity. Additional hypothesis tests of time varying risk premium, and constancy of risk premium are also tested to evaluate the impact of the exclusion of various features from the most general model employed. Estimates from the non-Gaussian and Gaussian

[3] Cyprus has been using Cypriot pound as its currency until January 2008 whereupon it switched to Euro as its national currency.

[4] France was among the eleven countries that adopted euro as their official currency in 1999.

Table 1
Data description.

Country	Exchange rates	Forward rates	Observations
Brazil	Brazilian real to US $ — exchange rate	Brazilian real to US $ 1 m — Fwd rate	1621
China	Chinese yuan to US $ — exchange rate	Chinese yuan to US $ 1 m — Fwd rate	2176
Cyprus	Cypriot pounds to US $ — exchange rate	Cypriot pound to Us $ 1 m — Fwd rate	1621
Denmark	Danish krone to US $ — exchange rate	Danish krone to US $ 1 m — Fwd rate	3510
Eurozone	Euro to US $ — exchange rate	Us $ to euro 1 m — Fwd rate	2988
France	French franc to US $ — exchange rate	French franc to US $ 1 m — Fwd rate	3510
India	Indian rupee to US $ — exchange rate	Indian rupee to US $ 1 m — Fwd rate	3296
Japan	Japanese yen to US $ — exchange rate	Japanese yen to US $ 1 m — Fwd rate	3510
Pakistan	Pakistan rupee to US $ — exchange rate	Pakistani rupee to US $ 1 m — Fwd rate	1623
United Kingdom	UK £ to US $ — exchange rate	US $ to UK £ 1 m — Fwd rate	3510

Notes on Table 1:
1. This table shows spot and forward exchange rates series employed in the present study.

state space models as well as their restricted versions are used for testing various hypotheses.

The likelihood ratio (LR) test statistic is used for testing all hypotheses that are constructed from log likelihood estimates of the unrestricted models and their restricted versions. The LR test statistics are constructed using Eq. (19).

$$LR = 2[\log L_{UR} - \log L_R] \sim \chi^2_q \tag{19}$$

where, Log L_{UR} is the log likelihood estimates obtained from the unrestricted models and Log L_R represents log likelihood estimates from the restricted model for a given hypothesis test. These hypothesis tests are elaborated in the following sub-sections.

4.1. Hypothesis tests 1: non-Gaussian model estimates

The chief hypothesis of the present study is 'no time varying risk premia' in each of the exchange rate series employed. However, other hypotheses of interest are also tested in all the series that include 'constancy in risk premia' and 'no heteroskedasticity' although additional hypotheses of interest are also tested in non-Gaussian framework. Hypothesis tests implemented using non-Gaussian state space models are explained in the following sub-sections.

4.1.1. No time varying risk premium

This hypothesis test is based on the LR test statistic that is constructed from the log likelihood estimates of the most general state space model and its restricted version (model 2) that restricts time varying risk premium ($\phi = c_\eta = 0$) in it. However, this test is not applicable in the present case because the LR test statistic for this test is non-standard since under the null hypothesis the signal to noise scale ratio (c_η) lies on the boundary of the admissible values for the asymptotic χ^2 distribution. For this test to be admissible, the likelihood function needs to be quadratic in the region in which the null hypothesis and the global optima lie, which is violated in the present study, so the standard asymptotic theory does not apply. Therefore, the small sample critical values are generated for this test using Monte Carlo simulations with initial values from the Gaussian homoskedastic versions of the null and alternative models.

Tables 2.1 to 2.10 present parameter estimates for non-Gaussian state space models and their restricted versions for Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euros, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates respectively. Using log likelihood estimates for all the series the LR test statistics for all the foreign exchange rates are constructed. These LR test statistic and the corresponding small sample critical values that are obtained from Monte Carlo simulations are placed beneath each statistic in parenthesis in the second column of Table 2.11. The results based on these test statistics show that the null hypothesis of no time varying risk premium is rejected in Brazilian real, Cypriot pounds, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar at 5% level of significance. The results on time varying risk premium for Chinese yuan and Eurozone euros forward foreign exchange rates do not prevail because the numerical algorithm for maximization of log likelihood function for the relevant models for these two series failed to converge.

Table 2.1
Non-Gaussian stable state space model estimates for Brazilian real.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.901	1.676	1.986	1.918	1.079	1.674
	(0.030)	(0.034)	(0.013)	(0.000)	(0.028)	(0.046)
μ	0.009	0.008			0.009	0.006
	(0.000)	(0.000)			(0.000)	(0.000)
ϖ	0.000	0.006	0.000			
	(0.000)	(0.000)	(0.000)			
β	0.896	0.0.007	0.926			
	(0.014)	(0.000)	(0.014)			
δ	0.058	0.0199	0.033			
	(0.009)	(0.0007)	(0.007)			
c				0.009	0.000	0.009
				(0.000)	(0.000)	(0.000)
c_η	0.016				0.000	
	(0.007)				(0.000)	
ϕ	0.223				1.055	
	(0.006)				(0.028)	
Log likelihood	5359.553	5184.698	4802.049	4675.344	4685.669	5184.461

Notes on Table 2.1:
1) The results for the model 1 shown in this table are generated by the most general stable state space model which is shown in the following Eqs. (1a)–(1c).

$$y_{t+1} = p_t + v_{t+1}, \quad v_{t+1} \sim c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid}\, S_\alpha(0.1)$$

$$(p_t - \mu) = \phi(p_{t-1} - \mu) + \eta_t \quad \eta_t \sim c_\eta c_t z_{2t}, \quad z_{2t} \sim \text{iid}\, S_\alpha(0.1)$$

$$c_t^\alpha = \varpi + \beta c_{t-1}^\alpha + \delta |y_t - E(y_t|y_1, y_2, \ldots, y_{t-1})|^\alpha$$

2) Model 2 is obtained by restricting time varying risk premium ($c_\eta = \phi = 0$) in the most general model i.e. model 1. Restricting constancy in the risk premium ($\mu = 0$) in model 2 gives model 3. Assuming homoskedasticity ($\beta = \delta = 0$) in model 3 gives model 4. Restricting conditional heteroskedasticity ($\beta = \delta = 0$) in model 1 results in model 5 and finally, restricting time varying risk premium ($c_\eta = \phi = 0$) in model 5 gives model 6.
3) Hessian based standard errors are reported in parentheses beneath each of the parameter estimates shown in this table.
4) Critical values for testing the hypothesis of 'no time varying risk premium' are shown in column 2 of Table 2.11. These LR test statistics are evaluated using small sample critical values that are generated from Monte Carlo simulations. These critical values are placed beneath each test statistic in parenthesis. The test statistics and the relevant critical values from Monte Carlo simulations for the remaining series are shown in a similar manner.

4.1.2. Constancy of risk premium

The null hypothesis of no risk premium is the second hypothesis that is tested in all the series for the present work. Here the null of no risk premium is tested against the alternative hypothesis of

Table 2.2
Non-Gaussian stable state space model estimates for Chinese yuan.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.811	–	1.986	1.744	1.987	1.651
	(0.019)	–	(0.007)	(0.028)	(0.006)	(0.003)
μ	−0.006	–			−0.000	0.002
	(0.000)	–			(0.000)	(0.000)
ϖ	0.000	–	0.000			
	(0.000)	–	(0.000)			
β	0.796	–	0.575			
	(0.016)	–	(0.027)			
δ	0.171	–	0.243			
	(0.015)	–	(0.019)			
c				0.002	0.000	−0.002
				(0.000)	(0.000)	(0.000)
c_η	0.773				0.555	
	(0.047)				(0.028)	
ϕ	0.989				0.246	
	(0.004)				(0.019)	
Log likelihood	12,751.753	–	11,032.647	9186.206	11,088.473	9619.734

Notes on Table 2.2:
1) The symbol '–'denotes missing numbers because the numerical algorithm for maximization of log likelihood function failed.
2) See notes on previous tables.

Table 2.3
Non-Gaussian stable state space model estimates for Cypriot pound.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.734	1.797	1.795	1.791	1.797	1.793
	(0.028)	(0.035)	(0.029)	(0.053)	(0.035)	(0.000)
μ	0.001	0.000			0.000	0.004
	(0.000)	(0.000)			(0.000)	(9.91e-05)
ϖ	6.48e-05	4.75e-05	4.80e-05			
	(8.92e-06)	(8.63e-06)	(0.000)			
β	0.001	9.552	4.65e-05			
	(0.000)	(2.313)	(5.50e-05)			
δ	0.002	0.080	0.011			
	(0.009)	(0.017)	(0.029)			
c				0.004	4.75e-05	0.000
				(0.000)	(8.63e-06)	(0.000)
c_η	0.009				6.29e-07	
	(0.004)				(1.05e-05)	
ϕ	0.269				0.011	
	(0.002)				(0.009)	
Log likelihood	5929.857	5924.588	5924.039	5923.081	5924.588	5923.606

Notes on Table 2.3.
1. See notes on previous tables.

Table 2.4
Non-Gaussian stable state space model estimates for Danish krone.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.824	1.876	1.891	1.889	1.884	1.884
	(0.030)	(0.026)	(0.031)	(0.000)	(0.018)	(0.029)
μ	−0.000	−0.001			−0.001	0.004
	(0.001)	(0.000)			(0.000)	(0.000)
ϖ	4.00e-05	0.0041	0.000			
	(4.03e-06)	(0.000)	(0.000)			
β	4.32e-07	0.0.002	0.003			
	(4.75e-05)	(0.001)	(0.027)			
δ	0.009	0.117	0.003			
	(0.008)	(0.001)	(0.006)			
c				0.004	0.000	−0.001
				(0.000)	(0.000)	(0.000)
c_η	0.057				0.003	
	(0.007)				(0.000)	
ϕ	0.978				0.007	
	(0.006)				(0.006)	
Log likelihood	12,912.086	12,822.682	12,812.037	12,811.053	12,818.287	12,817.441

Notes on Table 2.4:
1. See notes on previous tables.

Table 2.5
Non-Gaussian stable state space model estimates for Eurozone euro.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.763	1.882	1.885	1.885	1.963	1.881
	(0.019)	(0.025)	(0.018)	(0.000)	(0.014)	(0.001)
μ	0.001	2.37e-05			0.001	0.004
	(0.000)	(0.001)			(0.000)	(6.90e-05)
ϖ	6.25e-05	0.004	2.90e-05			
	(5.88e-06)	(6.74e-05)	(1.15e-05)			
β	7.98e-09	0.0.021	0.138			
	(4.75e-05)	(0.053)	(0.341)			
δ	0.001	0.008	4.30e-06			
	(0.006)	(0.769)	(0.000)			
c				0.004	1.08e−07	0.000
				(0.000)	(4.90e-08)	(0.000)
c_η	0.003				0.966	
	(0.002)				(0.005)	
ϕ	0.196				0.014	
	(0.001)				(0.002)	
Log likelihood	10,858.727	10,868.522	10,863.845	10,863.844	10,970.085	10,868.521

Notes on Table 2.5:
1. See notes on previous tables.

Table 2.6
Non-Gaussian stable state space model estimates for French franc.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.582	1.863	1.895	1.895	1.884	1.884
	(0.005)	(0.015)	(0.029)	(0.018)	(0.039)	(0.021)
μ	−0.001	−0.001			−0.001	0.004
	(5.14e-06)	(3.69e-05)			(0.000)	(6.30e-05)
ϖ	0.000	0.004	3.12e-05			
	(1.69e-05)	(6.39e-05)	(4.34e-06)			
β	0.038	0.012	2.25e-05			
	(0.116)	(0.002)	(7.18e-07)			
δ	1.36e-07	0.361	0.007			
	(2.12e-06)	(0.001)	(0.006)			
c				0.004	3.24e-05	−0.001
				(6.15e-05)	(0.0000)	(0.000)
c_η	0.004				0.007	
	(0.007)				(0.005)	
ϕ	0.844				0.005	
	(0.003				(0.005)	
Log likelihood	12,858.390	12,826.322	12,792.766	12,792.009	12,810.108	12,809.572

Notes on Table 2.6:
1. See notes on previous tables.

Table 2.7
Non-Gaussian stable state space model estimates for Indian rupee.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.633	1.400	1.930	1.855	1.632	1.526
	(0.0206)	(7.76e-08)	(3.32e-08)	(0.018)	(0.0284)	(0.027)
μ	0.003	0.003			0.003	0.002
	(0.000)	(0.000)			(4.83e-05)	(0.027)
ϖ	1.87e-07	0.001	6.57e-08			
	(0.000)	(2.39e-05)	(3.32e-08)			
β	0.826	0.167	0.825			
	(4.69e-08)	(0.012)	(0.017)			
δ	0.087	0.967	0.089			
	(0.009)	(0.004)	(0.009)			
c				0.003	1.32e-05	0.0027
				(4.37e-05)	(0.341)	(4.84e-05)
c_η	0.243				2.28e-06	
	(0.022)				(0.000)	
ϕ	0.948				0.399	
	(0.005)				(0.024)	
Log likelihood	15,716.384	15,100.752	13,538.970	13,115.026	14,434.179	14,115.874

Notes on Table 2.7:
1. See notes on previous tables.

Table 2.8
Non-Gaussian stable state space model estimates for Japanese yen.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.816 (0.000)	1.802 (0.042)	1.898 (0.016)	1.898 (0.024)	1.876 (0.024)	1.799 (0.035)
μ	−0.004 (0.0012)	0.005 (6.91e-05)			−0.003 (0.000)	0.005 (8.00e-5)
ϖ	4.60e-05 (0.000)	0.001 (2.39e-05)	0.000 (0.000)			
β	6.16e-06 (0.000)	0.0067 (0.055)	0.003 (0.021)			
δ	0.046 (0.009)	0.004 (0.034)	0.000 (0.000)			
c				0.005 (7.77e-05)	3.57e-07 (1.45e07)	−0.003 (0.000)
c_η	0.049 (0.007)				0.964 (0.008)	
ϕ	0.988 (0.005)				0.013 (0.003)	
Log likelihood	12,521.181	12,393.331	12,077.537	12,077.536	12,499.552	12,393.198

Notes on Table 2.8:
1. See notes on previous tables.

Table 2.9
Non-Gaussian stable state space model estimates for Pakistani rupees.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.880 (0.021)	1.824	1.983 (0.000)	1.883 (0.029)	1.824 (0.026)	1.142 (0.000)
μ	−0.003 (5.24e-05)	−0.003 (4.32e-05)			−0.005 (0.001)	−0.004 (0.003)
ϖ	6.95e-08 (1.72e-08)	4.58e-08 (1.48e-08)	6.96e-08 (2.91e-08)			
β	0.702 (0.033)	0.701 (0.031)	0.707 (0.031)			
δ	0.154 (0.021)	0.155 (0.019)	0.152 (0.018)			
C				0.004 (8.72e-05)	0.003 (5.27e-05)	0.002 (0.000)
c_η	0.018 (0.021)				0.001 (0.013)	
ϕ	0.336 (0.002)				0.003 (5.311)	
Log likelihood	7605.078	7471.055	6584.949	5956.206	6459.739	6457.962

Notes on Table 2.9:
1. See notes on previous tables.

Table 2.10
Non-Gaussian stable state space model estimates for British pound.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α	1.831 (0.029)	1.775 (0.024)	1.847 (0.029)	1.825 (0.021)	1.939 (0.019)	1.811 (0.021)
μ	−0.001 (0.0004)	−0.0009 (9.78e-05)			−0.001 (8.84e-05)	−0.001 (9.06e-5)
ϖ	2.85e-05 (4.39e-06)	0.0004 (8.81e-05)	2.93e-05 (5.56e-05)			
β	0.001 (0.000)	9.552 (2.3127)	1.15e-08 (7.18e-07)			
δ	0.046 (0.011)	0.081 (0.017)	0.052 (0.011)			
C				0.021 (5.67e-05)	1.20e-07 (5.03e-08)	-0.003 (0.000)
c_η	0.059 (0.007)				0.961 (0.006)	
ϕ	0.9737 (0.007)				0.016 (0.002)	
Log likelihood	13,262.646	13,197.516	13,145.768	13,124.416	13,353.019	13,178.654

Notes on Table 2.10:
1. See notes on previous tables.

Table 2.11
Hypothesis tests for non − Gaussian state space models.

	No time varying risk premium $LR(c_\eta = \phi = 0)$	Constancy in risk premium $LR(\mu = 2)$	No heteroskedasticity $LR(\beta = \delta = 0)$	Additional for time varying risk premium $LR(c_\eta = \phi = 0)$
Brazilian real	349.709	765.298	253.410	997.581
	(0.000)	(0.000)	(0.000)	(0.000)
Chinese yuan	—	—	3692.881	2937.476
			(0.000)	(0.000)
Cypriot pound	10.539	1.097	1.916	1.964
	(0.000)	(0.295)	(0.383)	(0.000)
Danish krone	178.808	21.290	1.968	1.692
	(0.000)	(0.000)	(0.374)	(0.000)
Eurozone euro	——	9.354	0.002	203.128
		(0.002)	(0.999)	(0.000)
French franc	64.136	100.668	1.514	1.072
	(0.003)	(0.000)	(0.469)	(0.002)
Indian rupee	1231.764	3123.564	847.888	636.610
	(0.000)	(0.000)	(0.000)	(0.000)
Japanese yen	255.700	631.546	0.000	212.708
	(0.000)	(0.000)	(0.999)	(0.000)
Pakistani rupees	268.054	1772.211	1257.487	15.554
	(0.000)	(0.000)	(0.000)	(0.001)
British pound	130.260	103.496	103.496	348.730
	(0.000)	(0.000)	(0.000)	(0.000)

Notes on Table 2.11.
1. See notes on previous tables.

constant risk premium in all the series using the LR test statistic that is constructed from log likelihood estimates for model 2 and its restricted version that restrict constancy in risk premium ($\mu = 0$) in it.

The LR test statistic and its corresponding p-values obtained from asymptotic χ_1^2 distribution for this test are shown in the third column of Table 2.11 beneath the test statistics in parentheses. The results based on this test show statistically significant evidence of constancy in risk premium in Brazilian real, Danish krone, Eurozone euros, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound exchange rates against US dollar at 5% level of significance. Inferences do not change even when the significance level is switched from 5 to 10% level. However, the results for constancy in risk premium for Chinese yuan are nonexistent and that of Cypriot pound are in sharp contrast.

4.1.3. No heteroskedasticity

The null hypothesis of no time varying volatility is tested using LR test statistics that are obtained from log likelihood estimates from the most general model (model 1) and its restricted version (model 4) that restricts time varying volatility ($\beta = \delta = 0$) in it. The LR test statistics for this test for all the series are shown in column 4 of Table 2.11. These test statistics are evaluated using respective p-values obtained from the asymptotic χ_2^2 distribution. The null of no time varying volatility overwhelmingly rejects the null hypothesis of no time varying volatility against volatility clustering in Brazilian real using p-values from χ_2^2 distribution. The LR test statistics for the remaining series that are also evaluated using p-values obtained from χ_2^2 distribution reveal statistically significant existence of volatility clustering in Chinese yuan, Indian rupee, Pakistani rupee, and British pound forward foreign exchange rates against US dollar at 5% level of significance. Inferences do not change substantially when significance level is switched from 5% to 10% level. Results for Cypriot pounds, Danish krone, Eurozone euro, French franc, and Japanese yen are in sharp contrast.

4.1.4. Additional test for time varying risk premium

Additional tests are implemented to understand the importance of time varying volatility, and to assess the consequences of ignoring conditional heteroskedasticity on the inferences drawn for the presence or absence of time varying or constant risk premium in Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euros, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar. In doing so, forecasts from models 5 and 6 that are homoskedastic versions of general model 1 and model 2 respectively are employed. Thus, the null hypothesis pertaining to constant risk premium is tested for each of the series using the LR test statistic for constant risk premium that is restricting time varying risk premium ($\phi = c_\eta = 0$) in model 5 which gives the restricted model (model 6). Test statistics for all the series along with the small sample critical values obtained from Monte Carlo simulations that are placed below each test statistic in parentheses in column 5 of the Table 2.11.

The results reveal statistically significant evidence of time varying risk premium in Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euro, French franc Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates at 5% level of significance. Although results did not change much when compared to the earlier results that are obtained from the most general non-Gaussian state space model and its restricted versions. However, ignoring conditional heteroskedasticity leads to artificial statistical inferences in favor of no time varying risk premium in Brazilian real, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward exchange rates against US dollar since the magnitude of the test statistics for additional tests for time varying risk premium for Brazil and UK intensifies, where the magnitude for the test statistics for Cyprus, Denmark, France, India, Japan, and Pakistan dampens. However, the results become insignificant for Cyprus, Denmark and France forward foreign exchange rates against US dollar.

4.2. Hypothesis tests 2: Gaussian model estimates

Like the non-Gaussian case, a number of hypothesis of interest are tested using log likelihood estimates from the most general Gaussian model and its restricted versions for each of the exchange rate series employed. These hypotheses include Gaussian no time varying risk premium, Gaussian constancy in risk premium, Gaussian test for normality, Gaussian test on lack of volatility clustering and Gaussian homoscedastic time varying risk premium. Each of these hypothesis tests is elaborated in the following sub-sections. Tables 3.1 to 3.10 show the parameter estimates for non-Gaussian state space models and their restricted versions for Brazilian real, Chinese yuan, Cypriot

Table 3.1
Gaussian stable state space model estimates for Brazilian real.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	0.008	0.008			0.008	0.008
	(004)	(0.000)			(0.000)	(0.000)
ϖ	0.000	0.000	8.42e-07			
	(0.000)	(0.000)	(4.4.e-07)			
β	0.866	481.068	0.916			
	(0.015)	(0.000)	(0.014)			
δ	0.082	0.051	0.038			
	(0.011)	(0.025)	(0.007)			
c				0.009	1.06e-06	0.008
				(0.000)	(3.49e-07)	(0.000)
c_η	0.023				0.849	
	(0.005)				(0.020)	
ϕ	0.999				0.067	
	(0.001)				(0.009)	
Log likelihood	5732.971	5012.195	4800.067	4634.069	5330.084	5010.145

Notes on Table 3.1:
1. See notes on the previous tables.
2. The most general Gaussian state space model shown in the following equations is used to obtain the estimates shown in this table:

$$y_{t+1} = p_t + v_{t+1}, \quad v_{t+1} \sim \sqrt{2} c_t z_{1t+1}, \quad z_{1t+1} \sim \text{iid}\, N(0.1)$$

$$(p_t - \mu) = \phi(p_{t-1} - \mu) + \eta_t \quad \eta_t \sim \sqrt{2} c_\eta c_t z_{2t}, \quad z_{2t+1} \sim \text{iid}\, N(0.1)$$

$$c_t^\alpha = \varpi + \beta c_{t-1}^2 + \delta \left| y_t - E\left(y_t | y_1, y_2, \ldots, y_{t-1}\right) \right|^2$$

3. Model 2 is obtained by restricting $c_\eta = \phi = 0$ in the most general model i.e. model 1. Restricting $\mu = 0$ in model 2 gives model 3. Setting $\beta = \delta = 0$in model 3 gives model 4. Restricting conditional heteroskedasticity ($\beta = \delta = 0$) in model 1 results in model 5 and finally, restricting $\phi = c_\eta = 0$ in model 5 gives model 6.
4. Hessian based standard errors are reported in parentheses beneath the parameter estimates.
5. Critical values for testing the hypothesis of "no time varying risk premium" are shown in the second column2 of the Table 3.11. These LR test statistics are evaluated using small sample critical values that are generated from Monte Carlo simulations. Test statistics and critical values from the Monte Carlo simulations for the remaining exchange rates series are presented in the last row of the table in subsequent columns in a similar manner.

pounds, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates respectively. Table 3.11 presents test statistics along with the relevant p-values for each of the test for all the series at Gaussian settings.

4.2.1. Gaussian no time varying risk premium

For testing the null hypothesis for the presence or absence of Gaussian time varying risk premium, constancy in risk premium ($\phi = c_\eta = 0$) is imposed in the most general Gaussian model for each of the series. The resulting model is Gaussian constant risk premium model (model 2). Using log likelihood estimates from the restricted and unrestricted models, the likelihood ratio (LR) test statistics are constructed for testing time varying risk premium in Brazilian real, Chinese yuan, Cypriot pound, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward exchange rates against US dollar. However, as mentioned earlier, critical values from χ^2 distribution cannot be used to evaluate the LR test statistics for this test therefore; small sample critical values are generated from Monte Carlo simulation for evaluating the LR test statistics for this test for each of the series. The ML estimates for models 1 and 2 are used to calculate the LR test statistics for this test for all the series that are evaluated using small sample critical obtained from Monte Carlo simulation that are shown in Table 3.11. These test statistics along with the p-values pertaining to Monte Carlo simulations are shown beneath each test statistic in parentheses.

Based on the results presented in Table 3.11, the null hypothesis is rejected in favor of the alternative hypothesis of no time varying risk premium for Brazilian real, Chinese yuan, Cypriot pound, Danish

Table 3.2
Gaussian stable state space model estimates for Chinese yuan.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−3.39e-05	−0.002			0.000	0.002
	(8.10e-06)	(0.000)			(0.000)	(3.39e-05)
ϖ	0.000	0.002	4.74e-06			
	(0.000)	(0.000)	(1.17e-05)			
β	0.598	0.000	0.029			
	(0.028)	(0.000)	(1.734)			
δ	0.238	0.997	0.000			
	(0.021)	(0.000)	(0.000)			
c				0.003	0.000	−0.002
				(3.93e-05)	(0.000)	(6.68e-05)
c_η	4.18e-06				0.000	
	(0.000)				(0.000)	
ϕ	−0.315				0.736	
	(1.244)				(0.029)	
Log likelihood	11,016.812	9493.918	9112.078	9112.078	10,641.530	9434.605

Notes on Table 3.2:
See notes on previous tables.

Table 3.3
Gaussian stable state space model estimates for Cypriot pound.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.001	0.000			0.000	0.005
	(3.25e-08)	(0.000)			(0.000)	(8.10e-05)
ϖ	4.88e-08	0.001	1.91e-05			
	(3.25e-08)	(0.0029)	(8.73e-05)			
β	0.9649	9.6481	1.42e-06			
	(0.005)	(59.859)	(0.000)			
δ	0.017	0.063	0.055			
	(0.003)	(0.027)	(0.018)			
c				0.005	1.91e-05	0.000
				(0.000)	(8.72e-05)	(0.000)
c_η	0.021				1.29e-05	
	(0.006)				(0.000)	
ϕ	0.996				0.054	
	(0.003)				(0.018)	
Log likelihood	6039.369	5857.140	5860.834	5852.905	5861.822	5853.969

Notes on Table 3.3.
1. See notes on previous tables.

Table 3.4
Gaussian stable state space model estimates for Danish krone.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.001	−0.001			−0.0004	0.0045
	(0.000)	(0.000)			(0.0001)	(0.0000)
ϖ	0.0000	0.005	1.96e-05			
	(0.000)	(0.000)	(5.68e-07)			
β	0.970	0.0.009	8.0e-05			
	(0.004)	(0.000)	(0.000)			
δ	0.015	0.075	0.025			
	(0.002)	(0.000)	(0.009)			
c				0.005	7.31e-08	−0.004
				(0.000)	(3.33e-08)	(0.000)
c_η	0.000				0.967	
	(0.000)				(0.004)	
ϕ	−0.436				0.015	
	(0.000)				(0.002)	
Log likelihood	12,907.315	12,739.674	12,742.225	12,736.471	12,911.029	12,739.683

Notes on Table 3.4:
1. See notes on previous tables.

krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward exchange rates against US dollar at 5% level of significance.

4.2.2. Gaussian constancy in risk premium

Again for testing the null hypothesis of no risk premium against the alternate hypothesis of constant risk premium at Gaussian settings, model 2 restricts $\mu = 0$ which results in a restricted model (model 3). The LR test statistics for this test for all the series are shown in Table 3.11 that are evaluated using corresponding p-values obtained from χ^2_1 distribution. The results reveal that the null hypothesis of no risk premium is rejected in favor of the alternative hypothesis of constant risk premium for Brazilian real, Chinese yuan, Danish Kroner, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani

Table 3.5
Gaussian stable state space model estimates for Eurozone euro.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	0.001	0.001			0.001	0.005
	(0.000)	(0.000)			(0.000)	(5.94e-05)
ϖ	5.84e-08	0.005	2.03e-05			
	(3.14e-08)	(0.000)	(6.34e-07)			
β	0.968	0.021	1.24e-08			
	(0.004)	(0.000)	(0.000)			
δ	0.015	0.008	0.021			
	(0.002)	(0.000)	(0.009)			
c				0.005	0.000	0.000
				(5.95e-05)	(0.000)	(0.000)
c_η	0.003				0.971	
	(0.000)				(0.004)	
ϕ	−0.192				0.015	
	(0.000)				(0.002)	
Log likelihood	10,962.661	10,803.011	10,803.845	10,800.302	10,960.205	10,803.019

Notes on Table 3.5:
1. See notes on previous tables.

Table 3.6
Gaussian stable state space model estimates for French franc.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.001	−0.001			−0.001	0.005
	(0.000)	(0.000)			(0.000)	(5.41e-05)
ϖ	7.01e-08	0.005	1.99e-05			
	(3.24e-08)	(5.14e-05)	(5.70e-05)			
β	0.968	7.41e-05	8.01e-05			
	(0.004)	(0.000)	(0.000)			
δ	0.015	-0.048	0.018			
	(0.002)	(0.000)	(0.009)			
c				0.005	0.000	0.001
				(5.43e-05)	(0.000)	(0.000)
c_η	0.019				0.971	
	(0.075)				(0.003)	
ϕ	0.089				0.015	
	(0.548)				(0.002)	
Log likelihood	12,906.783	12,738.433	12,729.932	12,726.419	12,810.108	12,738.453

Notes on Table 3.6:
1. See notes on previous tables.

Table 3.7
Stable state space model estimates for Indian rupee.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	0.004	0.003			0.003	0.003
	(0.000)	(0.000)			(4.36e-05)	(3.33e-05)
ϖ	6.85e-09	0.002	6.40e-06			
	(1.41e-09)	(3.17e-05)	(2.50e-07)			
β	0.952	0.189	9.34e-09			
	(0.003)	(0.018)	(4.30e-07)			
δ	0.029	0.962	0.216			
	(0.002)	(0.007)	(0.017)			
c				0.003	1.67e-08	0.003
				(4.07e-05)	(6.35e-09)	(6.66e-05)
c_η	0.190				0.855	
	(0.017)				(0.025)	
ϕ	0.946				0.085	
	(0.005)				(0.016)	
Log likelihood	15,212.893	14,050.525	13,213.671	13,001.339	14,704.319	13,663.793

Notes on Table 3.7:
1. See notes on previous tables.

rupee, and British pound forward exchange rates against US dollar at 5% level of significance.

4.2.3. Gaussian test on normality

The LR test statistics for Gaussian test for normality is computed using log likelihood estimates that are obtained from non-Gaussian model 3 and Gaussian model 3 (that restricts $\alpha = 2$ in non-Gaussian model 3) for Brazilian real, Chinese yuan, Cypriot pound, Danish Kroner, Eurozone euros, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward exchange rates against US dollar. However, this LR test statistic has a non-standard distribution since the null hypothesis for this test lies on the boundary of the admissible values for α, and hence the standard regularity conditions are not satisfied. Therefore, inferences for this test are based on small sample critical

Table 3.8
Gaussian stable state space model estimates for Japanese yen.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.003	−0.003			−0.003	0.003
	(0.000)	(0.000)			(0.000)	(6.23e-05)
ϖ	2.75e-07	0.005	2.71e-05			
	(8.68e-08)	(6.23e-05)	(8.47e-07)			
β	0.953	0.007	1.36e-08			
	(0.008)	(0.000)	(0.000)			
δ	0.019	0.008	0.067			
	(0.003)	(0.640)	(0.012)			
c				0.006	2.29e-05	−0.003
				(6.67e-05)	(6.91e-07)	(0.000)
c_η	1.84e-07				0.000	
	(0.000)				(0.000)	
ϕ	0.124				0.079	
	(1.055)				(0.014)	
Log likelihood	12,432.414	12,246.626	12,039.612	12,002.504	12,303.524	12,246.781

Notes on Table 3.8:
1. See notes on previous tables.

Table 3.9
Gaussian stable state space model estimates for Pakistani rupee.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.364	−8.33e-05			−0.004	−0.004
	(0.015)	(0.0001)			(0.000)	(0.000)
ϖ	1.42e-06	3.4063e-42	2.795e-06			
	(8.89e-08)	(0.0000)	(2.26e-07)			
β	2.53e-261	0.8604	8.259e-09			
	(0.000)	(0.0182)	(0.000)			
δ	0.558	0.0699	0.482			
	(0.043)	(0.009)	(0.028)			
C				0.004	1.02e-05	0.003
				(7.77e0)	(0.000)	(5.53e-05)
c_η	0.000				257.182	
	(0.000)				(0.000)	
ϕ	0.728				0.549	
	(7.09e-07)				(0.0206)	
Log likelihood	7351.067	6562.629	6387.494	5922.330	6764.732	6472.475

Notes on Table 3.9:
1. See notes on previous tables.

Table 3.10
Gaussian stable state space model estimates for British pound.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
μ	−0.001	−0.001			−0.001	0.004
	(0.001)	(0.000)			(8.96e-05)	(4.99e-05)
ϖ	1.36e-05	0.001	1.53e-05			
	(4.33e-07)	(0.002)	(0.000)			
β	2.24e-05	3.812	1.15e-08			
	(0.000)	(6.279)	(1.15e-08)			
δ	0.102	0.096	0.075			
	(0.011)	(0.027)	(0.011)			
C				0.004	1.18e-07	−0.001
				(5.05e-05)	(3.78e-08)	(9.91e-05)
c_η	0.019				0.957	
	(0.005)				(0.006)	
ϕ	0.997				0.018	
	(0.002)				(0.002)	
Log likelihood	13,130.097	13,035.165	13,026.656	12,980.298	13,335.176	13,020.963

Notes on Table 3.10:
1. See notes on previous tables.

Table 3.11
Hypothesis tests for Gaussian models.

	No time varying risk premium $LR(c_\eta = \phi = 0)$	Constancy in risk premium $LR(\mu = 2)$	No heteroskedasticity $LR(\beta = \delta = 0)$	Additional for time varying risk premium $LR(c_\eta = \phi = 0)$	Gaussian test on normality $LR(\alpha = 2)$
Brazilian real	1441.552	424.256	331.199	639.878	3.964
	(0.000)	(0.000)	(0.000)	(0.000)	(0.046)
Chinese yuan	3045.787	763.681	0.000	2413.850	3841.138
	(0.000)	(0.000)	(0.999)	(0.000)	(0.000)
Cypriot pound	364.457	—	15.859	15.707	126.409
	(0.000)		(0.000)	(0.001)	(0.000)
Danish krone	335.282	335.282	1.968	342.692	139.624
	(0.000)	(0.000)	(0.374)	(0.000)	(0.000)
Eurozone euro	319.300	9.354	0.002	203.128	120.000
	(0.000)	(0.002)	(0.999)	(0.000)	(0.000)
French franc	336.700	17.002	7.026	143.310	125.668
	(0.000)	(0.000)	(0.029)	(0.000)	(0.000)
Indian rupee	2324.736	1673.708	424.664	2081.052	650.598
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Japanese yen	371.576	414.028	74.212	113.446	75.850
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Pakistani rupees	1576.875	350.271	930.327	548.514	394.912
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
British pound	189.864	17.018	92.716	628.426	238.224
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)

Notes on Table 3.11:
1. See notes on previous tables.

values due to McCulloch (1997). Based on these critical values, the null hypothesis of normality for Brazilian real, Chinese yuan, Cypriot pound, Danish Kroner, Eurozone euros, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward exchange rates is rejected against the alternate hypothesis of non-normality.

4.2.4. Gaussian test for lack of volatility clustering

In order to understand if time varying volatility does exist in Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euros French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound against US dollar forward foreign exchange rates, the test for the lack of volatility clustering (homoskedasticity) can be formulated restricting time varying volatility ($\beta = \delta = 0$) in Gaussian model 3 which give the resulting null model (model 4). The LR test statistics for Gaussian test for lack of volatility clustering for all the series are shown in column 4 in Table 3.11 that are evaluated using p-value obtained from asymptotic χ_2^2 distribution. These results reveal that the null of no volatility clustering is statistically significant at the 5% level in Brazilian real, Cypriot pounds, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound. The results for no volatility clustering in Chinese yuan, Danish kroner, and Eurozone euros forward foreign exchange rates are in sharp contrast. Inferences do not change much when the significance level is switched from 5% to 10% level.

4.2.5. Gaussian homoskedastic time varying risk premium

Empirical research has established that time varying volatility does exist in forward foreign exchange rates. Therefore, for assessing the consequence of ignoring conditional heteroskedasticity on the inferences for the presence of constant risk premium in Gaussian models, LR test statistic is calculated from ML estimates for models 5 and 6 for testing the restriction of time varying risk premia ($\phi = c_\eta = 0$) in model 5 in all the series. The LR test statistic for Brazilian real, Chinese yuan, Cypriot pounds, Danish kroner, Eurozone euros, French francs, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar with the corresponding p-values that are generated from Monte Carlo simulations are shown in Table 3.11. These results reveal that constancy in risk premium in Brazilian real, Chinese yuan, Cypriot pound, Danish kroner, Eurozone euros, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates can easily be rejected in favor of time varying risk premium at 5% level of significance.

4.3. Discussions

The present research investigates the possible existence of risk premia in daily Brazilian real, Chinese yuan, Cypriot pound, Danish kroner, Eurozone euro, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollars using univariate state space models that were also employed by Wolff (1987, 2000), and Nijman et al. (1993) among others. The present work improves on the previous studies using forecasting models that take into account non-normality and time varying volatility in the forward foreign exchange rates that was also recommended in a number of studies on exchange rates forecasts. The contribution of the present study is three fold. First, it analyzes high frequency data, second it employs appropriate methodology dictated by the empirical literature and finally it uses cross-country exchange rate data series to study possible existence of risk premia in forward foreign exchange rates in the presence of non-normality and time varying volatility that are prevalent in foreign exchange rate series employed.

The results based on non-Gaussian state space models reveal the existence of time varying risk premium in Brazilian real, Cypriot pound, Danish kroner, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound and time varying volatility in Brazilian real, Danish kroner, Eurozone euros, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar. Moreover, non-normality does exist in all the series and time varying volatility prevails in most series even at Gaussian settings. Therefore, the study results reveal that the univariate signal plus noise models with stable distributions and GARCH-like affects show statistically significant evidence of time varying risk premium in Brazilian real, Cypriot pound, Danish kroner, France franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound and as well as constancy in risk premium in Brazilian real, Danish kroner, Eurozone euro, France franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound versus US dollar forward foreign exchange rates series.

The study results obtained from state space models with conditionally heteroskedastic non-normal set-up are in line with Wolff (1987) who used a similar methodology in Gaussian setting using weekly data, and Bidarkota (2004) who employed non-Gaussian state space models using monthly data.

The results also reveal that ignoring conditional heteroskedasticity leads to artificial statistical inferences in favor of time varying risk premium in most forward foreign exchange rate series against US dollar. In addition, the study results in Gaussian framework are not much different to those obtained from the non-Gaussian models. This divulges that high frequency data helped in obtaining better forecast of foreign exchange rates for all the series under investigation. Therefore, the present study focuses on the need for methodological improvements among other things in cross-country study pertaining to exchange rate forecast.

To understand the link between exchange rate markets and the economy it is imperative to focus on the type of policy action that would be appropriate with a given exchange rate regime. According to Keynes (1936), the underlying elements of monetary policy can affect output and inflation in the economy since it is operating through government spending and tax cuts as well as transfer payments such as unemployment insurance and medical coverage. Usually monetary policy works better with floating exchange rates where fiscal policy is more persuasive with fixed exchange rates (Freenstra and Taylor, 2008; Krugman and Obstfeld's, 2000). Conversely, fiscal policy is equally good with fixed or a floating exchange rates provided the central bank did not like to increase interest rate in response to an expansionary monetary policy. On the other hand the monetary policy will be redundantly important when exchange rates are fixed and international capital mobility is permitted in the presence of the interest rates that would be kept constant at world interest rate level. However, the floating rate that is more effective with monetary policy does not pose such restrictions. Nevertheless, many economists including Blanchard (2009) demonstrated that under the normal circumstances monetary policy should be used as the stabilization policy for stabilizing inflation and output leaving no room for fiscal policy as stabilization policy over the medium term.

Policymakers in open economies often face what is known as a macroeconomic trilemma which consists of stabilizing exchange rates, free international capital mobility, and engagement in monetary policy oriented towards domestic goals. However, only two of these three goals being mutually dependable, policymakers have no choice but to pick only two of the three goals. Major empirical challenges confront anyone seeking empirical measures of the three economic objectives underlying the trilemma. The first element, the exchange rate, is perhaps the simplest to measure. Therefore, exchange rate predictions and identification of appropriate regime are imperative.

While efforts to identify appropriate regime using various macro-financial variables have been made in the literature, the International Monetary Fund (IMF) publishes countries' exchange rate regime choices which may not base on their self-reported status but depends on the preferred approach of analysis that examines what countries do rather than what they report (Calvo and Reinhart, 2002; Obstfeld and Rogoff, 1995; Reinhart and Rogoff, 2004).

Abell (1990) showed that the budget deficits and the current account deficits are linked through appreciation of the exchange rate among other things. Thus, exchange rates being pivotal for stabilization of the economy exchange rate predictions are imperative for policymakers to work with twin deficits in the economy that was prevalent in the US as well as in the European economies in 1990s. For instance, during this period Germany and Sweden had rising budget deficit that was accompanied with deteriorating current account balance because of real appreciation in their national currencies of these countries (Ibrahim and Kumah, 1996).

5. Conclusion

The present research investigates the possible existence of risk premia in daily Brazilian real, Chinese yuan, Cypriot pound, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollars using univariate state space models that were also employed by Wolff (1987, 2000), and Nijman et al. (1993) among others. The present study improves on the methodologies used by the previous studies using models that include features to account for non-normality and time varying volatility that may be present in the series that was also recommended in a number of studies on exchange rates.

The study results based on non-Gaussian state space model show statistically significant evidence of time varying risk premium in Brazilian real, Cypriot pound, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound forward foreign exchange rates against US dollar. The results fail to reveal statistically significant evidence of constancy in risk premium in Brazilian real, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound at 5% level of significance. The results pertaining to statistically significant evidence of time varying risk premia in Brazilian real, Danish krone, Eurozone euro, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound exchange rates against US dollar do not change even at Gaussian settings. However, the additional tests on time varying risk premium and constancy in risk premium reveal that excluding conditional heteroskedasticity from the employed models leads to artificial statistical inferences. These results are in line with Wolff (1987) who employed signal extraction approach in Gaussian settings using weekly data.

A test of homoskedasticity or lack of volatility clustering based on non-Gaussian state space or unobserved component models show statistically significant evidence of time varying volatility in Brazilian real, Chinese yuan, Indian rupee, Pakistani rupee, and British pound forward foreign exchange rates against US dollar. Likewise, the normality hypothesis is gotten rejected in all the series at Gaussian settings at 5% level of significance showing persistence of non-normality in all and presence of volatility clustering in most of the forward foreign exchange rate series.

Based on the study results it can be concluded that the univariate state space or signal plus noise models with stable distributions and conditional heteroskedasticity show statistically significant evidence of time varying risk premium in Brazilian real, Cypriot pound, Danish krone, French franc, Indian rupee, Japanese yen, Pakistani rupee, and British pound versus US dollar forward foreign exchange rates as well as constancy in risk premium in the most series employed. Therefore, future work in this area may employ additional nonlinear models including feed forward and recurrent artificial neural networks for forecasting risk premium in forward foreign exchange rate markets.

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