



# An optimization approach to aircraft dispatching strategy with maintenance cost – A case study



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## ABSTRACT

This paper presents an optimization approach to identify aircraft dispatching strategy at a flight training school. The strategy adopted by the school was to dispatch the aircraft which is closest to its scheduled maintenance. This strategy was examined and compared with other potential dispatching strategies. The paper presents a mixed integer linear programming model to identify the strategy that minimizes the total cost of scheduled maintenance. The analysis shows that the optimization approach can save 2%–5% on annual maintenance cost compared with other strategies. The model can equally be applied to rental cars or trucking companies.

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## 1. Introduction

Embry-Riddle Aeronautical University (ERAU) at its Daytona Beach campus offers various aviation degree programs including flying single and multi-engine aircraft. The University has more than 1000 flight students, 41 Cessna 172 single engine and 6 Diamond multi engine aircraft as well as diverse simulators in its flight school. All of these aircraft are leased. The school has qualified crew who perform scheduled and unscheduled maintenance programs on all types of aircraft. On average, there are about 2700 flight training sessions for single engine aircraft per month. Each training session takes about 1.6–1.8 h. However, all flight training sessions are reserved on a 2-h time blocks to accommodate briefing, reports, etc. The flight Dept. at ERAU, is responsible for training, operation and maintenance of aircraft. It has set the scheduled maintenance program for every 50 flying hours for the Cessna 172 single engine aircraft. Of course, the maintenance scope and cost differ for each of these scheduled maintenance.

The current strategy of the flight Dept. for dispatching an aircraft to students is to utilize the one closest to its 50-h scheduled maintenance. The Dept. is, however, interested in exploring other assignment strategies resulting in lower maintenance cost and/or higher aircraft availability to students. This paper attempts to present a mathematical modeling approach to identify a dispatching strategy which results in minimum total annual maintenance cost and increased aircraft availability. This strategy is compared and contrasted with other strategies including the

existing dispatching strategy. Section 2 provides a literature review of existing models applied to similar industries. Section 3 introduces parameters specific to this case. Sections 4–6 present the mathematical model, computation analyses and performance evaluations of various strategies. Finally, section 7 concludes this paper.

## 2. Literature review

This case study has some similarities to multiple asset management which has been extensively studied in the literature. In multiple asset management, the focus is to manage multiple resources to meet demand typically at different locations. Examples include rental cars (see for example Pachon et al., 2006, Li and Tao, 2010), rail cars (Papier and Thonemann, 2007; Bojovic, 2002) and truck assignments (Miao et al., 2009). In these research streams the focus is to assign multiple resources to a number of customers at different locations and at minimum cost. These studies do not address varying cost of maintenance with usage. They adopt a variety of network optimization models to satisfy demand at different locations. The literature on multiple resource allocation with varying cost is very scarce. The study by Hertz et al. (2009) considers varying maintenance cost in a rental car company. However, the scope of the research is completely different from this study. They propose an inventory control model to purchase new cars by examining the existing fleet to satisfy the demand.

Other related research studies include parallel machines scheduling (see for example Cheng et al., 2011 or Kubzin and Strusevich, 2006) where only one maintenance activity is allowed throughout the makespan.

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The commercial airlines' aircraft routing in the literature is primarily focused at maximizing utilization and route aircraft to hubs for schedule maintenance (Bazargan, 2010). In these models, maintenance is included as a side constraint to insure the aircraft is at the right hub for maintenance after certain number of flight hours (See for example Barnhart et al., 1998 or Li and Wang, 2005).

Although the above research works provide some information on standardized problems, they do not capture the scope and nature of the current case study where maintenance cost varies with usage. To the best of our knowledge, we are not aware of any similar study, where maintenance cost is the driving force to utilize a resource among multiple resources.

### 3. Aircraft maintenance cost

Maintenance activities are the backbone of successful aircraft operations. In the aviation industry, the role of maintenance is to provide safe and airworthy aircraft every day. The Cessna 172 aircraft are popular and extensively used for training flight students at different flight schools worldwide. The ERAU's flight Dept. utilizes 41 of these aircraft for training flight students.

Similar to cars, the types and scopes of scheduled maintenance programs vary with usage. This variation in maintenance programs lead to different cost depending on usage. In the aviation industry, a typical metric for aircraft usage, is tach times. Tach times, broadly defined as the number of hours that the aircraft engine(s) have been running. Throughout this paper, when hours are mentioned they are meant to be the tach times.

The flight Dept. requires scheduled maintenance to be performed for every 50 h of usage for Cessna 172 aircraft. It should be noted that the school's maintenance program is more stringent than those recommended by the manufacturer. The manufacturer's recommended maintenance program for Cessna 172 does not require scheduled maintenance for every 50 h of flight.

Fig. 1 presents the cost of scheduled maintenance (in US\$) for this fleet of aircraft against these 50 h utilizations. The cost of maintenance includes labor and parts. As an example, the 50 h scheduled maintenance cost is \$1085.38, for 100 h it is \$1693.24, etc. This figure provides the maintenance costs for up to 2250 h. These cost figures repeat themselves every 2250 h. The flight Dept. continues to use these aircraft up to 7200 h where they are replaced with new ones.

There are currently 41 Cessna 172 aircraft available for training sessions. For any scheduled training there may be more than one aircraft available. In that case, the strategy that the flight Dept. adopts for aircraft dispatching is to utilize the aircraft which is closest to its scheduled maintenance. This algorithm is programmed into the flight dispatching system and automatically

selects the aircraft for training based on their current flight hours. As an example, if there are two aircraft with 1587.2 and 2461.6 flight hours available for dispatching, the system dispatches the former since this aircraft has 12.8 h (1600–1587.2) while the latter has 38.4 h (2500–2461.6) left to its scheduled maintenance. The scope of this study is therefore to identify and evaluate other strategies. In particular, those strategies that minimize total maintenance cost and/or maximize aircraft availability. It should be noted that at peak times some training sessions are canceled because there is no aircraft available.

### 4. Mathematical model

This section presents the mathematical approach to derive the strategies that lead to minimum dispatching cost and/or maximize aircraft availability as discussed earlier. The first model attempts to identify aircraft to be dispatched in an effort to minimize the total maintenance cost over the planning period. This model is then modified to address maximizing aircraft availability.

In this model we use the term *planning period* to signify the desired period of time for aircraft dispatching (for example in one year). This is the time period included in the model to minimize maintenance cost and maximize aircraft availability.

The description of the mathematical model is as follows:

Index:

$J$  = index for aircraft ( $j = 1, \dots, A$ )

$k$  = index for maintenance program ( $k = 1, \dots, K$ )

Decision variables:

$x_j$  = hours added to aircraft  $j$  at the end of planning period

$$y_{k,j} = \begin{cases} 1 & \text{aircraft } j \text{ has reached maintenance } k \\ 0 & \text{Otherwise} \end{cases}$$

$H_j$  = Total hours on aircraft  $j$  at the end of the planning period

Parameters

$I_j$  = Initial total flight time hours on aircraft  $j$  at the beginning of planning period

$C_{k,j}$  = Cost of maintenance (in dollars) for maintenance  $k$  on aircraft  $j$

$M_{k,j}$  = Scheduled hours for maintenance  $k$  on aircraft  $j$

$p_{k,j}$  = Taking a value of 1 if maintenance type  $k$  has been performed on aircraft  $j$  at the beginning of planning period and 0 otherwise

$T$  = Average flying time of a flight

$ACT$  = Number of flight activities in the planning period

$M$  = An arbitrary large positive number

Mathematical Model:

$$\text{Minimize } \sum_{k=1}^K \sum_{j=1}^A (y_{k,j} - p_{k,j}) c_{k,j} \tag{1}$$

Subject to:

$$H_j = I_j + x_j \quad \forall j \tag{2}$$

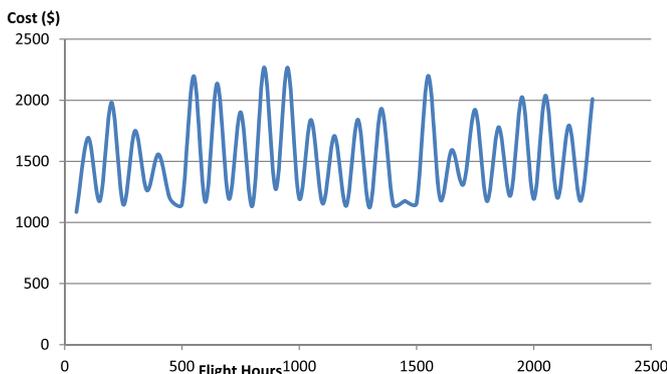


Fig. 1. Scheduled maintenance cost versus aircraft flight times.

$$\sum_{j=1}^A x_j \geq ACT \times T \tag{3}$$

$$H_j - M_{k,j} \leq My_{k,j} \quad \forall k,j \tag{4}$$

$$H_j - M_{k,j} \geq M(y_{k,j} - 1) \quad \forall k,j \tag{5}$$

$$I_j - M_{k,j} \leq Mp_{k,j} \quad \forall k,j \tag{6}$$

$$I_j - M_{k,j} \geq M(p_{k,j} - 1) \quad \forall k,j \tag{7}$$

$$y_{k,j} \in \{0, 1\} \tag{8}$$

The objective function (1) attempts to minimize the total maintenance cost over the planning period. The set of constraints (2) determines the total hours on aircraft  $j$  after the planning period. Constraints (3) insure that the total number of hours flown on all aircraft is greater or equal to the required number of aircraft hours. Constraints (4) and (5) identify what maintenance programs are needed for the total hours on any aircraft during the planning period. Finally, constraints (6) and (7) identify what maintenance programs are performed on any aircraft before the planning period. The purpose of constraints (4)–(7) is to insure that only those maintenance programs that were done during the planning period are included in the objective function. Finally, constraint (8) specifies the binary status of variable  $y_{k,j}$ .

The arbitrary large number ( $M$ ) in the constraints (4)–(7) is intended to impose the right logic within these constraints. These are referred to as ‘If-then logic’ constraints, and are utilized to make sure the binary decision variable  $y_{k,j}$  takes the right value 1 or 0.

It should be note that the two decision variables  $x_j$  and  $H_j$  represent the total hours added to and total hours on the aircraft after the planning period. As an example, assume that at the beginning of the planning period the total hours on an aircraft is 1655 h. After the planning period the total hours on that aircraft is 5482 h. In that case, this total hours on the aircraft (5482 h) is designated by  $H_j$  and the total hours added to the aircraft ( $5482 - 1655 = 3827$  h) during the planning period is represented by  $x_j$ .

The above mathematical model and its objective function attempts to minimize the total maintenance cost. However, aircraft unavailability due to maintenance is also identified as an undesirable outcome which should be minimized. To address this concern, the objective function for the above mathematical model is minimally changed as follows.

$$\text{Minimize } \sum_{k=1}^K \sum_{j=1}^A (y_{k,j} - p_{k,j}) \tag{9}$$

This change will minimize the total number of scheduled maintenances performed on all aircraft. The revised objective function utilizes all aircraft in an effort to maximize aircraft availability through reducing number of maintenance performed.

### 5. Computational analysis

The parameters needed to run the above models were collected from the flight Dept. as follows:

- The planning period for this study is set to 1 year.
- The total number of flight activities in one year is 32,000 ( $ACT$ ).

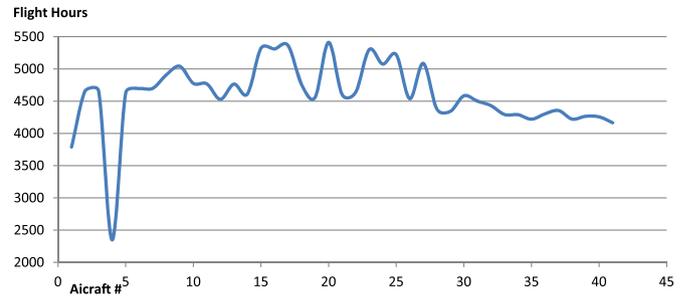


Fig. 2. Single engine aircraft flying hours at the beginning of the planning period.

- The average flight time ( $T$ ) is about 1.6 h within the two hour block time. The rest of the two-hour time block is spent on briefing, questions and answers and filling forms.
- Cost for each scheduled maintenance ( $c_{k,j}$ ) were compiled as explained in Section 3 and Fig. 1.
- Scheduled maintenance programs ( $M_{k,j}$ ) are set to 50 h for Cessna 172 aircraft.
- The only other parameter needed for the model is the initial flight time hours on each aircraft at the beginning of the year or planning period ( $I_j$ ). This parameter is explained in the next section.

#### 5.1. Minimum cost solution

The flight Dept. provided us with flight hours on each aircraft at the beginning of the academic year 2012–2013. Fig. 2 presents the flight time (in hours) on each of 41 single engine aircraft at the beginning of planning period. As the figure implies, the ages and hours on aircraft vary significantly. As mentioned in Section 3, when the aircraft reach to 7200 h they are replaced with new ones.

The mathematical model to minimize total maintenance cost has more than 12,000 and 24,000 variables and constraints respectively. The model was solved using Cplex<sup>1</sup> Solver. Fig. 3 presents the total hours on each aircraft after one year of planning period under current and optimum strategies. By current, we mean the existing strategy to dispatch the aircraft closest to their 50 h scheduled maintenance program. The optimum strategy reflects the solution from the mathematical model with minimum total maintenance cost. It should be noted that under the optimum solution, 7 aircraft are replaced by reaching to 7200 flight hours during the planning period. This number of replaced aircraft for the current strategy is 5. The optimum solution requires 1037 scheduled maintenance in one year to be performed on all 41 aircraft for a total maintenance cost of \$1,521,685. These figures for the current strategy are 1037 scheduled maintenance programs and a total maintenance cost of \$1,589,927 respectively. These strategies will be discussed in more details in Section 6.

Fig. 4 shows the total number of hours added to each aircraft during the one year planning period under current and optimum strategies.

#### 5.2. Maximize aircraft availability

For any scheduled maintenance, the aircraft is grounded for one day independent of the type and/or scope. Accordingly, the aircraft remains unavailable for the same period of time for any type of scheduled maintenance.

<sup>1</sup> [www.ibm.com](http://www.ibm.com).

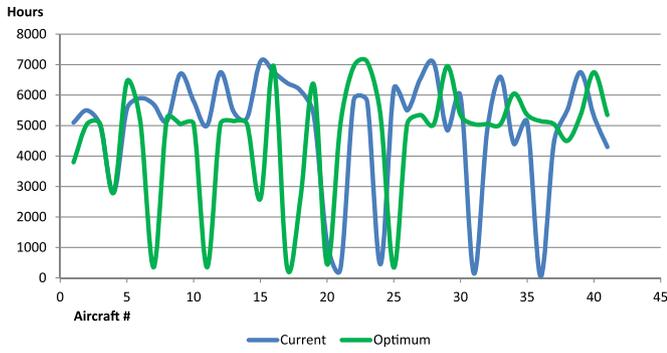


Fig. 3. Total number of hours ( $H_j$ ) on each aircraft under current and optimum strategies.

We ran the model with the revised objective function (9) in Section 4, to maximize aircraft availability. Fig. 5 presents the total hours added to each aircraft over one year for optimum solutions based on minimum cost (min cost generated in Section 5.1) and minimum number of maintenance referred to as min Mx. The total number of maintenance programs under Min Mx is 1025 compared with 1037 for minimum maintenance cost. The total maintenance costs under the two strategies are \$1,566,362 and \$1,521,685 respectively.

The average number of hours added to each aircraft is 1264 under both solutions. However, as shown by the chart, the Min cost solution has more variability than Min Mx. The coefficient of variation (standard deviation/mean) for Min cost is 0.92 while this metric for Min Mx is 0.18. These metrics clearly imply that more significant variations exist among hours added to each aircraft under min cost compared to min Mx. Under min Mx strategy no aircraft reaches to 7200 h to be replaced over the entire one year planning period.

6. Performance evaluations

Section 5 introduced two optimum strategies based on cost of maintenance and aircraft availability. In this section, other potential dispatching strategies are introduced and compared with the optimum solutions. These potential strategies include:

- Closest to maintenance – This is the current strategy adopted by the flight Dept. where the aircraft closest to its scheduled 50-h maintenance is dispatched;

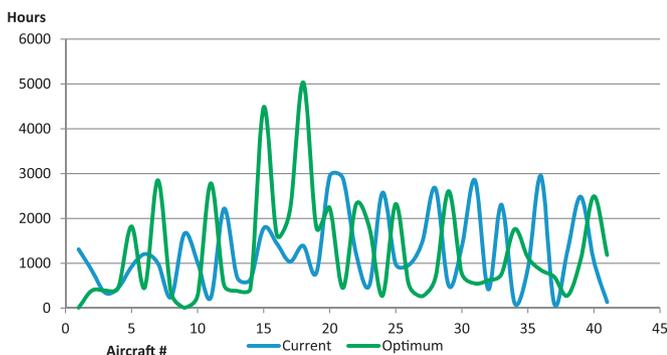


Fig. 4. Total number of flight hours added ( $x_j$ ) to each aircraft under current and optimum strategies.

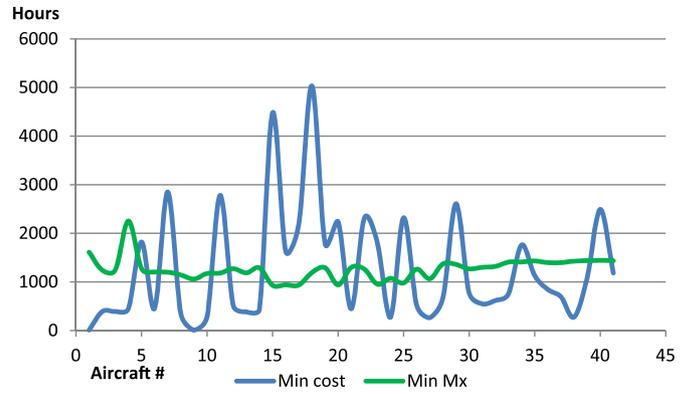


Fig. 5. Total number of hours ( $x_j$ ) added to each aircraft under optimum solutions for minimum cost and minimum number of maintenance strategies.

- Farthest to maintenance: This strategy is opposite to the above where the aircraft which is farthest to its next 50-h scheduled maintenance is dispatched;
- Random: This strategy as the name implies randomly selects an aircraft for dispatching.
- Cheapest next maintenance: This strategy dispatches the aircraft which has the cheapest upcoming scheduled maintenance;
- Equal utilization: The aircraft with lowest utilization is dispatched. At the end of planning period all aircraft have roughly the same utilizations.

Computer programs were developed to represent and generate data for each of the above 5 strategies. Total cost and number of different scheduled maintenance programs performed on each and all of 41 aircraft under each strategy were compiled. Table 1 presents the total maintenance cost, number of maintenance programs performed and their performance under each strategy. The column ‘% higher than the optimum’, as the name implies, presents the percentage of cost higher than min cost solution.

As the table suggests, the strategy generated by min cost optimization provides 4.29% cost reduction over the existing practice, closest to maintenance. This cost reduction is realized while the number of total maintenance programs performed remains the same (1037) under both strategies.

The results were shared with the Flight Dept. The solutions for the two optimization models with min cost and min Mx were found to be of interest. In particular considering the cost savings and aircraft availability, the Flight Dept. favored Min Mx since:

- It generates 1.48% cost saving over the current practice;
- It generates 12 less scheduled maintenance (1037–1025) in a year implying more aircraft availability;

Table 1 Aircraft dispatching strategies.

Strategy	Total cost	Total number of maintenance performed	% Higher than optimum
Closest to maintenance	1,589,927	1037	4.29
Farthest to maintenance	1,552,156	1038	1.96
Random	1,595,537	1038	4.63
Cheapest next maintenance	1,587,713	1037	4.16
Equal utilization	1,596,072	1036	4.66
Min cost (optimization)	1,521,685	1037	0
Min number of Mx. (optimization)	1,566,362	1025	2.85

- It utilizes the aircraft in a more uniform and steady manner than other strategies. This uniform and steady utilization of aircraft can be helpful in man-power planning at the maintenance hangar.

It should be noted that one major advantage of non-optimization strategies (the first top 5 strategies in Table 1) is their simplicity to implement. These strategies clearly sort and highlight the aircraft that need to be dispatched on a day by day basis. The dispatching algorithms for these strategies are easily incorporated into the IT system. The optimization models, on the other hand, provide the total hours added to each aircraft by the end of planning period. It does not provide the number of hours added to each aircraft on a daily basis.

We examined the optimization solutions for both min cost and min Mx on number of hours added to each aircraft over the planning period to see if there is any pattern which can be used to derive the daily usage. However, the search was inconclusive and no specific pattern was observed. After discussing this issue with the flight Dept., it was decided to uniformly spread the total hours added to each aircraft on a monthly basis based on their historical demand. The cost saving and increase in aircraft availability was found to justify this somewhat non-trivial dispatching strategy.

## 7. Conclusion

This study was initiated by a flight school to examine their current aircraft dispatching practices and potentially propose enhanced strategies. These enhanced strategies may include cost reduction and/or improved aircraft availability. The literature review shows that available models do not capture the unique nature of this case study where maintenance cost varies with usage. The paper introduced mathematical models to minimize the maintenance cost and increase aircraft availability. The solutions to these models were compared and contrasted with existing practices and

other potential dispatching strategies. It was shown that the optimization models provide moderate cost savings of 2%–4.6% and improved aircraft availability over other strategies. The solution with minimum total number of maintenance activities was preferred to the minimized cost. This strategy generates both cost saving and improved aircraft availability over the current practices while maintaining a steady utilization on all aircraft.

The mathematical model presented in this paper can easily be adapted to rental cars and trucking industries where multiple resources are available and maintenance cost vary with usage. Naturally, these industries use mileage (or kilometers) as utilization metric and the models can be revised to accommodate this metric instead of flight times. Additional side constraints maybe needed for demand at different locations.

## References

- Barnhart, C., Boland, N., Johnson, E., Nemhauser, G., Sheno, R., 1998. Flight string models for aircraft fleet and routing. *Transp. Sci.* 32 (3), 208–220.
- Bazargan, M., 2010. *Airline Operations & Scheduling*, second ed. Ashgate Publishing Group.
- Bojovic, N., 2002. A general system theory approach to rail freight car fleet sizing. *Eur. J. Oper. Res.* 136 (1), 136–172.
- Cheng, T., Hsu, C., Yang, D., 2011. Unrelated parallel-machine scheduling with deteriorating maintenance activities. *Comput. Oper. Res.* 60, 602–605.
- Hertz, A., Schindl, D., Zufferey, N., 2009. A solution method for a car fleet management problem with maintenance constraints. *J. Heuristics* 15 (5), 425–450.
- Kubzin, M., Strusevich, V., 2006. Planning machine maintenance in two-machine shop scheduling. *Oper. Res.* 54 (4), 789–800.
- Li, Y., Wang, X., 2005. Integration of fleet assignment and aircraft routing. *Transp. Res. Rec.* 1915, 79–84.
- Li, Z., Tao, F., 2010. On determining optimal fleet size and vehicle transfer policy for a car rental company. *Comput. Oper. Res.* 37 (2), 341–350.
- Miao, Z., Lim, A., Ma, H., 2009. Truck dock assignment problem with operational time constraint within crossdocks. *Eur. J. Operational Res.* 192 (1), 105–115.
- Pachon, J., Iakovou, E., Chi, I., 2006. Vehicle fleet planning in the car rental industry. *J. Revenue Pricing Manag.* 5 (3), 221–236.
- Papier, F., Thonemann, U., 2007. Queuing models for sizing and structuring rental fleets. *Transp. Sci.* 42 (3), 302–317.