# Mathematical Modeling of Upflow Anaerobic Sludge Blanket Reactor in Domestic Wastewater Treatment

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**Abstract** - This paper introduces a dynamic model to describe an Upflow Anaerobic Sludge Blanket (UASB) reactor. The models of a UASB reactor are discussed in order to modify their drawbacks and propose new improved model with less complexity and more reliability. The developed model is a combination of two recent models introduced in Sweden. According to this model, UASB reactor is divided hydraulically into three compartments with integration of a kinetic model. Simulations were performed to investigate the validity of the developed model which indicates a good agreement with experimental data. Moreover, the developed model appears to be sensitive to variation of hydraulic and kinetic parameters through sensitivity analysis.

**Index Terms** - modeling; UASB reactor; sewage treatment; sensitivity analysis.

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$$\ldots + R_S Y$$

$$\phi = \sqrt{\ldots}$$


Fig.1. Substrate concentration degradation at (a) various dispersion coefficients, and (b) different factions of reactor volume occupied by granules

[illegible]

## B. Model proposed by Parsamehr, Sweden

[illegible]

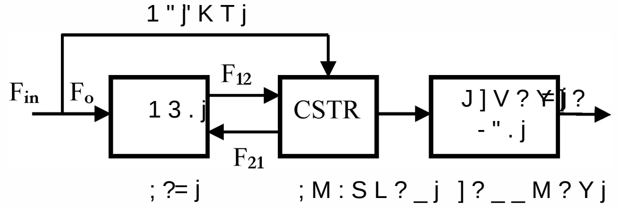


Fig.2. A schematic block diagram for hydraulic model of the UASB reactor

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$$V_1 \frac{dC_{X1}}{dt} = F_o C_{Xin} - \eta_{dr} x q_{gb} C_{X1} + A C_{X2} v_s + R_{S1} Y V_1$$

$$V_2 \frac{dC_{X2}}{dt} = \eta_{dr} x q_{gb} C_{X1} - A C_{X2} v_s - (1-\eta) F_{in} C_{X2} + R_{S2} Y V_2 \quad (6)$$

$$V_1 \frac{dC_{S1}}{dt} = F_o C_{Sin} + F_{21} C_{S2} - F_{12} C_{S1} - R_{S1} V_1$$

$$V_2 \frac{dC_{S2}}{dt} = F_{12} C_{S1} + SF\, F_{in} C_{Sin} - F_{21} C_{S2} - F_{in} C_{S2} - R_{S2} V_2$$

$$R_S = (\mu - K_d)\frac{C_X}{Y}, \quad \frac{dC_{S3}}{dt} = D\frac{\partial^2 C_{S3}}{\partial z^2} - W\frac{\partial C_{S3}}{\partial z}$$

$$F_{in} = SF\, F_{in} + F_o, \quad F_{12} = F_{21} + F_o$$

$$F_{21} = q_{gb}\left(\frac{10}{10 + h_2 + h_3}\right)$$

$$R_{CH4} = f_{CH4}\frac{(1-Y)\mu_1 C_{X1}}{Y}$$

$$C_{HS} = \frac{10^{-pH} C_S}{K_A + 10^{-pH}} \quad (13)$$

[illegible]

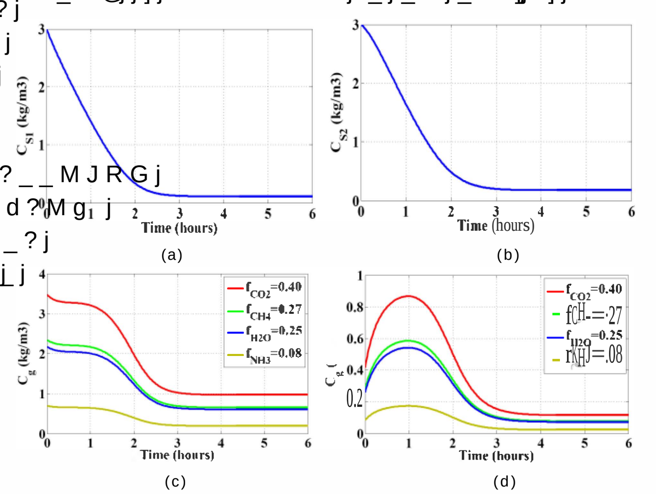
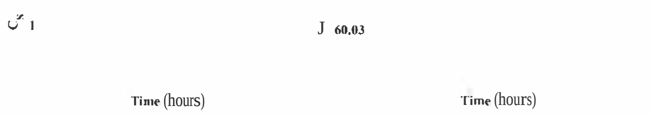



Fig.3. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time, and gases concentration produced (c) in reactor bed zone versus time, and (d) in reactor blanket zone versus time




(a) (b)

Fig.4. (a) Substrate concentration degradation in the UASB reactor versus time, and (b) The effective biomass concentration in the UASB reactor versus time

[illegible]

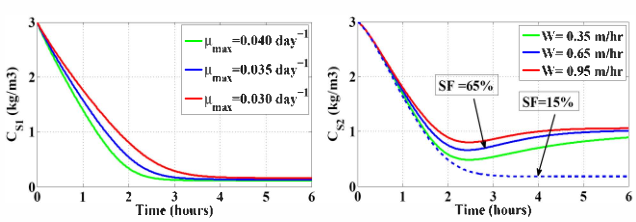
[illegible]

A. Previous experimental results

[illegible]

$E_{COD}$ = [illegible]

$E_{COD}$ = [illegible]




(a) (b)

Fig.5. (a) Effect of bacterial growth rate on reactor bed zone performance, and (b) Effect of upflow velocity and bypass flow faction on reactor blanket zone performance

B. Developed model for VASE reactor

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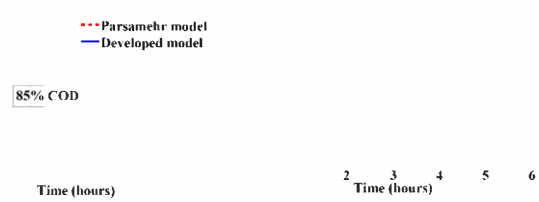
C. Sensitivity analysis for developed model

[illegible]




(a) (b)

Fig.6. Substrate concentration degradation (a) in reactor bed zone versus time, and (b) in reactor blanket zone versus time

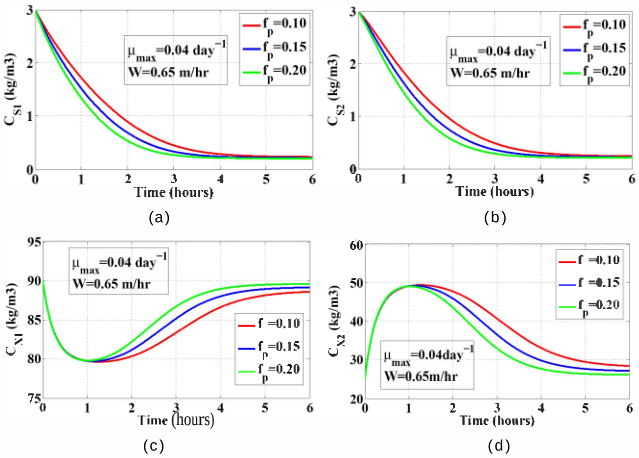



Fig.7. Substrate concentration in the UASB reactor (a) bed zone versus time and (b) blanket zone versus time, and Biomass concentration in the UASB reactor (c) bed zone versus time and (d) blanket zone versus time

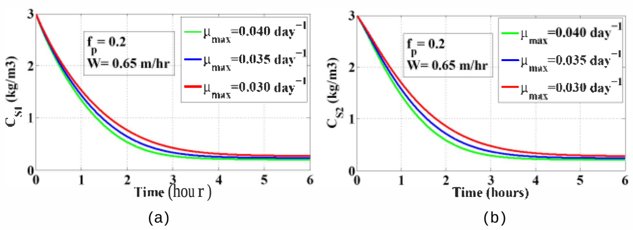



Fig.8. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

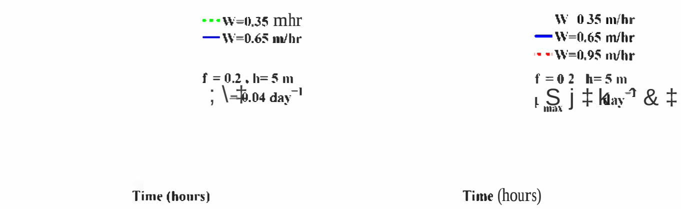



Fig.9. Substrate concentration in the UASB reactor (a) bed zone versus time, and (b) blanket zone versus time

$6 j+* '41$+*1j

3H?BjGaYU?@H=??d?M=UOVU?=?UM@j1 jY?:<_UYj]HUe j Y?OUd:Mj?@UB@JH?Rj<gjjaj;]_Y:<_U?R<?R_Y: JUR @U3YU@jHUaYjjUj@jj<O:jR=j'jU@jLGj+ O =:gj eHJ<NjVYU:VNjYUfJOY?:]?ajMj_]jH:U_@j & j 3H?=j?d?M=UOVU?=?MjV?:_YUjj;?Yj?jMJ:<M?ajYO_U?‡?Mj @U=Y?j<YJ4JERGjY?:<_VU?Yj@UYO:R<? j

ABBREVIATIONS

| Symbol | Definition | Unit |
|---|---|---|
| 5‡ | cross sectional area of reactor | $m^2$ |
| CHSS, ' CSo, CSm | unionized, variant and inlet substrate concentration in reactor | $kg/m^3$ |
| Cx, CXo l | various, initial and inlet biomass concentration in reactor | $kg/m^3$ |
| D, DA | dispersion and effective diffusion coefficients | $m^2/hr$ |
| E | substrate removal efficiency | % |
| Fin | influent wastewater discharge | $m^3/hr$ |
| J, ¢ | fraction of volume of reactor occupied by granules and Thiele modulus | —— |
| V‡ | height of reactor | m |
| K, k | chemical reaction constant and substrate volumetric conversion rate | $hr^{-1}$ |
| $k_L$ | mass transfer coefficient | m/hr |
| $K_A$, $K_1$, $K_s$ | substrate equilibrium, inhibition and Monod half saturation constants | $kg/m^3$ |
| Kd U‡U‡‡ | microorganism decay, specific growth and maximum specific growth rates | $hr^{-1}$ |
| >‡ | organic loading rate as BOD | $kg/m^3$ day |
| r, drr | dragging and settling efficiencies | % |
| A‡ | radius of the biological granule | m |
| $R_{CH4}$, $R_s$ | methane production, biomass reaction (substrate utilization) rates | $kg/m^3$ hr |
| s | sludge thickness | cm |
| e, ‡ | various hydraulic retention time | hours |
| G‡ | volume of reactor | $m^3$ |
| Hc J‡ | settling and upflow velocities | m/hr |
| x | drag coefficient of biomass base | $m^3/m^3$ |
| h‡ | yield coefficient of biomass per substrate | kg/kg |
| z | vertical height of reactor | m |

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