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Solitary Wave Solutions of a Fractional Boussinesq Equation

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Abstract

In this paper, we aaply certain mapping methods to solve Fractional Boussinesq equation. We derive Jacobi elliptic function solutions and deduce the solitary wave solutions and the singular wave solutions when the modulus of the elliptic functions approaches 1. The correctness of the SWS for some particular values has been checked using the tanh method. Solitary wave solutions have been plotted for different values of the fractional order of the derivatives and for the power of the dependent variable.

Keywords: Travelling wave solutions, mapping methods, Jacobi elliptic functions, fractional PDEs, solitary wave solutions, singular wave solutions

1 Introduction

One of the sought after topics of research in Applied Mathematics is the study of nonlinear phenomena in different physical situations. There has been a significant progress in the investigation of exact solutions of nonlinear evolution equations (NLEEs) [1-6] in the past few decades.

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NLEEs are the governing equations in various areas of physical, chemical, biological and geological sciences. For example in Physics, NLEEs appear in the study of nonlinear optics, plasma physics, fluid dynamics etc. and in geological sciences in the dynamics of magma.

The important question that arises is about the integrability of these NLEEs. Several methods have been developed for finding exact solutions. Some of these commonly used techniques are tanh method [7], extended tanh method [8,9], exponential function method [10], G'/G expansion method [11-13], Mapping methods [14-18].

In this paper, we derive periodic wave solutions (PWSs) of a fractional Boussinesq equation in terms of Jacobi elliptic functions (JEFs) [19] and deduce their infinite period counterparts in terms of hyperbolic functions such as solitary wave solutions (SWSs) and singular wave solutions using mapping methods. The mapping methods employed in this paper give a variety of solutions which other methods cannot.

The paper is organised as follows: In section 2, we give a definition of Riemann-Liouville fractional derivative [20] and a mathematical analysis of the mapping methods, In section 3, an introduction to JEFs in sections 4-6 we give a description of the Fractional Boussinesq equation and its solutions using mapping methods, in section 7, tanh mehod has been employed to find a SWS and section 8 gives the conclusion.

2 Mathematical Analysis of Mapping Methods

In this section, we give an analysis of the mapping method which will be employed in this paper.

We consider the nonlinear FPDE

$$F(u, D_t^{\alpha}u, D_x^{\alpha}u, D_x^{2\alpha}u, D_x^{3\alpha}u, ...) = 0, \ \ 0 < \alpha \leq 1. \tag{1}$$

where the unknown function u depends on the space variable x and time variable t.

Here, the Riemann-Liouville fractional derivatives are given by

$$D_t^{\alpha}t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}t^{r-\alpha}, \ \ D_x^{\alpha}x^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}x^{r-\alpha}. \tag{2}$$

We consider the TWS in the form

$$u(x,t) = u(\xi), \ \ \xi = \frac{x^{\alpha}}{\Gamma(1+\alpha)} - \frac{c\,t^{\alpha}}{\Gamma(1+\alpha)}, \tag{3}$$

where c is the wave speed.

Substituting eq. (3) into eq. (1), the PDE reduces to an ODE and then we search for the solution of the ODE in the form

$$u(\xi) = \sum_{i=0}^{n} A_i f^i(\xi), \tag{4}$$

where n is a positive integer which can be determined by balancing the linear term of the highest order with the nonlinear term. A_i are constants to be determined.

Here, f satisfies the equation

$$f'^2 = pf + qf^2 + rf^3, \tag{5}$$

where p, q, r are parameters to be determined.

After substituting eq. (4) into the reduced ODE and using eq. (5), the constants A_i, p, q, r can be determined.

The mapping relation is thus established through eq. (4) between the solution to eqn. (5) and that of eq. (1).

The motivation for the choice of f is from the fact that the squares of the first derivatives of JEFs satisfy eq. (5) and so we can express the solutions of eq. (1) in terms of those functions.

3 Jacobi Elliptic Functions

Consider the function

$$F(\phi, m) = \xi = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - m^2 \sin^2\theta}}. \tag{6}$$

Letting $t = \sin\theta$, we obtain

$$\xi = \int_0^{\sin\phi} \frac{dt}{\sqrt{(1-t^2)(1-m^2t^2)}}. \tag{7}$$

This is called Legendre's standard elliptical integral of the first kind.

When $m = 0, \xi = \displaystyle\int_0^x \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} x$, where $x = \sin\phi$.
When $m = 1, \xi = \displaystyle\int_0^x \frac{dt}{1-t^2} = \tanh^{-1} x$, where $x = \sin\phi$.

For $0 < m < 1$, we define ξ as the inverse of a function which is known as Jacobi Sine elliptic function, expressed in the form $\xi = \mathrm{sn}^{-1}x$ so that $x = \mathrm{sn}\xi$ or $\mathrm{sn}\xi = \sin\phi$. Here, m is known as the modulus of the JEFs.

We define two other elliptic functions as

$$\mathrm{cn}\xi = \sqrt{1-x^2} = \sqrt{1-\mathrm{sn}^2\xi}, \tag{8}$$

$$\mathrm{dn}\xi = \sqrt{1-m^2x^2} = \sqrt{1-m^2\mathrm{sn}^2\xi}. \tag{9}$$

$\mathrm{cn}\xi$ is known as Jacobi cosine elliptic function and $\mathrm{dn}\xi$ is known as JEF of the third kind.

Some properties of JEFs are:

$$\begin{aligned}
&\mathrm{sn}^2\xi + \mathrm{cn}^2\xi = 1,\ \ \mathrm{dn}^2\xi + m^2\mathrm{sn}^2\xi = 1\\
&\mathrm{ns}\,\xi = \frac{1}{\mathrm{sn}\,\xi}, \mathrm{nc}\,\xi = \frac{1}{\mathrm{cn}\,\xi}, \mathrm{nd}\,\xi = \frac{1}{\mathrm{dn}\,\xi}\\
&\mathrm{sc}\,\xi = \frac{\mathrm{sn}\,\xi}{\mathrm{cn}\,\xi}, \mathrm{sd}\,\xi = \frac{\mathrm{sn}\,\xi}{\mathrm{dn}\,\xi}, \mathrm{cd}\,\xi = \frac{\mathrm{cn}\,\xi}{\mathrm{dn}\,\xi}\\
&\mathrm{cs}\,\xi = \frac{\mathrm{cn}\,\xi}{\mathrm{sn}\,\xi}, \mathrm{ds}\,\xi = \frac{\mathrm{dn}\,\xi}{\mathrm{sn}\,\xi}, \mathrm{dc}\,\xi = \frac{\mathrm{dn}\,\xi}{\mathrm{cn}\,\xi}
\end{aligned} \tag{10}$$

The derivatives of JEFs are given by

$$(\mathrm{sn}\,\xi)' = \mathrm{cn}\,\xi\,\mathrm{dn}\,\xi,\ \ (\mathrm{cn}\,\xi)' = -\mathrm{sn}\,\xi\,\mathrm{dn}\,\xi,\ \ (\mathrm{dn}\,\xi)' = -\mathrm{m}^2\mathrm{sn}\,\xi\,\mathrm{cn}\,\xi. \tag{11}$$

When $m \to 0$, the JEFs degenerate to the triangular functions, that is,

$$\mathrm{sn}\,\xi \to \sin\xi,\ \ \mathrm{cn}\,\xi \to \cos\xi,\ \ \mathrm{dn}\,\xi \to 1 \tag{12}$$

and when $m \to 1$, the JEFs degenerate to the hyperbolic functions, that is,

$$\mathrm{sn}\,\xi \to \tanh\xi,\ \ \mathrm{cn}\,\xi \to \mathrm{sech}\,\xi,\ \ \mathrm{dn}\,\xi \to \mathrm{sech}\,\xi. \tag{13}$$

4 Fractional Boussinesq Equation

We Consider the fractional Boussinesq equation

$$D_t^{2\alpha}q - k^2D_x^{2\alpha} + aD_x^{2\alpha}(q^{2n}) + b_1D_x^{4\alpha}q + b_2D_x^{2\alpha}D_t^{2\alpha}q = 0, \tag{14}$$

where a, b_1 and b_2 are arbitrary constants.

We are looking for TWSs of eq. (14) in the form

$$q(x,t) = U(s), \ \ s = \frac{x^{\alpha}}{\Gamma(1+\alpha)} - \frac{vt^{\alpha}}{\Gamma(1+\alpha)}. \tag{15}$$

Substituting eq. (15) into eq. (14), we get

$$(v^2 - k^2)U'' + a(U^{2n})'' + (b_1 + b_2 v^2)U'''' = 0, \tag{16}$$

where prime denotes differentiation with respect to s.

Integrating eq. (16) twice with respect to s and keeping the integration constants equal to 0, we obtain

$$(v^2 - k^2)U + aU^{2n} + (b_1 + b_2 v^2)U'' = 0. \tag{17}$$

Letting $U(s) = [u(s)]^{1/2n-1}$, eq. (17) can be written as

$$(2n-1)^2(v^2 - k^2)u^2 + a(2n-1)^2 u^3 + (b_1 + b_2 v^2)[2(1-n)u'^2 + (2n-1)uu''] = 0. \tag{18}$$

We rewrite eq. (18) as

$$Au'^2 + Buu'' + Cu^2 + Du^3 = 0, \tag{19}$$

where,

$$A = 2(1-n)(b_1 + b_2 v^2), \tag{20}$$

$$B = (2n-1)(b_1 + b_2 v^2), \tag{21}$$

$$C = (2n-1)^2(v^2 - k^2), \tag{22}$$

$$D = a(2n-1)^2. \tag{23}$$

5 Solution by Mapping Method

We apply the mathematical analysis of the mapping method to eq. (19). Here, we can see that $l = 1$. So, we assume the solution of eq. (19) in the form

$$u(s) = A_0 + A_1 f(s), \tag{24}$$

where f satisfies eq. (3).

Substituting eq. (24) into eq. (19) and equating the coefficients of powers of f to zero, we arrive at the following set of algebraic equations:

$$f^3: \quad AA_1^2 r + \frac{3}{2}BA_1^2 r + DA_1^3 = 0, \tag{25}$$

$$f^2: \quad AA_1^2 q + \frac{3}{2}BA_0A_1 r + BA_1^2 q + CA_1^2 + 3DA_0A_1^2 = 0, \tag{26}$$

$$f: \quad AA_1^2 p + BA_0A_1 q + \frac{1}{2}BA_1^2 p + 2CA_0A_1 + 3DA_0^2A_1 = 0, \tag{27}$$

$$f^0: \quad \frac{1}{2}BA_0A_1 p + CA_0^2 + DA_0^3 = 0. \tag{28}$$

From eqs. (25) and (26), we get

$$A_0 = -\frac{(Aq + Bq + C)(2A + 3B)}{6D(A + B)}, \quad A_1 = -\frac{r}{2D}(2A + 3B). \tag{29}$$

From eqs. (27) and (28), we obtain the constraint condition

$$AA_1 p + BA_0 q + CA_0 + 2DA_0^2 = 0. \tag{30}$$

Case 1. $f(s) = \tanh^2(s)$. In this case, we can see that $p = 4,\ q = -8,\ r = 4$.
So, we get

$$A_0 = \frac{(8A + 8B - C)(2A + 3B)}{6D(A + B)}, \quad A_1 = -\frac{2}{D}(2A + 3B). \tag{31}$$

Thus, the solution of eq. (14) can be written as

$$q(x,t) = \left[\frac{2A+3B}{D}\left\{\frac{8A+8B-C}{6(A+B)} - 2\tanh^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}, \tag{32}$$

with the constraint condition

$$16(A+B)^2 + C(8A+8B+C) = 0. \tag{33}$$

Case 2. $f(s) = \operatorname{sech}^2(s)$. In this case, we can see that $p = 0,\ q = 4,\ r = -4$ so that we get

$$A_0 = -\frac{(4A+4B+C)(2A+3B)}{6D(A+B)}, \quad A_1 = \frac{2}{D}(2A+3B). \tag{34}$$

So, the solution of eq. (14) becomes

$$q(x,t) = \left[-\frac{2A+3B}{D}\left\{\frac{4A+4B+C}{6(A+B)} - 2\operatorname{sech}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}, \tag{35}$$

with the constraint condition

$$4B + C + 2DA_0 = 0. \tag{36}$$

With the value of A_0, the condition given by eq. (36) reduces to

$$C = 8(A+B). \tag{37}$$

Case 3. $f(s) = \operatorname{cs}^2(s)$. Here, we have $p = 4-4m^2,\ q = 8-4m^2,\ r = 4$ so that we obtain

$$A_0 = -\frac{[(8-4m^2)(A+B)+C](2A+3B)}{6D(A+B)}, \quad A_1 = -\frac{2}{D}(2A+3B). \tag{38}$$

So, the solution of eq. (14) can be written as

$$q(x,t) = \left[-\frac{2A+3B}{D}\left\{\frac{(8-4m^2)(A+B)+C}{6(A+B)} + 2\operatorname{cs}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}, \tag{39}$$

When $m \to 1$, solution given by eq. (39) gives rise to the singular solution

$$q(x,t)=\left[-\frac{2A+3B}{D}\left\{\frac{4(A+B)+C}{6(A+B)}+2\,\mathrm{csch}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-\frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}, \tag{40}$$

with the same constraint condition given by eq. (37).

Case 4. $f(s)=\mathrm{sn}^2(s)$. Here, we have $p=4,\ q=-(4+4m^2),\ r=4m^2$ so that we get

$$A_0=\frac{[(4+4m^2)(A+B)-C](2A+3B)}{6D(A+B)},\quad A_1=-\frac{2m^2}{D}(2A+3B). \tag{41}$$

So, the solution of eq. (14) can be written as

$$q(x,t)=\left[\frac{2A+3B}{D}\left\{\frac{(4+4m^2)(A+B)-C}{6(A+B)}-2\,m^2\mathrm{sn}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-\frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}, \tag{42}$$

As $m\to 1$,we obtain the same solution and the constraint condition given by eq. (32.

Case 5. $f(s)=\mathrm{cn}^2(s)$. So, we have $p=4(1-m^2),\ q=8m^2-4,\ r=-4m^2$ so that we arrive at

$$A_0=\frac{[(4-8m^2)(A+B)-C](2A+3B)}{6D(A+B)},\quad A_1=\frac{2m^2}{D}(2A+3B). \tag{43}$$

Thus, the solution of eq. (12) can be written as

$$q(x,t)=\left[\frac{2A+3B}{D}\left\{\frac{(4-8m^2)(A+B)-C}{6(A+B)}+2\,m^2\mathrm{cn}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-\frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right\}\right]^{1/2n-1}. \tag{44}$$

When $m\to 1$,we obtain the same solution and the constraint condition given by eq. (37).

6 Solution by modified mapping method

In this method, we assume the solution of eq.(19) in the form

$$u(s)=A_0+A_1f(s)+B_1f^{-1}(s), \tag{45}$$

where f^{-1} is the reciprocal of f.

Substituting eq.(45) into eq.(19) and using eq.(3), we arrive at a set of algebraic equations by equating the coeffients of various powers of f to zero given by

$$f^3: \quad AA_1^2r + \frac{3}{2}BA_1^2r + DA_1^3 = 0, \tag{46}$$

$$f^2: \quad AA_1^2q + \frac{3}{2}BA_0A_1r + BA_1^2q + CA_1^2 + 3DA_0A_1^2 = 0, \tag{47}$$

$$\begin{aligned} f: \quad & AA_1^2p - 2AA_1B_1r + BA_0A_1q + 2BA_1B_1r \\ & + \frac{1}{2}BA_1^2p + 2CA_0A_1 + 3DA_1^2B_1 + 3DA_0^2A_1 = 0, \end{aligned} \tag{48}$$

$$\begin{aligned} f^0: \quad & -2AA_1B_1q + \frac{1}{2}BA_0B_1r + 2BA_1B_1q + 6DA_0A_1B_1 \\ & + CA_0^2 + \frac{1}{2}BA_0A_1p + 2CA_1B_1 + DA_0^3 = 0, \end{aligned} \tag{49}$$

$$\begin{aligned} f^{-1}: \quad & -2AA_1B_1p + AB_1^2r + \frac{1}{2}BB_1^2r + BA_0B_1q \\ & + 2BA_1B_1p + 2CA_0B_1 + 3DA_1B_1^2 + 3DA_0^2B_1 = 0, \end{aligned} \tag{50}$$

$$f^{-2}: \quad AB_1^2q + BB_1^2q + \frac{3}{2}BA_0B_1p + CB_1^2 + 3DA_0B_1^2 = 0, \tag{51}$$

$$f^{-3}: \quad AB_1^2p + \frac{3}{2}BB_1^2p + DB_1^3 = 0. \tag{52}$$

From eqs. (46), (47), (51) and (52), we derive the following:

$$\begin{aligned} & A_1 = -\frac{r}{2D}(2A+3B), \;\; B_1 = -\frac{p}{2D}(2A+3B) \\ & rB_1 = pA_1, \;\; A_0 = \frac{A_1(Aq+Bq+C)}{3r(A+B)}. \end{aligned} \tag{53}$$

Eqs. (48), (49) and (50) give rise to a very cumbersome constraint condition which is omitted in the paper.

Case 1. $f(s) = \tanh^2(s)$. In this case, we can see that $p = 4, \; q = -8, \; r = 4$.

So, we get

$$A_0 = \frac{(2A+3B)(8A+8B-C)}{6D(A+B)}, \quad A_1 = B_1 = -\frac{2(2A+3B)}{D}. \tag{54}$$

Thus the solution of eq. (14) is

$$q(x,t) = \left[A_0 + A_1 \tanh^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right) + B_1 \coth^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right]^{1/2n-1}, \tag{55}$$

where A_0, A_1 and B_1 are given by eq. (54).

Case 2. $f(s) = \mathrm{cs}^2(s)$. In this case, we can see that $p = 4-4m^2,\ q = 8-4m^2,\ r = 4.$

Here, we get

$$A_0 = -\frac{(2A+3B)[(8-4m^2)(A+B)+C]}{6D(A+B)}, \quad A_1 = -\frac{2(2A+3B)}{D}, \quad B_1 = \frac{(2m^2-2)(2A+3B)}{D}. \tag{56}$$

Thus the solution of eq. (14) can be written as

$$q(x,t) = \left[A_0 + A_1 \,\mathrm{cs}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right) + B_1 \,\mathrm{sc}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right]^{1/2n-1}, \tag{57}$$

where A_0, A_1 and B_1 are given by eq. (56).

As $m \to 1$, solution given by eq. (57) reduces to eq. (40).

Case 3. $f(s) = \mathrm{cn}^2(s)$. In this case, we can see that $p = 4-4m^2,\ q = 8m^2-4$ $r = -4m^2$.

In this case, we get

$$A_0 = -\frac{(2A+3B)[(8m^2-4)(A+B)+C]}{6D(A+B)}, \quad A_1 = \frac{2m^2(2A+3B)}{D}, \quad B_1 = \frac{(2m^2-2)(2A+3B)}{D}. \tag{58}$$

Thus the solution of eq. (14) becomes

$$q(x,t) = \left[A_0 + A_1 \,\mathrm{cn}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right) + B_1 \,\mathrm{nc}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right]^{1/2n-1}, \tag{59}$$

where A_0, A_1 and B_1 are given by eq. (58).

As $m \to 1$, solution given by eq. (59) reduces to eq. (35).

7 Solution by Tanh Method

In this method, the Tanh function is introduced as a new variable since all derivatives of tanh can be represented as a polynomial in tanh itself. We apply this method to eq. (19) with $A = 1, B = -2, C = 4, D = 1$. With these values of the parameters, eq. (19) reduces to

$$u'^2 - 2uu'' + 4u^2 + u^3 = 0. \tag{60}$$

With the assumption that TWSs can be expressed in terms of $\tanh(s)$, we introduce $y = \tanh(s)$ as the new dependent variable and consider solutions of the form

$$u(s) = S(y) = \sum_{n=0}^{N} a_n y^n, \;\; y = \tanh(s). \tag{61}$$

So, we can write

$$\frac{d}{ds} = (1 - y^2)\frac{d}{dy}, \tag{62}$$

$$\frac{d^2}{ds^2} = (1 - y^2)\left[-2y\frac{d}{dy} + (1 - y^2)\frac{d^2}{dy^2}\right]. \tag{63}$$

Now, eq. (60) can be written as

$$\left[(1 - y^2)\frac{dS}{dy}\right]^2 - 2S(1 - y^2)\left[-2y\frac{dS}{dy} + (1 - y^2)\frac{d^2S}{dy^2}\right] + 4S^2 + S^3 = 0. \tag{64}$$

By substituting eq. (61) into eq. (64) and balancing the highest degree terms in y, we get $N = 2$ so that we can assume the solution of eq. (64) in the form

$$S(y) = a_0 + a_1 y + a_2 y^2. \tag{65}$$

By substituting eq. (65) into eq. (64) and equating the coefficients of different powers of y to zero, we obtain $a_0 = -8, a_1 = 0, a_2 = 8$.

Thus the solution of eq. (60) is

$$u(s) = -8\,\mathrm{sech}^2(s). \tag{66}$$

Thus the solution of eq. (14) for the special values of A, B, C and D can be written as

$$q(x,t) = -8\left[\mathrm{sech}^2\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{vt^\alpha}{\Gamma(1+\alpha)}\right)\right]^{1/2n-1} \tag{67}$$

which confirms the correctness of the solution given by eq. (32) with the constraint condition given by eq. (33) also satisfied.

Fig.(1)-(8) represent the solution for Eq(14),by taking different vales for α and n

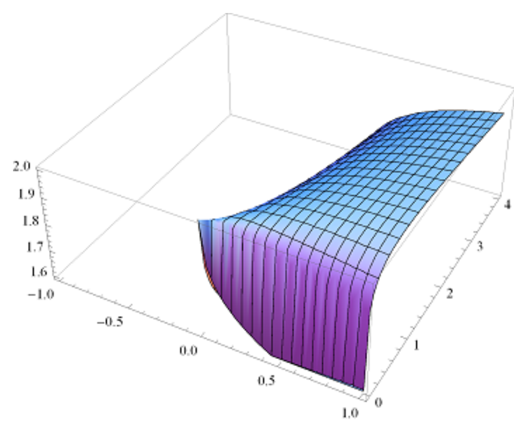

Figure 1: Solution for $q(x,t)$ when $n = 2, \alpha = 0.2$

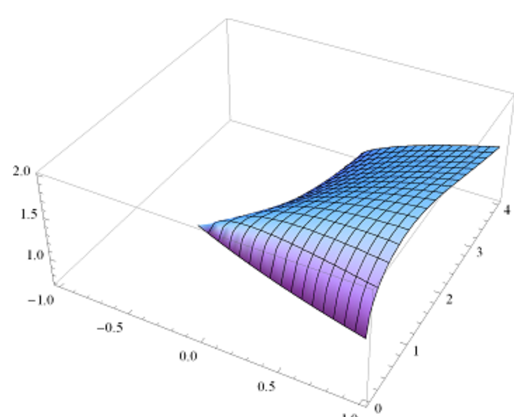

Figure 2: Solution for $q(x,t)$ when $n = 2, \alpha = 0.5$

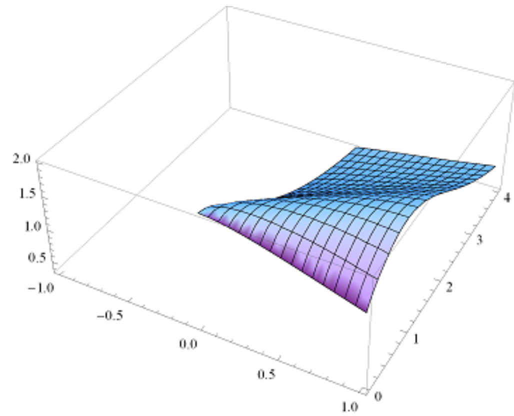

Figure 3: Solution for $q(x,t)$ when $n=2, \alpha=0.8$

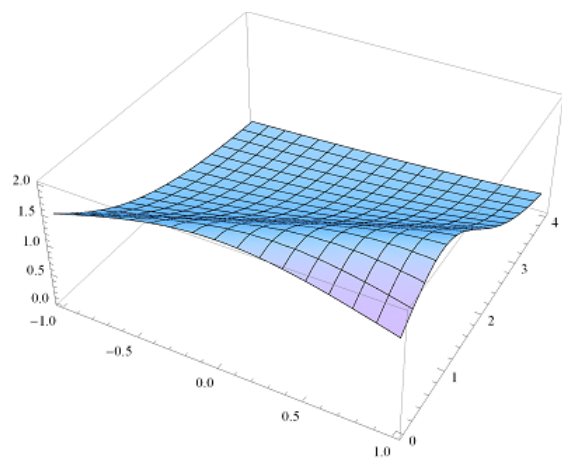

Figure 4: Solution for $q(x,t)$ when $n=2, \alpha=1$

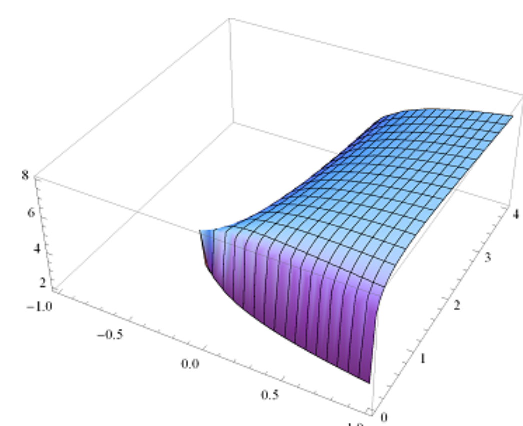

Figure 5: Solution for $q(x,t)$ when $n=1, \alpha=0.2$

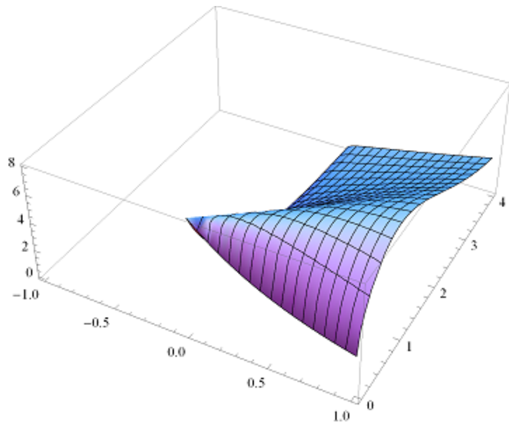

Figure 6: Solution for $q(x,t)$ when $n=1, \alpha=0.5$

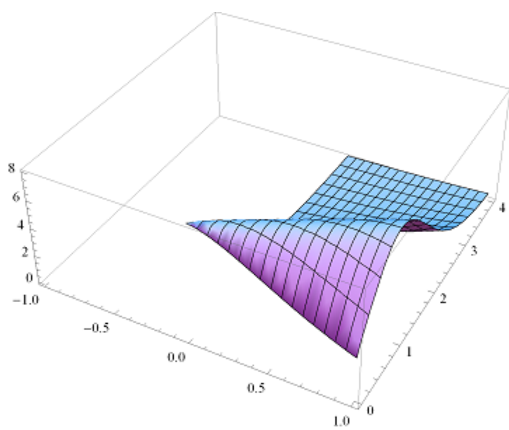

Figure 7: Solution for $q(x,t)$ when $n=1, \alpha=0.8$

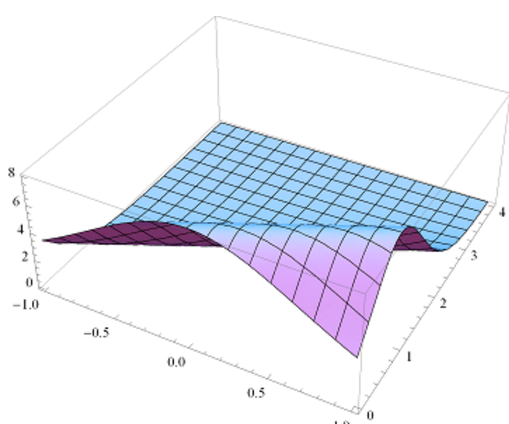

Figure 8: Solution for $q(x,t)$ when $n=1, \alpha=1$

8 Conclusion

The Fractional Boussinesq equation has been solved using mapping methods solutions of which are expressed in terms of JEFs. When the modulus m of the elliptic functions approaches 1, it leads to SWSs as well as singular wave solutions. The correctness of the SWS has been checked using the tanh method. SWSs for different values of n and α have been graphically illustrated.

References

[1] W. Hereman, P.P. Banerjee, A. Korpel, G. Assanto, A. Van Immerzeele and A. Meerpoel, Exact solitary wave solutions of non-linear evolution and wave equations using a direct algebraic method, *Journal of Physics A*, **19** (1986), 607-628. https://doi.org/10.1088/0305-4470/19/5/016

[2] M.W. Coffey, On series expansions giving closed-form solutions of Korteweg-de Vries-like equations, *SIAM Journal of Applied Mathematics*, **50** (1990), 1580-1592. https://doi.org/10.1137/0150093

[3] T. Ying-Hui, C. Han-Lin and L. Xi-Qiang, New exact solutions to long-short wave interaction equations, *Communications in Theoretical Physics*, **46** (2006), 397-402. https://doi.org/10.1088/0253-6102/46/3/004

[4] C. Deng and Y. Shang, Construction of exact periodic wave and solitary wave solutions for the long-short wave resonance equations by VIM, *Communications in Nonlinear Science and Numerical Simulation*, **14** (2009), 1186-1195. https://doi.org/10.1016/j.cnsns.2008.01.005

[5] A. Biswas and H. Triki, 1-soliton solution of the Klein-Gordon-Schrodinger's equation with power law nonlinearity, *Applied Mathematics and Computation*, **217** (2010), 3869-3874. https://doi.org/10.1016/j.amc.2010.09.046

[6] M. Labidi, H. Triki, E.V. Krishnan and A. Biswas, Soliton Solutions of the Long-Short Wave Equation with Power Law Nonlinearity, *Journal of Applied Nonlinear Dynamics*, **1** (2012), 125-140. https://doi.org/10.5890/jand.2012.05.002

[7] W. Malfliet and Willy Hereman, The tanh method: 1. Exact solutions of nonlinear evolution and wave equations, *Physica Scripta*, **54** (1996), 563-568. https://doi.org/10.1088/0031-8949/54/6/003

[8] S. Shukri and K. Al-Khaled, The extended tanh method for solving systems of nonlinear wave equations, *Applied Mathematics and Computation*, **217** (2010), 1997-2006. https://doi.org/10.1016/j.amc.2010.06.058

[9] M. Alquran and K. Al-Khaled, Mathematical methods for a reliable treatment of the (2+1)-dimensional Zoomeron equation, *Mathematical Sciences*, **6** (2012), 11. https://doi.org/10.1186/2251-7456-6-11

[10] K.R. Raslan, The Application of He's Exp-function Method for MKdV and Burgers Equations with Variable Coefficients, *International Journal of Nonlinear Science*, **7** (2009), 174-181.

[11] M. Alquran and A. Qawasmeh, Soliton solutions of shallow water wave equations by means of G'/G expansion method, *Journal of Applied Analysis and Computation*, **4** (2014), 221-229.

[12] A. Qawasmeh and M. Alquran, Reliable study of some fifth-order nonlinear equations by means of G'/G expansion method and rational sine-cosine method, *Applied Mathematical Sciences*, **8** (2014), 5985-5994. http://dx.doi.org/10.12988/ams.2014.48669

[13] A. Qawasmeh and M. Alquran, Soliton and periodic solutions for (2+1)-dimensional dispersive long water-wave system, *Applied Mathematical Sciences*, **8** (2014), 2455-2463. https://doi.org/10.12988/ams.2014.43170

[14] Y. Peng, Exact periodic wave solutions to a new Hamiltonian amplitude equation, *Journal of the Physical Society of Japan*, **72** (2003), 1356-1359. https://doi.org/10.1143/jpsj.72.1356

[15] E.V. Krishnan and Y. Peng, A new solitary wave solution for the new Hamiltonian amplitude equation, *Journal of the Physical Society of Japan*, **74** (2005), 896-897. https://doi.org/10.1143/jpsj.74.896

[16] Y.Z. Peng and E.V. Krishnan, Exact travelling wave solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation, *Acta Physica Polonica*, **108** (2005), 421-428. https://doi.org/10.12693/aphyspola.108.421

[17] Muna Al Ghabshi, E.V. Krishnan, Kamel Al-Khaled and Marwan Alquran, Exact and approximate solutions of a system of partial differential equations, *International Journal of Nonlinear Science*, **23** (2017), 11-21.

[18] E.V. Krishnan and K.S. Al Ghafri, Jacobi elliptic elliptic function solutions of a K(n,n)-type equation, *International Journal of Mathematical Analysis*, **10** (2016), 1229-1243. https://doi.org/10.12988/ijma.2016.68101

[19] D.F. Lawden, *Elliptic Functions and Applications*, Springer Verlag, 1989. https://doi.org/10.1007/978-1-4757-3980-0

[20] E.V. Krishnan, Jacobi elliptic function solutions of a fractional nonlinear evolution equation, *International Journal of Mathematical Analysis*, **10** (2016), 127-138. https://doi.org/10.12988/ijma.2016.511282

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